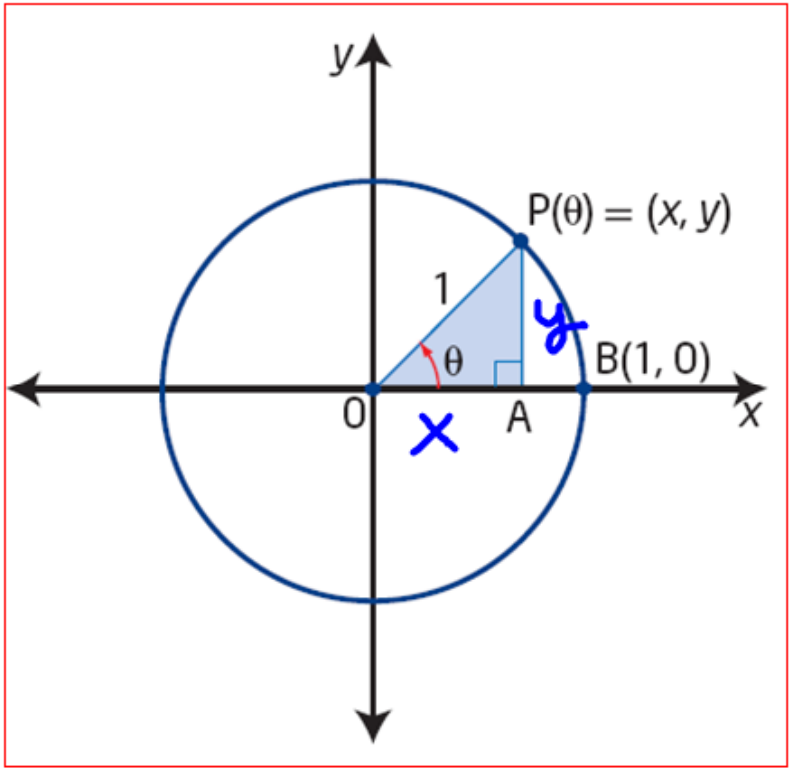
4.3 Trigonometric Ratios

Coordinates in Terms of Primary Trigonometric Ratios

If $P(\theta) = (x, y)$ is the point on the terminal arm of angle θ that intersects the unit circle, notice that



What does $\cos\theta$ =?

$\frac{x}{1} = x$

What does $\sin\theta$ =?

$\frac{y}{1} = y$

What does $\tan\theta$ =?

$\frac{y}{x}$

Reciprocal Trigonometric Ratios

Three other trigonometric ratios are defined: they are the reciprocals of sine, cosine, and tangent. These are **cosecant**, **secant**, and **cotangent**.

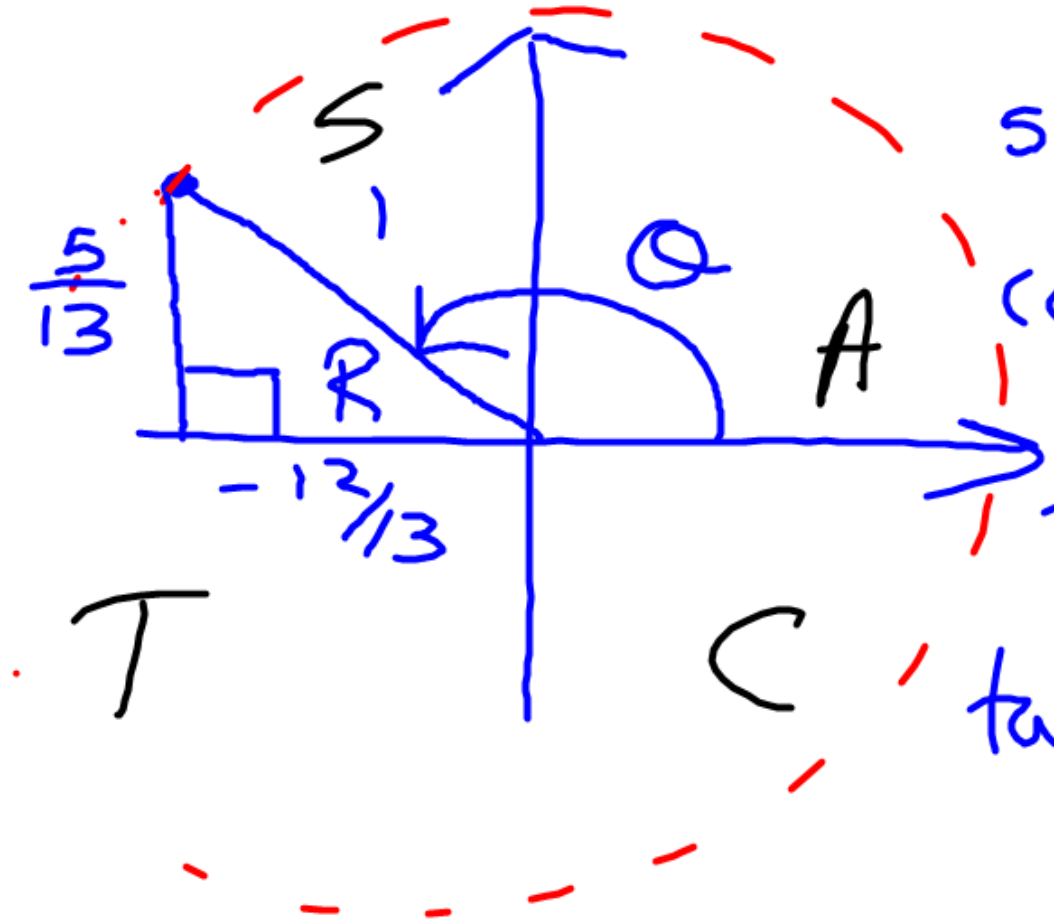
By definition, $\csc\theta = \frac{1}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, and $\cot\theta = \frac{1}{\tan\theta}$.

Finding Trig Ratios of Points on the Unit Circle

The point $A\left(-\frac{12}{13}, \frac{5}{13}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle θ in standard position.

a) Draw the diagram to model the situation.

b) Determine the six trigonometric ratios of θ. Express your answer in lowest terms



$\sin\theta = \frac{5}{13}$

$\cos\theta = -\frac{12}{13}$

$\tan\theta = \frac{\left(\frac{5}{13}\right)}{\left(-\frac{12}{13}\right)}$

$\tan\theta = -\frac{5}{12}$

$\csc\theta = \frac{13}{5}$

$\sec\theta = -\frac{13}{12}$

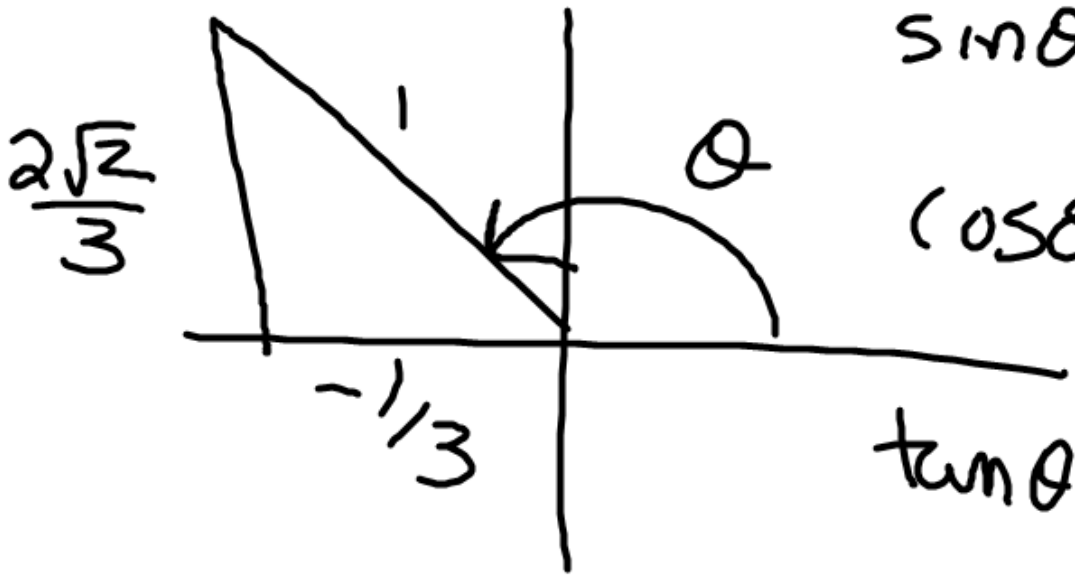
$\cot\theta = -\frac{12}{5}$

Your Turn

The point $B\left(-\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$ lies at the intersection of the unit circle and the terminal arm of an angle θ in standard position.

a) Draw a diagram to model the situation.

b) Determine the values of the six trigonometric ratios for θ. Express your answers in lowest terms.



$\sin\theta = \frac{2\sqrt{2}}{3}$

$\cos\theta = -\frac{1}{3}$

$\tan\theta = \frac{\left(\frac{2\sqrt{2}}{3}\right)}{\left(-\frac{1}{3}\right)} = -2\sqrt{2}$

$\csc\theta = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$

$\sec\theta = -3$

$\cot\theta = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4}$

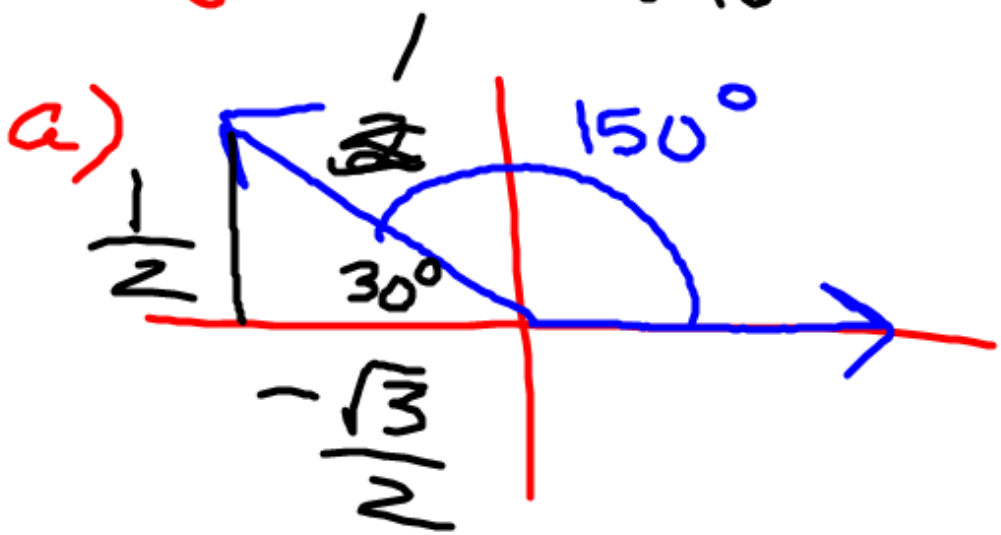
Calculating Exact Values of Trigonometric Ratios

Example 1: Determine the exact value of each. Draw a diagram to illustrate your answers. Verify your solution wih your unit circle!

a) $\cos\frac{5\pi}{6}$ b) $\sin\left(-\frac{4\pi}{3}\right)$ c) $\tan\frac{7\pi}{4}$ d) $\cos\pi$ e) $\csc\frac{7\pi}{6}$

(150°) −240°

a)

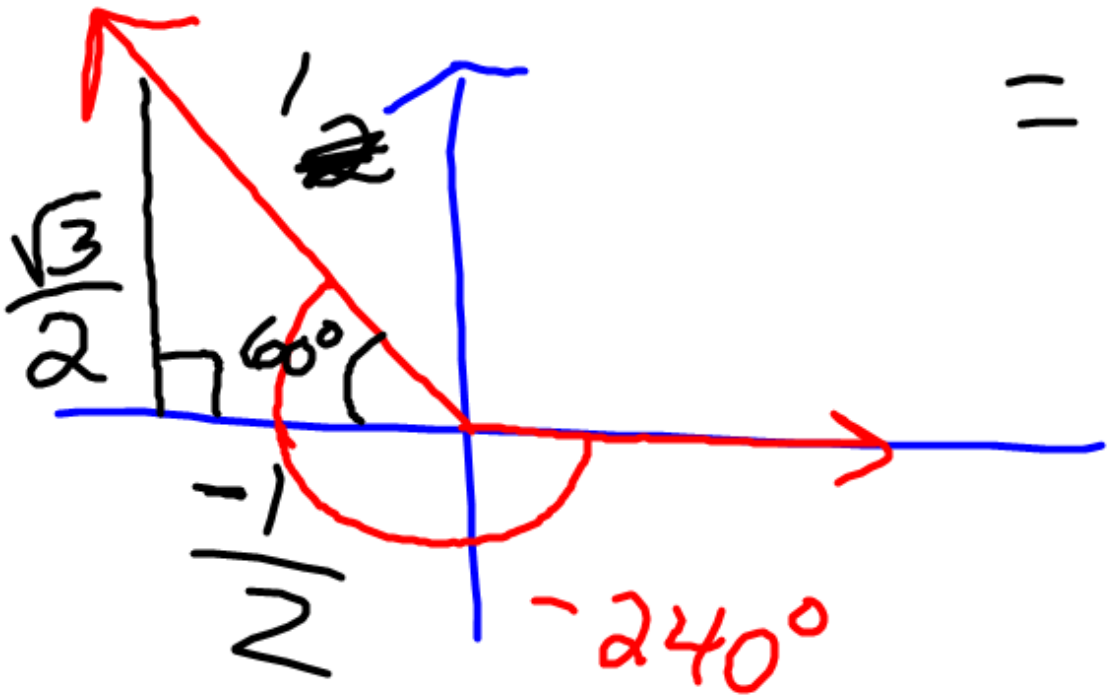


b)

$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

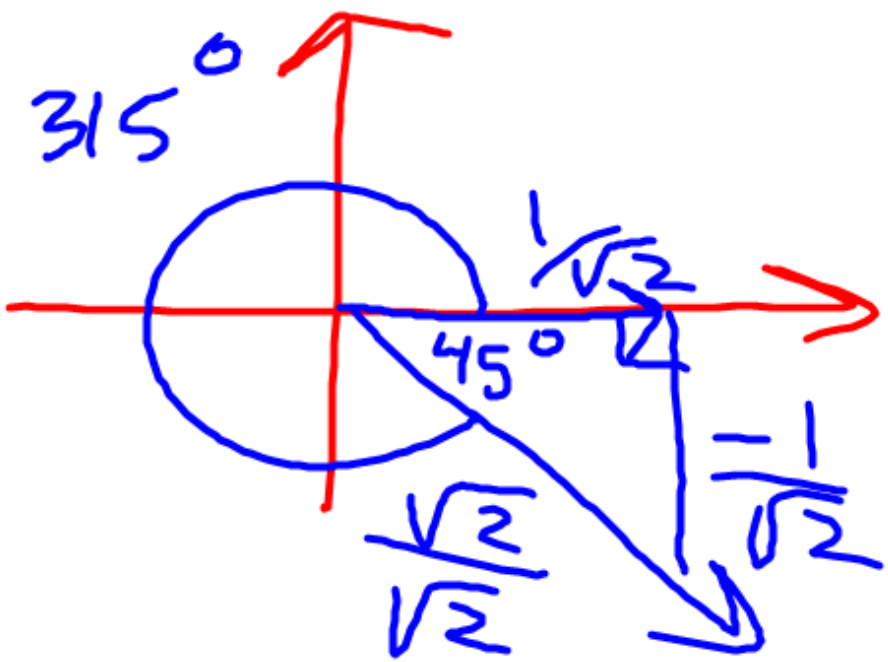
b) $\sin\left(-4\pi/3\right)$

$(-240°)$

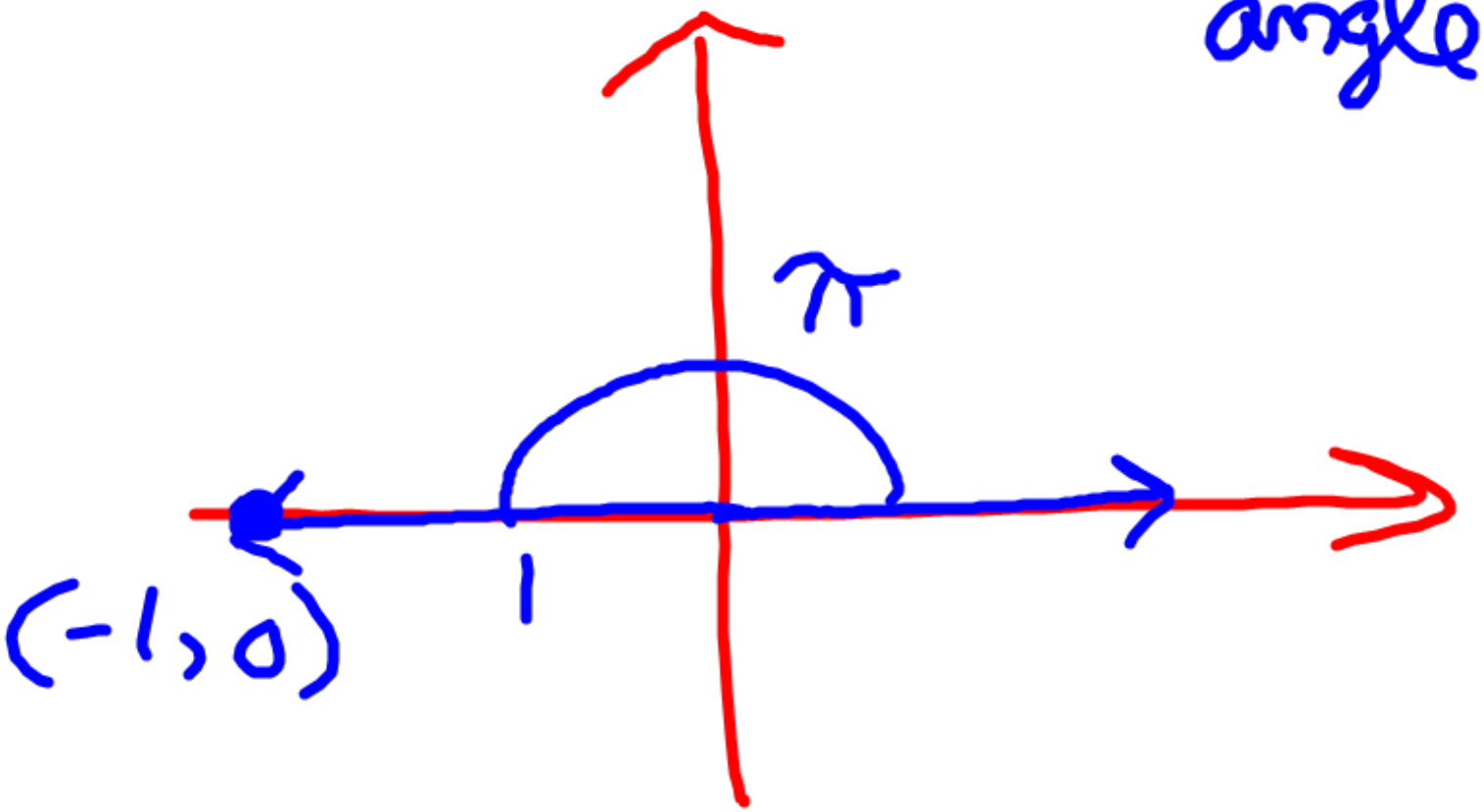


$= \frac{\sqrt{3}}{2}$

c) $\tan \frac{7\pi}{4}$ (315°) $= \frac{-1}{1} = -1$

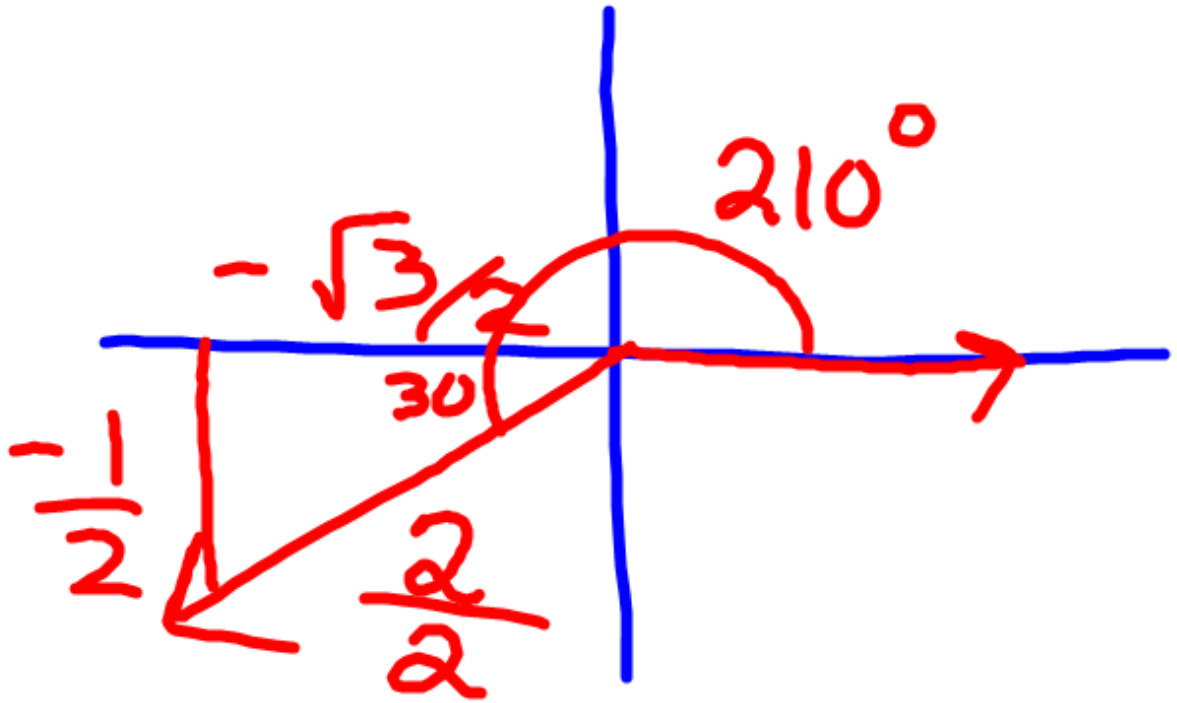


a) $\cos \pi = -1$ quadrantal angles



e) $\csc 7\pi/6 = \frac{2}{-1} = -2$

(210°)



Approximate Values of Trigonometric Ratios

You can determine approximate values for sine, cosine, and tangent using a scientific calculator. You can determine these values in either **degrees** or **radians!**

Example 2: Determine the approximate value for each trig ratio. Give your answers to four decimal places.

a) $tan\frac{7\pi}{5}$ b) $\cos 250°$ c) $\csc(-75°)$

a) 3.0777 b) -0.3420

c) $\csc(-75^\circ)$

$$\frac{1}{\sin(-75^\circ)} \equiv 1.0353$$

$$\frac{1}{(\sin -75^\circ)}$$

~~$\sin^{-1}\left(\frac{1}{-75^\circ}\right)$~~

b) 0.8391

c)

Your Turn

What is the approximate value for each trigonometric ratio? Round answers to four decimal places. Justify the sign of each answer.

a) sin 1.92
b) tan (−500°)
c) sec 85.4°
d) cot 3

$$\frac{1}{\cos(85.4^\circ)} = 12.4690$$

a) $\sin 1.92 = 0.9396$

d) $\cot 3 = \frac{1}{\tan 3} = -7.0153$

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#1a,c,eg,i,k, 2a-f,8

Evaluating Exact Values of Angles

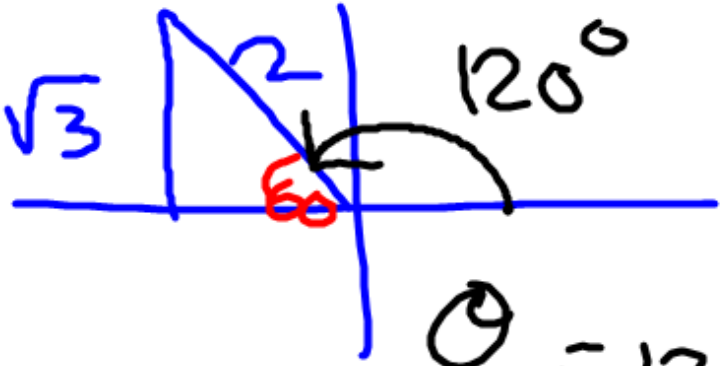
a) $sin\theta = \frac{\sqrt{3}}{2}$ in the domain $0 \leq \theta < 2\pi$

S	A
T	C

domain

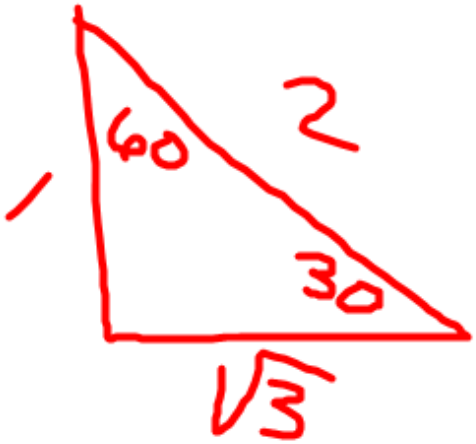
$\theta_1 = 60°$

$= \pi/3$



$\theta_2 = 120°$

$= 2\pi/3$

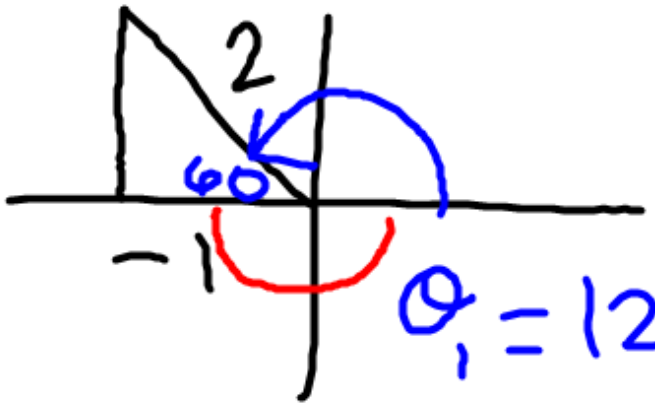


b) $\cos\theta = -\frac{1}{2}$ in the domain $-180° \leq \theta < 360°$

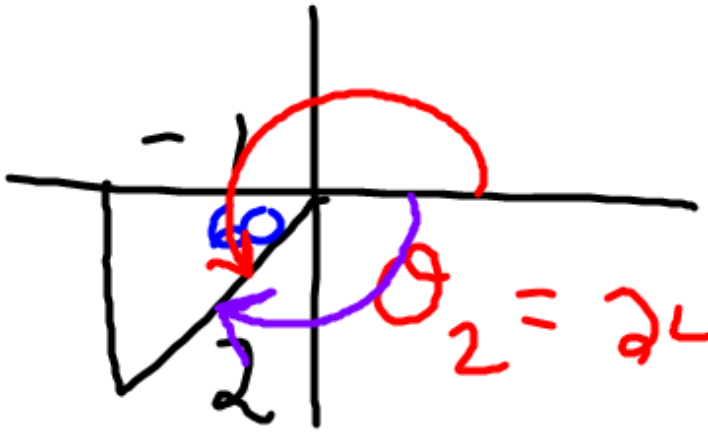
x

r

S	A
T	C



$\theta_1 = 120°$



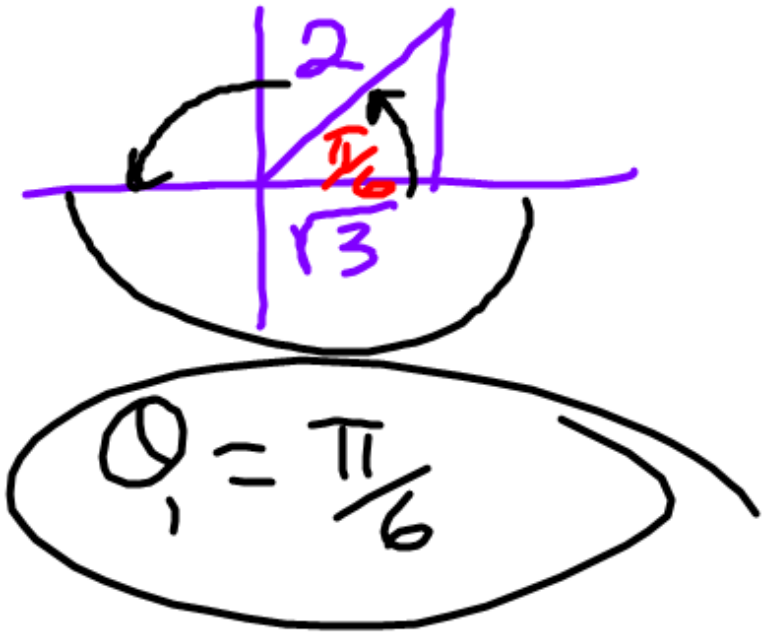
$\theta_2 = 240°$

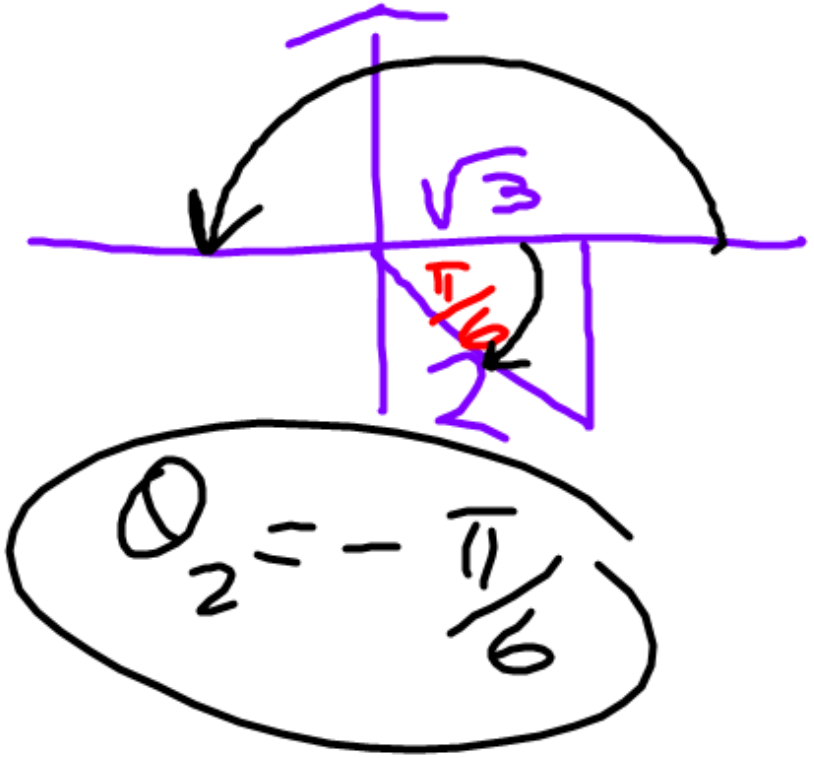
$\theta_3 = -120°$

c) $\sec\theta = \frac{2}{\sqrt{3}}$ in the domain $-\pi \leq \theta < \pi$

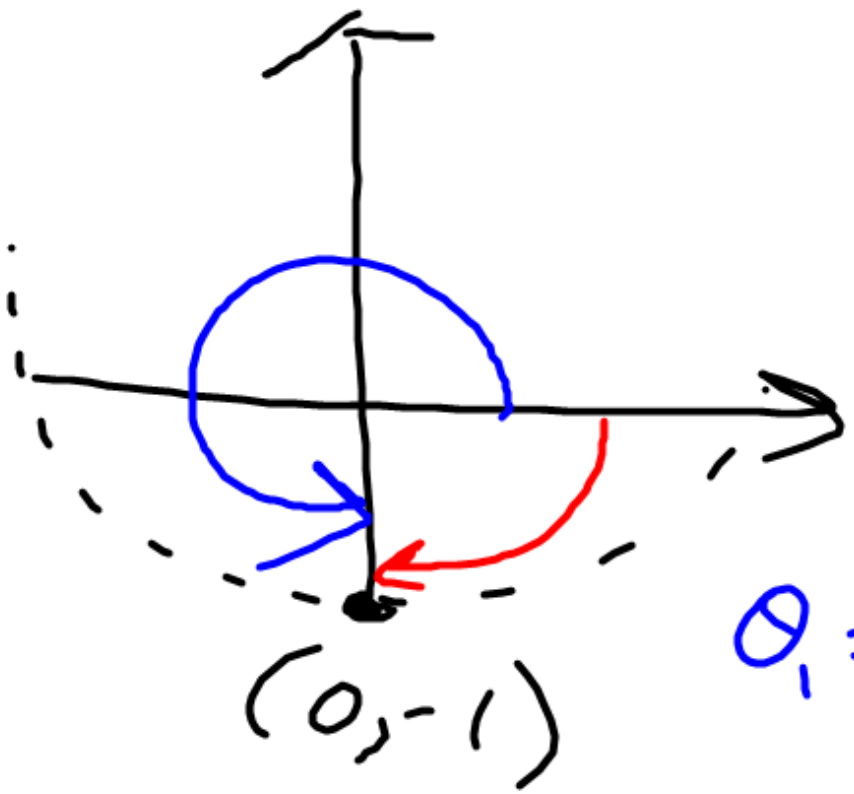
$\cos\theta = \frac{\sqrt{3}}{2} \quad \frac{x}{r}$

S	A
T	C





d) $\sin\theta = -1$ in the domain $-360° \leq \theta < 360°$



Sin
Cos — -1, 0, 1
tan — 0

$\theta_1 = 270°$

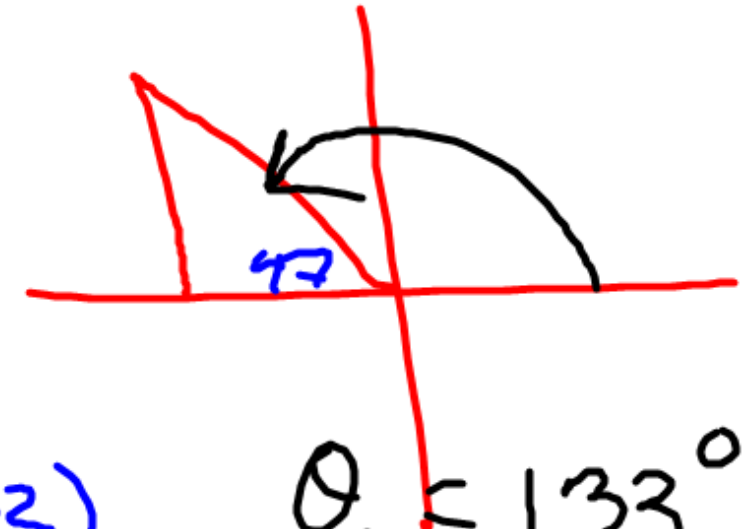
$\theta_2 = -90°$

Evaluating Approximate Values of Angles

need calculator.

Example 3: Determine the measure of all angles that satisfy each of the following , use diagrams to show possible answers.

a) $sin\theta = 0.732$ *in the domain* $0° \leq \theta \leq 360°$.



$\theta = 47°$

$\theta_R = \sin^{-1}(0.732)$

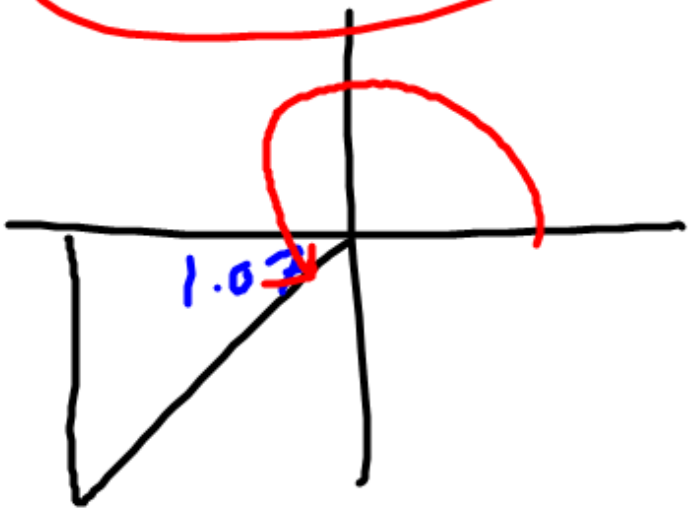
$\theta_2 = 133°$

$\theta_R = 47°$

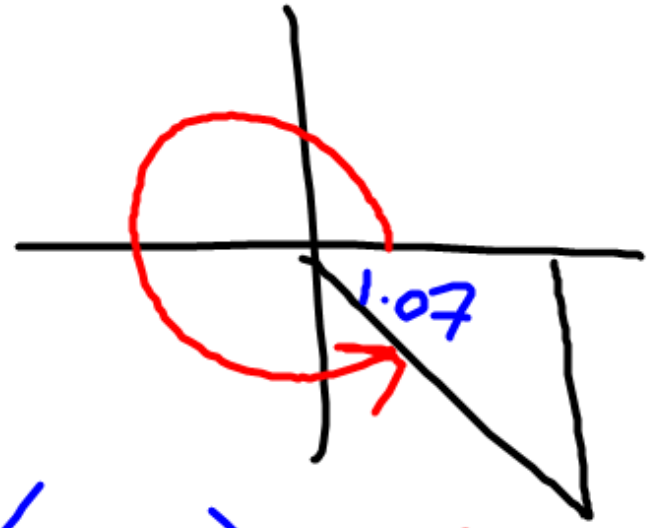
b) $\sin\theta = -0.879$ *in the domain* $0 \le \theta \le 2\pi$.

$\theta_1 = \pi + 1.07$

$= 4.21$



S	A
T	C



$\theta_R = \sin^{-1}(.879)$

$\theta_R = 1.07$

$\theta_2 = 2\pi - 1.07$

$\theta_2 = 5.21$

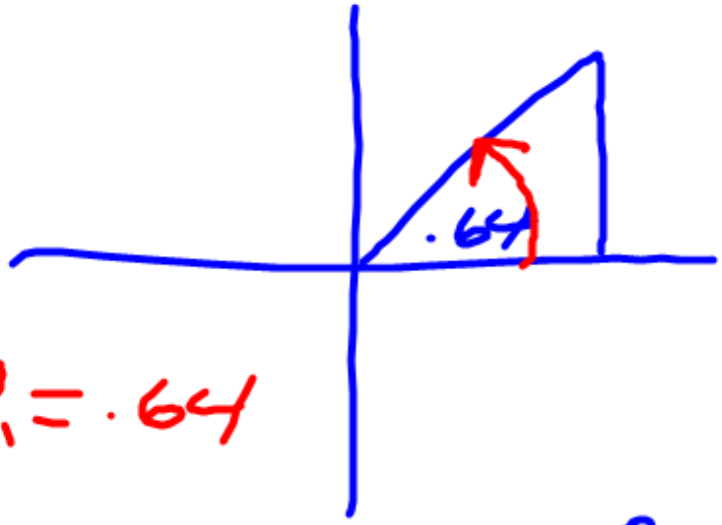
c) $cos\theta = -.843$ $for\ the\ domain\ of\ -360° \leq \theta \leq 180°$, to the nearest tenth of a degree.

d) $\cot\theta = 1.345$ *in the domain of* $-\pi \le \theta \le \pi$

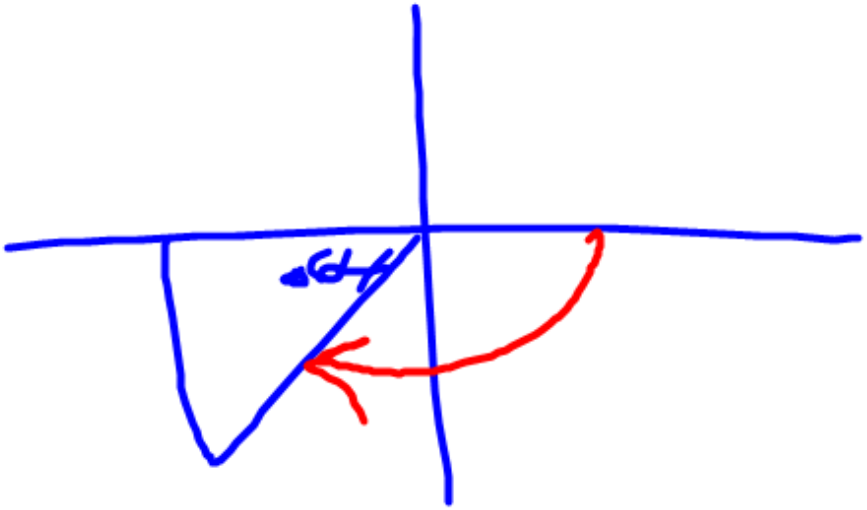
$\tan\theta = \frac{1}{1.345}$

$= 0.7435$

S	A
T	C



$\theta_1 = .64$



$\theta_R = \tan^{-1}(1.345)$

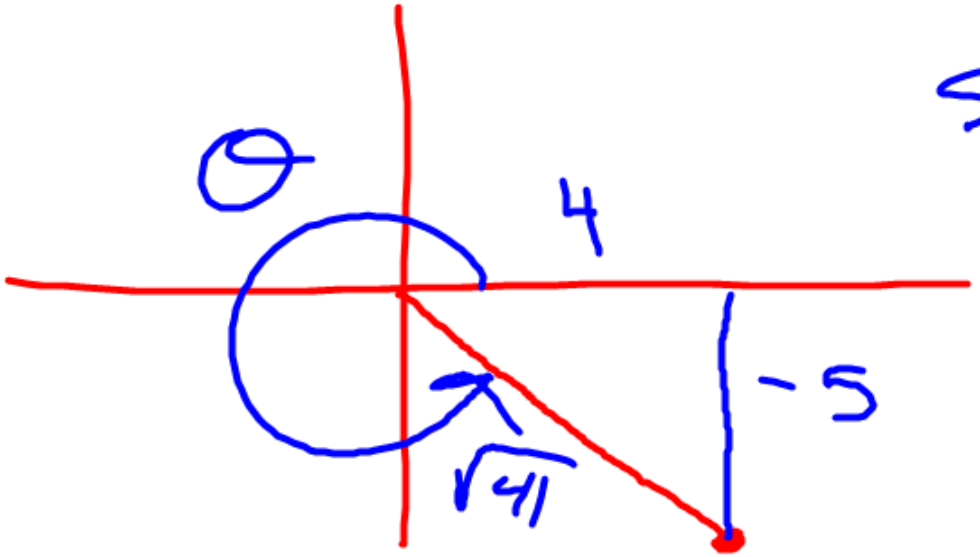
$\theta_R = .64$

$\theta_2 = -(\pi - .64)$

$= -2.50$

Calculating Trig Values of Points Not on the Unit Circle

Ex.4 Given $(4,-5)$ is a point on the terminal arm of an angle θ drawn in standard position. Calculate the primary trig ratios.



$\sin\theta = \frac{y}{r} = -\frac{5}{\sqrt{41}}$

$\cos\theta = \frac{x}{r} = \frac{4}{\sqrt{41}}$

$\tan\theta = -\frac{5}{4}$

$(4)^2 + (-5)^2 = r^2$

$41 = r^2$

$\pm\sqrt{41} = r$

Your Turn

The point D(−5, −12) lies on the terminal arm of an angle θ in standard position. What is the exact value of each trigonometric ratio for θ?

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#'s 10,11,12,13,