4.4 Fractional Exponents and Radicals

Lesson Focus

Relation relational exponents and radicals

What happens if our exponent is a rational number?

$49^{\frac{1}{2}} = \sqrt[2]{49} = 7$

$27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

Fractional Exponents

Powers with Rational Exponents with Numerator 1

When n is a natural number and x is a rational number, $x^{\frac{1}{n}} = \sqrt[n]{x}$

Example

Rewrite each power as a radical then evaluate

a) $27^{\frac{1}{3}}$ **b)** $0.49^{\frac{1}{2}}$ **c)** $(-64)^{\frac{1}{3}}$ **d)** $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

a) $\sqrt[3]{27}$ b) $\sqrt{0.49}$ c) $\sqrt[3]{-64}$ d) $\sqrt{\frac{4}{9}}$

$= 3$ $= .7$ $= -4$ $= \frac{2}{3}$

Example – Your Turn

Evaluate each power without using a calculator.

a) $1000^{\frac{1}{3}}$

$= \sqrt[3]{1000}$

$= 10$

b) $0.25^{\frac{1}{2}}$

$\sqrt{.25}$

$= 0.5$

c) $(-8)^{\frac{1}{3}}$

$\sqrt[3]{-8}$

$= -2$

d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

$= \sqrt[4]{\frac{16}{81}}$

$= \frac{2}{3}$

Homework

P. 227-228

3, 5, 6

Fractional Exponents

Powers with Rational Exponents

When m and n are natural numbers, and x is a rational number,

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$$

and

$$x^{\frac{m}{n}} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

Shortcut – the numerator of the fractional exponent is a power, the denominator is a root

You can choose which to evaluate first

Example

a) Write $40^{\frac{2}{3}}$ in radical form in 2 ways.

b) Write $\sqrt{3^5}$ and $\left(\sqrt[3]{25}\right)^2$ in exponent form.

a) $\left(\sqrt[3]{40}\right)^2$ $\qquad$ $\sqrt[3]{40^2}$

b) $3^{5/2}$ $\qquad$ $25^{2/3}$

Example

Evaluate.

a) $0.04^{\frac{3}{2}}$

$(\sqrt{0.04})^3$

$(.2)^3 = .008$

b) $27^{\frac{4}{3}}$

$= (\sqrt[3]{27})^4$

$= (3)^4 = 81$

c) $(-32)^{0.4}$

$(-32)^{2/5} = (\sqrt[5]{-32})^2 = (-2)^2 = 4$

Example – Your Turn

Evaluate.

a) $0.01^{\frac{3}{2}}$

$(\sqrt{0.01})^3$

$(0.1)^3$

$= .001$

b) $(-27)^{\frac{4}{3}}$

$(\sqrt[3]{-27})^4$

$= (-3)^4$

$= 81$

c) $81^{\frac{3}{4}}$

$(\sqrt[4]{81})^3$

$= (3)^3$

$= 27$

Homework

P. 227-228

4, 7, 10, 11, 12*, 13

Example

Biologists use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass, b kilograms, of a mammal with body mass m kilograms. Estimate the brain mass of each animal.

a) a husky with a body mass of 27 kg

b) a polar bear with a body mass of 200 kg

$b = 0.01(200)^{2/3}$

$= 0.01\left(\sqrt[3]{200}\right)^2$

$= 0.01(5.848)^2 = 0.342\ \text{Kg}$

Example

Use the formula $b = 0.01m^{\frac{2}{3}}$ to estimate the brain mass of each animal.

a) a moose with a body mass of 512 kg

b) a cat with a body mass of 5 kg

$b = 0.01(5)^{2/3}$

$b = 0.01(\sqrt[3]{5})^2$

$= 0.01(1.71)^2$

≈ 0.029 kg

Homework

P. 227-228

17, 18, 19, 21