

$$2x - 3 = 0$$

$$2x = 3$$

$$x = 3/2$$

## 4.4 Solving Trigonometric Equations

$$x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0$$

$$x - 3 = 0 \quad \text{OR} \quad x + 2 = 0$$

$$x = 3$$

$$x = -2$$

Ex.1 Solve the following:

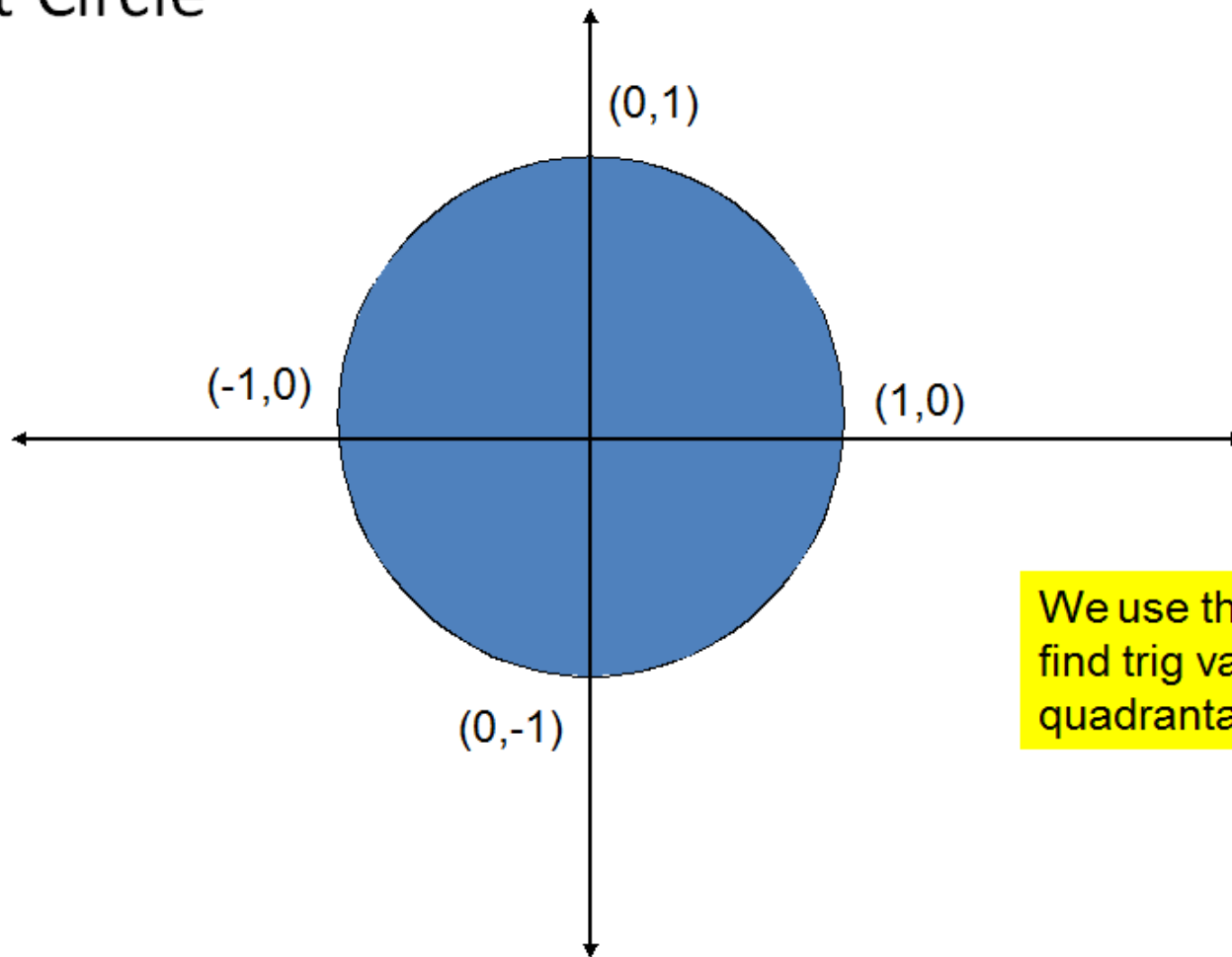
$$2x^2 + x = 0$$

Ex.2 Solve the following:

$$x^2 + 7x + 10 = 0$$

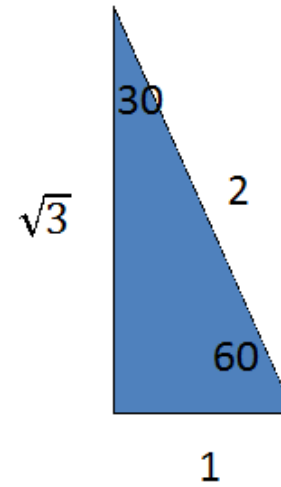
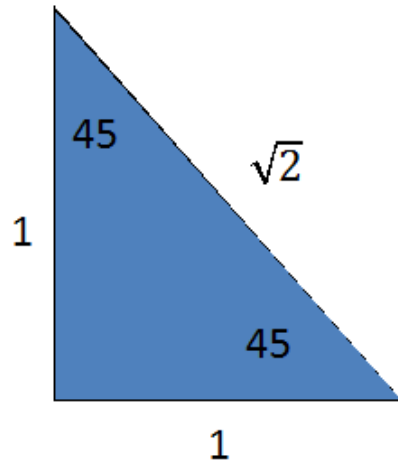
To solve trig equations we will need the following:

# Unit Circle



We use this unit circle to find trig values of quadrantal angles!

# The special triangles:



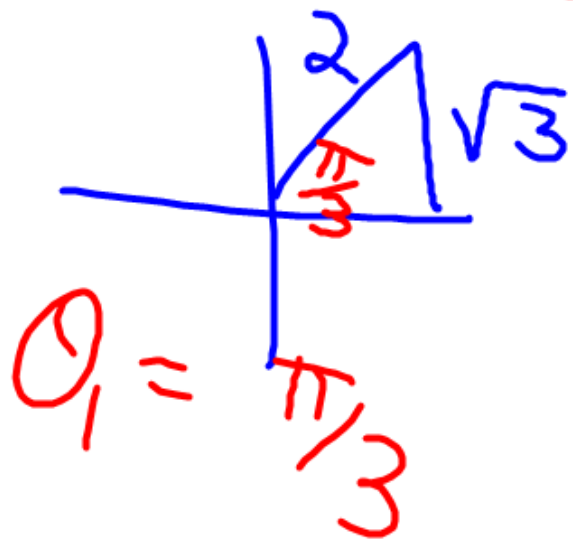
Example 1: Solve each trigonometric equation in specified domain:

a)  $2\sin x - \sqrt{3} = 0, 0 \leq x \leq 2\pi$

These are exact roots. How can we tell?

$$\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{\sqrt{3}}{2} \quad \begin{matrix} \text{y} \\ \text{r} \end{matrix}$$

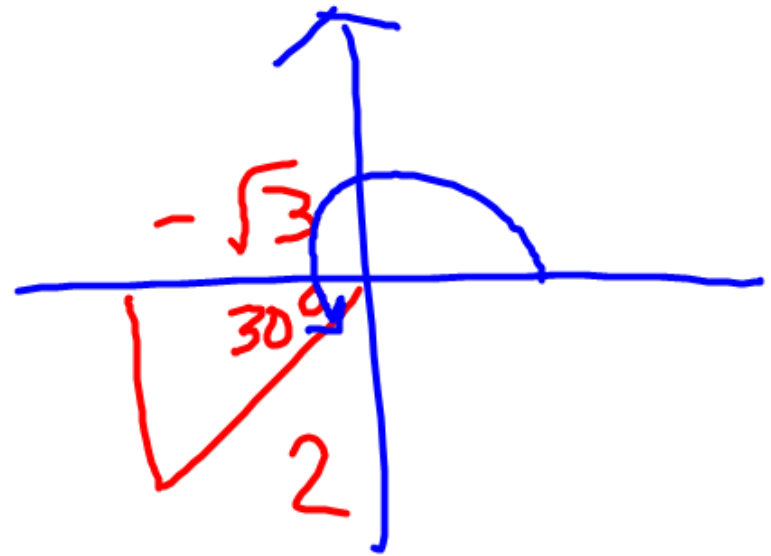
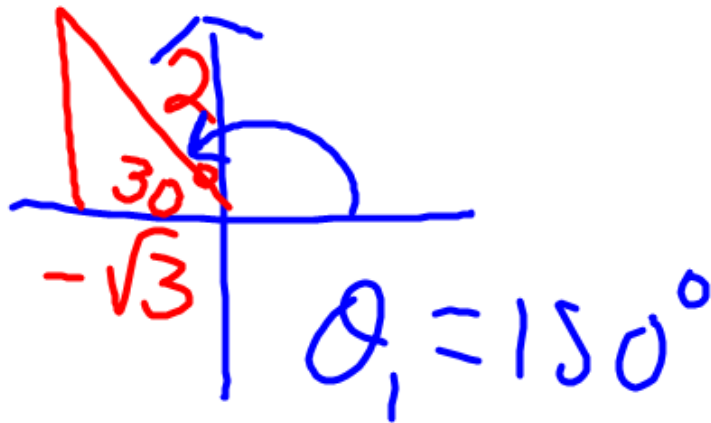


b)  $\sqrt{3}\sec x + 2 = 0, 0^\circ \leq x \leq 360^\circ$

Is this an exact value?

$$\sqrt{3}\sec x = -2$$
$$\sec x = -\frac{2}{\sqrt{3}}$$

$$\cos x = -\frac{\sqrt{3}}{2} \quad \begin{matrix} x \\ r \end{matrix}$$





### Your Turn

Solve each trigonometric equation in the specified domain.

a)  $3 \cos \theta - 1 = \cos \theta + 1, -2\pi \leq \theta \leq 2\pi$

b)  $4 \sec x + 8 = 0, 0^\circ \leq x < 360^\circ$

$$\begin{aligned} \text{a) } 2 \cos \theta &= 2 \\ \cos \theta &= 1 \end{aligned}$$

$$\theta_1 = 0$$

$$\theta_2 = 2\pi$$

$$\theta_3 = -2\pi$$

$$4 \sec x + 8 = 0$$

$$4 \sec x = -8$$

$$\sec x = -\frac{8}{4}$$

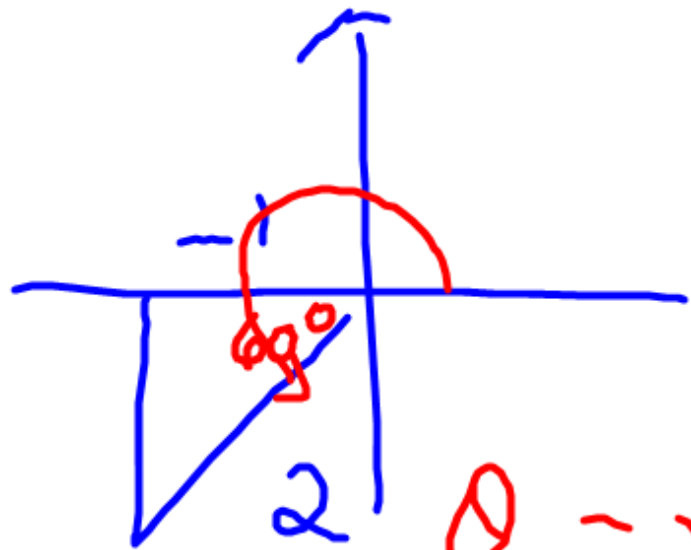
$$\sec x = -2$$

$$\cos x = -\frac{1}{2}$$



$$\theta_1 = 120^\circ$$

$$0^\circ \leq x < 360^\circ$$



$$\theta_2 = 240^\circ$$

When solving trigonometric equations some of our solutions may be exact values and some may be only approximate values. Our next example illustrates this fact!

Example 2: Solve for  $\theta$  if:

$$3\cos^2\theta + \cos\theta - 2 = 0, 0 \leq \theta \leq 360^\circ$$

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

$$(3\cos\theta - 2)(\cos\theta + 1) = 0$$

$$3\cos\theta - 2 = 0$$

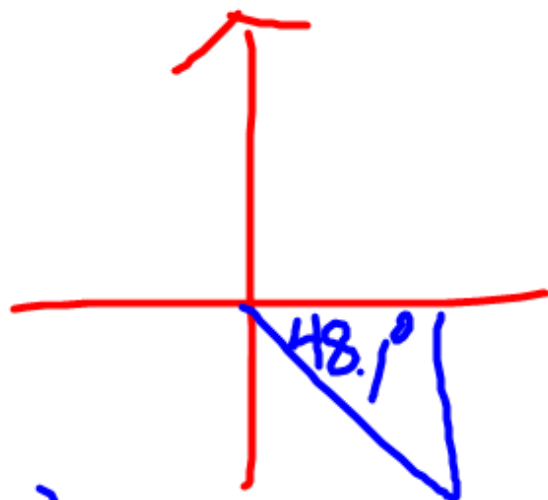
$$\text{OR } \cos\theta + 1 = 0$$

$$\cos\theta = \frac{2}{3}$$

approx

$$\cos\theta = -1$$

$$\cos \theta = 2/3$$



$$\cos^{-1}(2/3) = \theta$$

$$\theta_1 = 48.1^\circ$$

$$\theta_2 = 311.9^\circ$$

$$\cos \theta = -1$$

$$\theta = 180^\circ$$

## Finding General Solutions to Trigonometric Equations

Example 3:

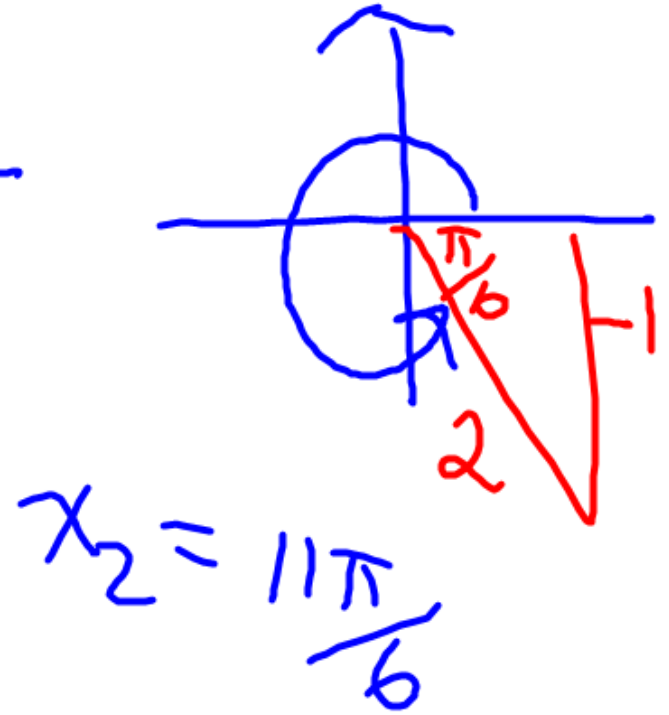
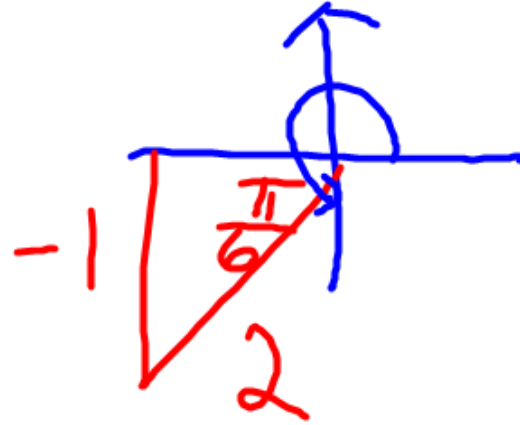
- a) If  $2\sin x + 1 = 0$ , solve for  $x$  in the domain  $0 \leq x \leq 2\pi$ . Give solutions as exact values.
- b) Determine the general solutions for  $2\sin x + 1 = 0$ , where the domain is real numbers measured in radians.

$$a) 2\sin x + 1 = 0$$

$$2\sin x = -1$$

$$\sin x = -\frac{1}{2}$$

$$x_1 = \frac{7\pi}{6}$$



b) General solution

$$x_1 = \frac{7\pi}{6} \pm 2\pi n, n \in \mathbb{N}$$

$$x_2 = \frac{11\pi}{6} \pm 2\pi n, n \in \mathbb{N}$$

## Your Turn

- a)** If  $\cos^2 x - 1 = 0$ , solve for  $x$  in the domain  $0^\circ \leq x < 360^\circ$ .  
Give solutions as exact values.
- b)** Determine the general solution for  $\cos^2 x - 1 = 0$ , where the domain is real numbers measured in degrees.

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3a,b,d,4,5,7,18