4.5 Elementary Differentiation Rules

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Learning Targets:

1. SWBAT find the derivative using the constant rule.
2. SWBAT find the derivative using the power rule.
3. SWBAT find the derivative using the sum and difference rule.

It would be time consuming and tedious if we had to always compute derivatives from the definition of a derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

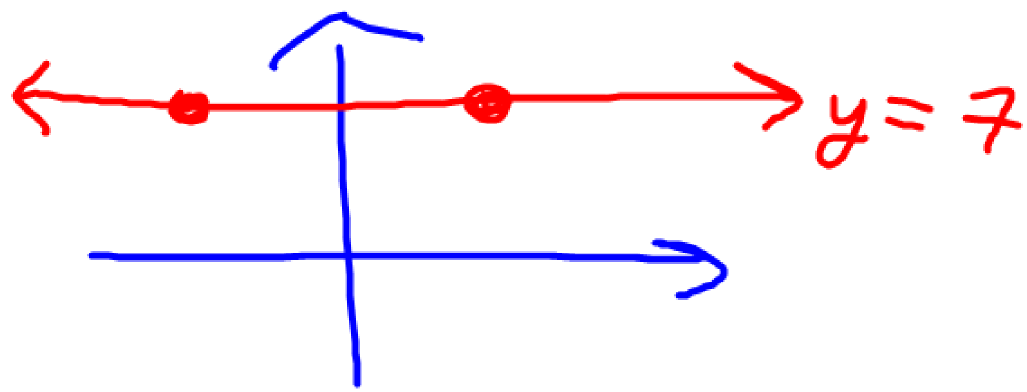
slope tangent line

1. Constant Rule

If f is a contsant function, $f(x)=c$,
then $f'(x)=0$.

$$\boxed{\frac{d(c)}{dx}=0}$$

$y=7$



Your Turn #1

Give the value of $f'(x)$ or $\frac{dy}{dx}$ for each function.

(a) $y = 17$ $y' = 0$

(b) $f(x) = \sqrt{3}$ $f'(x) = 0$

(c) $y = 3e$ where e is the irrational number 2.71828…. $y' = 0$

(d) $f(x) = 2^{10}$ $f' = 0$

2. Power Rule

If $f(x) = x^n$, where n is a positive integer,
then $f'(x) = nx^{n-1}$.

Ex.2 Find the derivative of the following:

$$a)\ f(x) = x^7$$

$f' = 7x^6$

$$b)\ y = x^{100}$$

$$y' = 100x^{99}$$

$$c)\ \frac{d}{du}\left(u^9\right)$$

$$= 9u^8$$

Ex.3 Find the **equation** of the **tangent line** to the curve $y = x^6$ at the point (-2,64).

$y' = 6x^5$

$y'(-2) = 6(-2)^5 = 6(-32) = -192$

$y - y_1 = m(x - x_1)$

$y - 64 = -192(x+2)$

$y - 64 = -192x - 384$

$y = -192x - 320$

3. General Power Rule

If n is any real number, then

$$\frac{d\left(x^n\right)}{dx} = nx^{n-1}$$

Your Turn #2

State the derivative.

(a) $y = x^5$

(b) $y = x^{-7}$

(c) $y = x^{5/4}$

(d) $f(x) = x^{5/6}$

(e) $f(x) = \dfrac{1}{x^4}$

(f) $f(x) = \sqrt[4]{x}$

(g) $y = \sqrt[5]{x^4}$

(h) $y = \dfrac{1}{\sqrt[3]{x^2}}$

a) $y = x^5$

$y' = 5x^4$

b) $y = x^{-7}$

$y' = -7x^{-8}$

c) $y = x^{5/4}$

$y' = \frac{5}{4}x^{1/4}$

d) $y = x^{5/6}$

$y' = \frac{5}{6}x^{-1/6}$

e) $f(x) = \dfrac{1}{x^4}$

$f(x) = x^{-4}$

$f'(x) = -4x^{-5}$

$= \dfrac{-4}{x^5}$

f) $f(x) = \sqrt[4]{x}$

$f(x) = x^{\frac{1}{4}}$

$f' = \frac{1}{4}x^{-3/4}$

$= \dfrac{1}{4x^{3/4}}$

g) $y = \sqrt[5]{x^4} = x^{4/5}$

$y' = \frac{4}{5}x^{-1/5}$

h) $y = \frac{1}{\sqrt[3]{x^2}}$

$y = x^{-2/3}$

$y' = -\frac{2}{3}x^{-5/3}$

4. Constant Multiple Rule

If $g(x) = cf(x)$, then

$g'(x) = cf'(x)$.

Your Turn #3

Find the derivative of each of the following:

(a) $f(x) = 10x^4$

(b) $y = -3x^{-7}$

(c) $y = \dfrac{11}{x}$

(d) $f(x) = -\dfrac{2}{x^8}$

(e) $f(x) = 4\sqrt{x}$

a) $f' = 40x^3$

b) $y' = 21x^{-8}$

c) $y = 11x^{-1}$

$y' = -11x^{-2}$

d) $y = -2x^{-8}$

$y' = 16x^{-9}$

e) $f(x) = 4x^{1/2}$

$f'(x) = 2x^{-1/2}$

Your Turn #4

Find the derivative of each of the following:

(a) $f(x) = 10x^{1.2}$

(b) $y = \sqrt{10x}$

(c) $f(x) = \sqrt[4]{3x^3}$

(d) $f(x) = \dfrac{6}{\sqrt{x}}$

(e) $y = \left(2x^4\right)^{-3}$

a) $f' = 12x^{.2}$

* b) $y = \sqrt{10} \cdot \sqrt{x}$

$y = \sqrt{10}\, x^{1/2}$

$y' = \sqrt{10} \cdot \frac{1}{2} x^{-1/2}$

$y' = \frac{\sqrt{10}}{2} x^{-1/2} = \frac{\sqrt{10}}{2\sqrt{x}}$

e) $y = (2x^4)^{-3}$

$$y = 2^{-3} \cdot x^{-12}$$

$$y = \frac{1}{8} \cdot x^{-12}$$

$$y' = \frac{-12}{8} x^{-13} = -\frac{3}{2} x^{-13}$$

Assignment Page 185
#'s 2-56 (Even)
Handout 4.5 Constant Power Rule
#'s 2 Left side, 3, 4, 7, 8

$$y = x^7$$
$$y' = 7x^6$$

2a) $f(x) = 8x^{12}$

$f' = 96x^{11}$

e) $y = \frac{1}{x^4}$

$y = x^{-4}$

$y' = -4x^{-5}$

g) $g(t) = (2t)^3$

$g(t) = 8t^3$

$g'(t) = 24t^2$

l)

$$y = \frac{3}{\sqrt[4]{x}}$$

$$y = \frac{3}{x^{1/4}}$$

$$Y = 3x^{-1/4}$$

$$Y' = -\frac{3}{4}x^{-5/4}$$

3 e)

$$y = \sqrt{x^3} \qquad x = 8$$

$$y = x^{3/2}$$

$$y' = \frac{3}{2} x^{1/2} = \frac{3}{2}\sqrt{x}$$

$$y'(8) = \frac{3}{2}\sqrt{8} = \frac{3}{\cancel{2}}\cancel{2}\sqrt{2} = \underline{3\sqrt{2}}$$

4

c) $xy = 1$ $\left(5, \frac{1}{5}\right)$

$y = \frac{1}{x}$

$y'(5) = \frac{-1}{(5)^2} = -\frac{1}{25}$

$y = x^{-1}$

$y - y_1 = m(x - x_1)$

$y' = -x^{-2}$

$y - \frac{1}{5} = -\frac{1}{25}(x - 5)$

$y' = \frac{-1}{x^2}$

$\left(y - \frac{1}{5} = -\frac{1}{25}x + \frac{5}{25}\right)25$

$25y - 5 = -x + 5$

$x + 25y - 10 = 0$

7 $\boxed{\text{Point}}$

$m = 24$

$y = 3x^2$

$y' = 6x$

$24 = 6x$

$4 = x$

$y = 3(4)^2$

$y = 48$

8 Point

$m = \underline{6}$

$y = x\sqrt{x}$

$\parallel 6x - y = 4$

$y = x^1 \cdot x^{1/2}$

$y = mx + b$

$\boxed{6x - 4 = y}$

$y = x^{3/2}$

$m = 6$

$y' = \frac{3}{2}\sqrt{x}$

$6 = \frac{3}{2}\sqrt{x}$

$12 = 3\sqrt{x}$

$4 = \sqrt{x}$

$16 = x$

$y = 16\sqrt{16}$

$y = 64$

5. Sum and Difference Rule

Sum Rule /Difference

If both $f(x)$ and $g(x)$ are differentiable functions, then if $y = f(x) \pm g(x)$, then

$$y' = f'(x) \pm g'(x)$$

Difference Rule

If both $f(x)$ and $g(x)$ are differentiable functions, then if $y = f(x) - g(x)$, then $y' = f'(x) - g'(x)$

Your Turn #5

Find the derivative of each of the following functions. First, be sure to write each as a sum/difference of terms in the form cx^n.

(a) $y = 9x^8 - 8x^7 + 4^5$

(b) $f(x) = \frac{3}{x} - \frac{5}{x^3} + \frac{7}{5x^5}$

(c) $f(x) = (5x-3)^2$

(d) $y = (x+4)(2x-5)$

(e) $y = \frac{(3x+5)(3x-5)}{x^5}$

(f) $f(x) = \sqrt{\frac{x}{3}} - \frac{2}{\sqrt{x}} + 6$

a) $y' = 72x^7 - 56x^6$

b) $y = 3x^{-1} - 5x^{-3} + \frac{7}{5}x^{-5}$

$y' = -3x^{-2} + 15x^{-4} - 7x^{-6}$

d) $y = 2x^2 + 3x^1 - 20$

$y' = 4x + 3$

*e) $y = \dfrac{(3x+5)(3x-5)}{x^5}$

$$y = \frac{9x^2 - 25}{x^5}$$

$$y = \frac{9x^2}{x^5} - \frac{25}{x^5}$$

$$y = 9x^{-3} - 25x^{-5}$$

$$y' = -27x^{-4} + 125x^{-6}$$

f)

$$f(x) = \sqrt{\frac{x}{3}} - \frac{2}{\sqrt{x}} + 6$$

$$f(x) = \frac{1\sqrt{x}}{\sqrt{3}} - 2x^{-1/2} + 6$$

$$f(x) = \frac{1}{\sqrt{3}} x^{1/2} - 2x^{-1/2} + 6$$

$$f'(x) = \frac{1}{2\sqrt{3}} x^{-1/2} + x^{-3/2}$$

Your Turn #6

(a) If $y = 2x^2 - 7x$, find y'.

(b) Find $\dfrac{d}{dx}\left(x^6 - 2x\right)$.

(c) If $A = \pi r^2$, find $\dfrac{dA}{dr}$.

(d) Find $\dfrac{d}{dq}\left(\pi q^3 - \pi\right)$.

Your turn #7

Find the coordinates of the two points on the graph of $f(x) = x^3 + 3x^2 - 3x$ at which the slope of the tangent line is 6.

$$f'(x) = \underbrace{3x^2 + 6x - 3}_{\text{slope tangent line}}$$

$$6 = 3x^2 + 6x - 3$$

$$0 = 3x^2 + 6x - 9$$

$$0 = 3(x^2 + 2x - 3)$$

$0 = 3(x+3)(x-1)$

$x = -3$

$f(-3)$

$= (-3)^3 + 3(-3)^2 - 3(-3)$

$= \cancel{-27} + \cancel{27} + 9$

$= 9$

$(-3, 9)$

$x = 1$

$f(1)$

$= (1)^3 + 3(1)^2 - 3(1)$

$= 1$

$(1, 1)$

Your turn #8

Explain why a line with a slope of -2 could never be tangent to the curve

$$y = \frac{1}{3}x^3 + x.$$

$$y' = x^2 + 1$$

$$-2 = x^2 + 1$$

$$-3 = x^2$$

Not possible

$\therefore m = -2$.

Not possible

Example: The find point on the equation $y = x^2 + 6x$ where the slope of the tangent line is parallel to the line $4x + 2y = 8$

$m_{tan} = -2$

$2y = -4x + 8$

$y = -2x + 4$

$m = -2$

$y' = 2x + 6$

$-2 = 2x + 6$

$-8 = 2x$

$-4 = x$

$y = (-4)^2 + 6(-4)$

$= -8$

$(-4, -8)$

Assignment
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#'s 1, 2 a c e , 3 - 5
Handout
#'s 3, 6, 7, 8