Boardwork

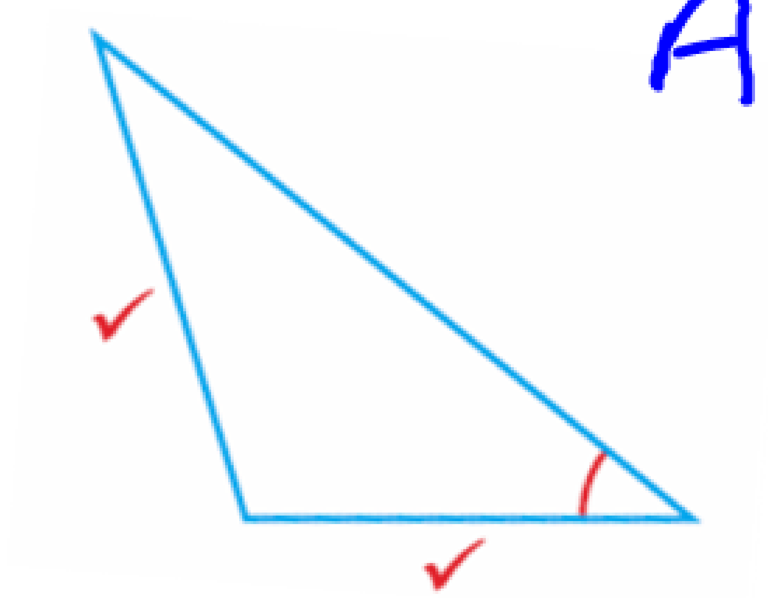
Given ΔABC such that $a=12$, $c=15$ and $\angle A=53°$, can you find **two different values** for $\angle C$?

What would these two triangles look like?

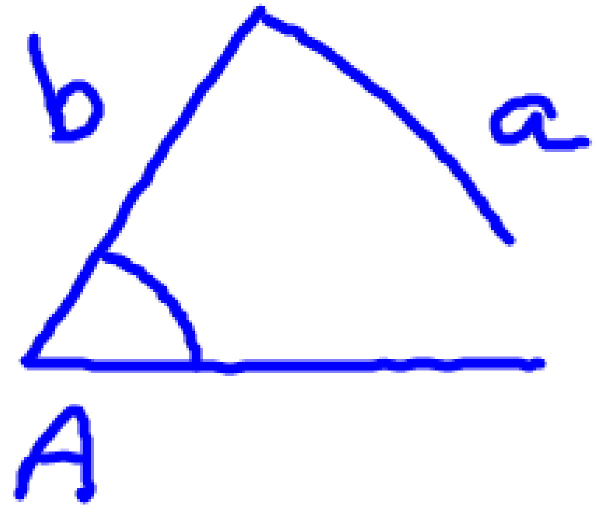
4.3 Ambiguous Case of the Sine Law

*A situation in which **zero, one or two triangles** can be drawn, given the available information*
The ambiguous case may occur only when information is SSA

ASS



What's SSA backwards?



Make sure to learn this stuff!
Don't make an ass of yourself!

Given angle is obtuse

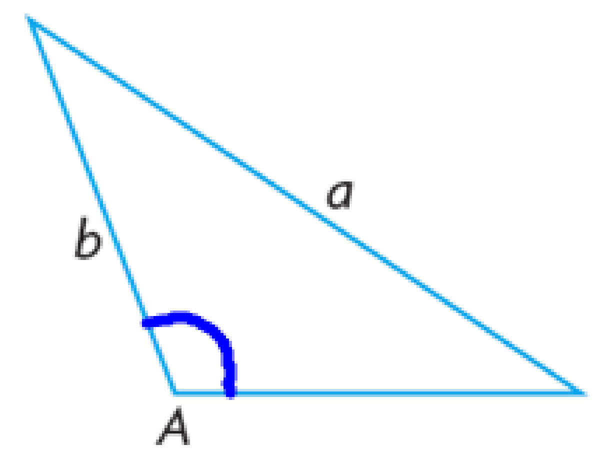
If the given angle in a SSA is obtuse, there are ***two different cases***

Obtuse Case 1

The given angle is **obtuse** and the side opposite that angle is greater than the other given side

How many possible triangles?

1 triangle



When a > b
ONE triangle

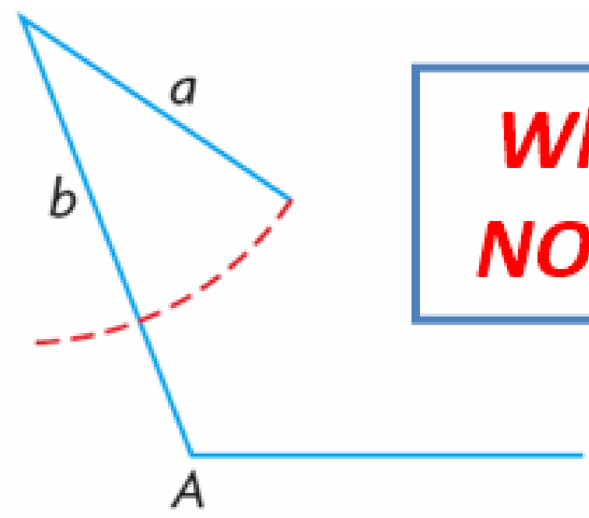
Obtuse Case 2

The given angle is **obtuse** and the side opposite that angle is less than or equal to the other given side

How many possible triangles?

0 triangles

Opposite of the biggest angle must be the longest side



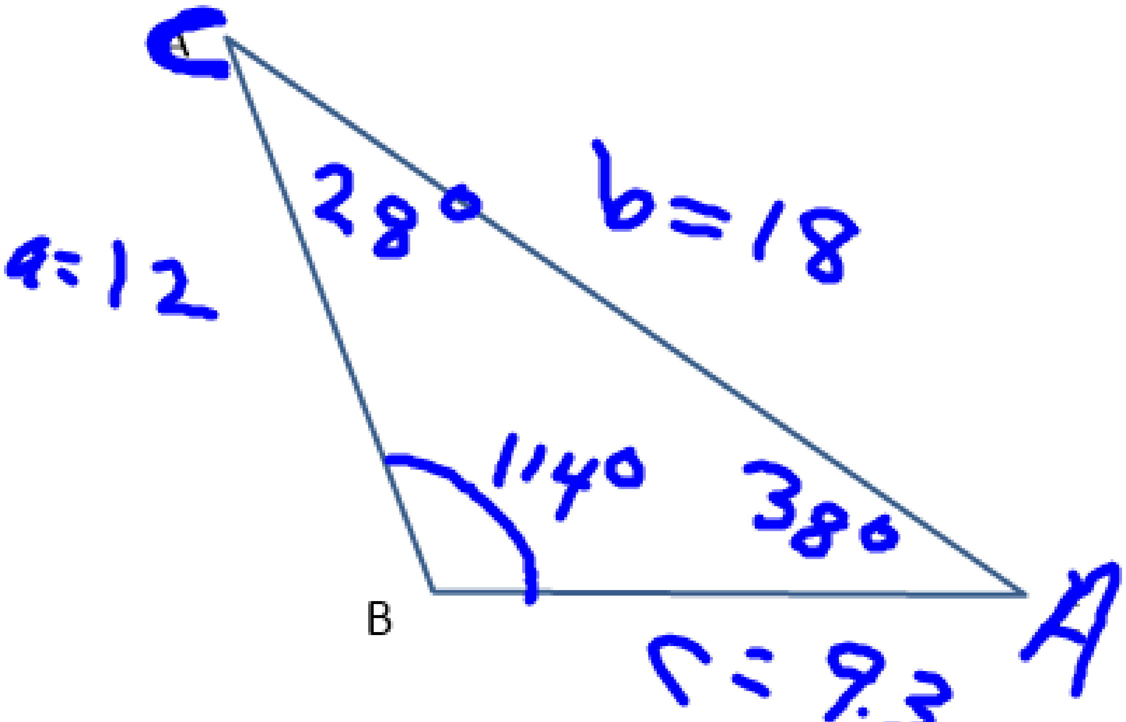
When a ≤ b
NO triangles

Obtuse Example

<B = 114°, b = 18, a = 12

How many triangles? 1 △

Solve for c



$$\frac{\sin A}{12} = \frac{\sin 114^\circ}{18}$$

$\sin A = .6090$

$A = \sin^{-1}(.6090)$

$A = 38^\circ$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

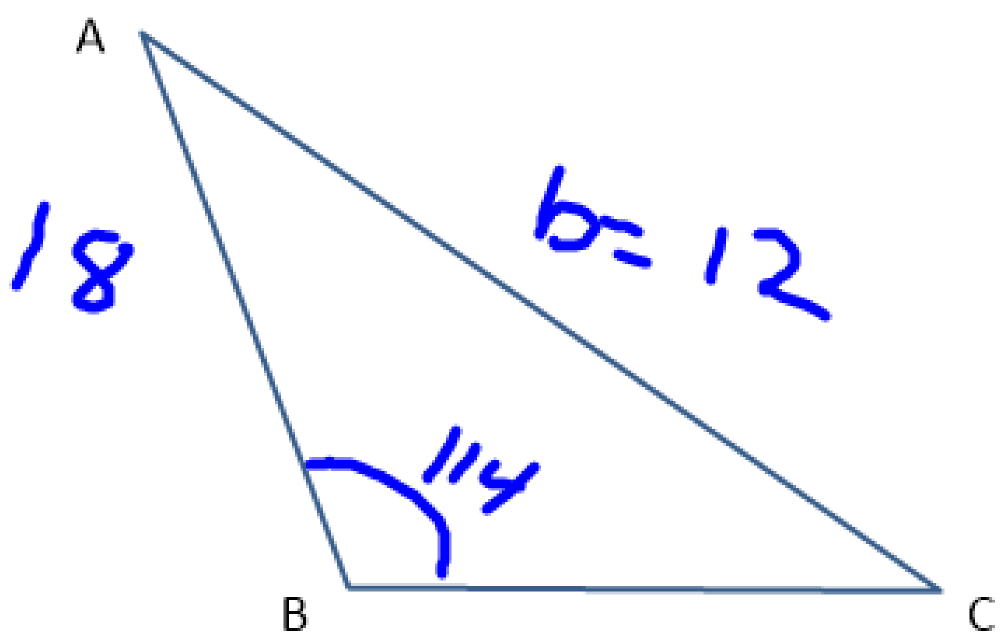
$$\frac{c}{\sin 28^\circ} = \frac{18}{\sin 114^\circ}$$

$$c = 9.3$$

Obtuse Example

<B = 114°, b = 12, c = 18

How many triangles?



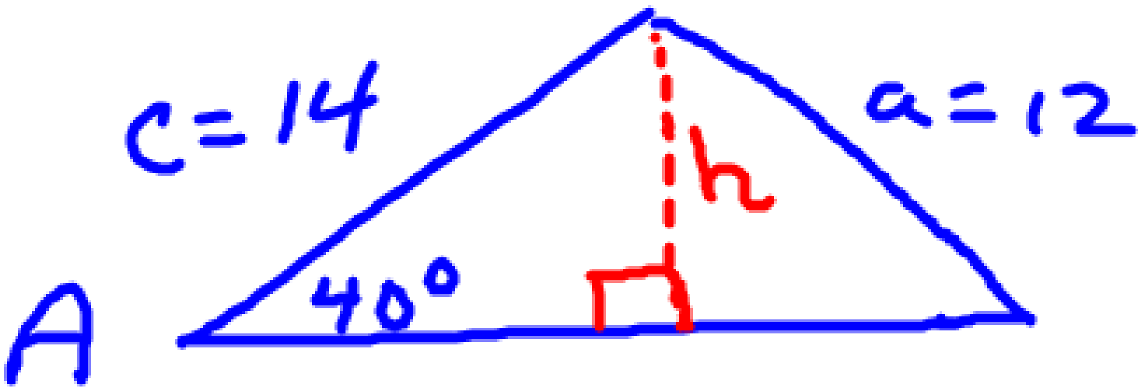
Given angle is Acute

*If the given angle is **acute**, there are **four different cases***

An easy way to check, is to calculate the height of the triangle opposite the given angle

Height

To calculate the height, use SOH



Given <A = 40°, and a = 12 and c = 14, calculate the height of the triangle

$$\sin 40^\circ = \frac{h}{14}$$

$$14 \sin 40^\circ = h$$

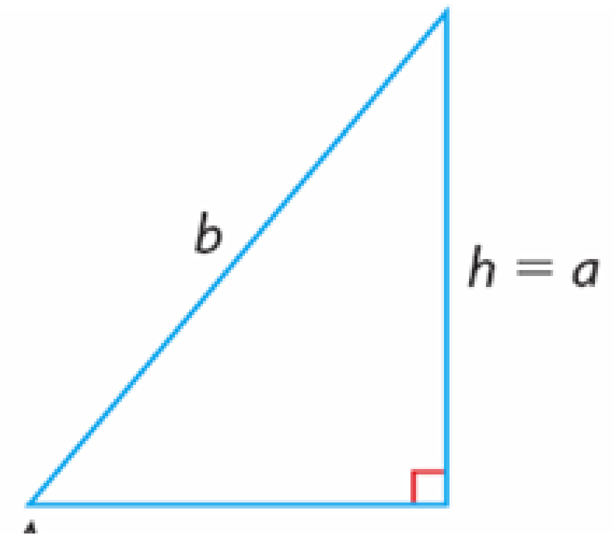
$$9.00 = h$$

Acute Case 1

If the opposite side of the given angle is **equal to** the height, how many triangles are possible?

When a = h
ONE triangle

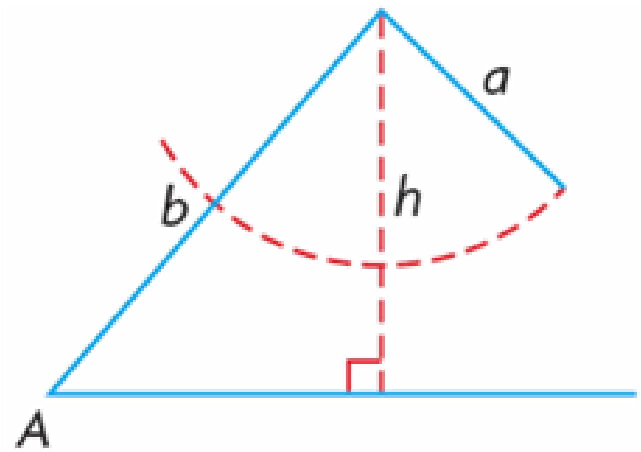
Only **one possible triangle**



Acute Case 2

If the opposite side of the given angle is **less then** the height, how many triangles are possible?

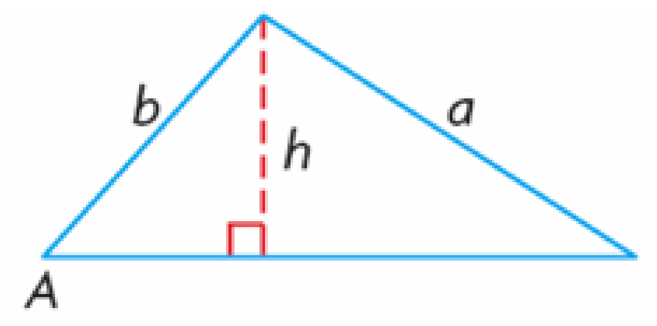
There are **NO**



When a < h
NO triangles

Acute Case 3

If the opposite side of the given angle is **greater then or equal to** the other given side, how many triangles are possible?



When $a \geq b$
ONE triangle

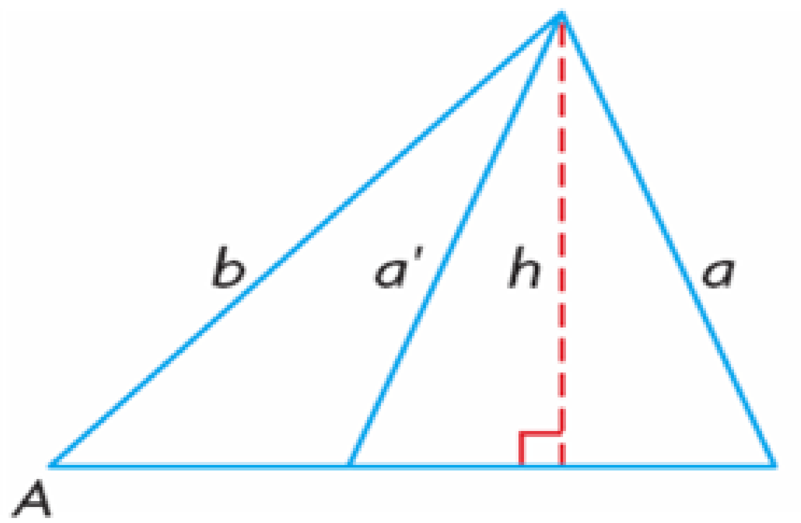
There is **one possible triangle**

NOW IT GETS TRICKY

What if the opposite side is longer then the height, but shorter than the other given side?

Acute Case 4

If the opposite side of the given angle is **greater then** the height, but smaller than the other given side, how many triangles are possible?



When h < a < b
TWO triangles

There are **TWO possible triangle**

Example 1 (p. 177)

Given each *SSA* situation for $\triangle ABC$, determine how many triangles are possible.

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

b) $\angle A = 30°$, $a = 6$ m, and $b = 12$ m

c) $\angle A = 30°$, $a = 8$ m, and $b = 12$ m

d) $\angle A = 30°$, $a = 15$ m, and $b = 12$ m

Hint* <*A* = 30 and *b* = 12, use SOH to find the height, *h*
Then compare *h* to *a*

a)

height

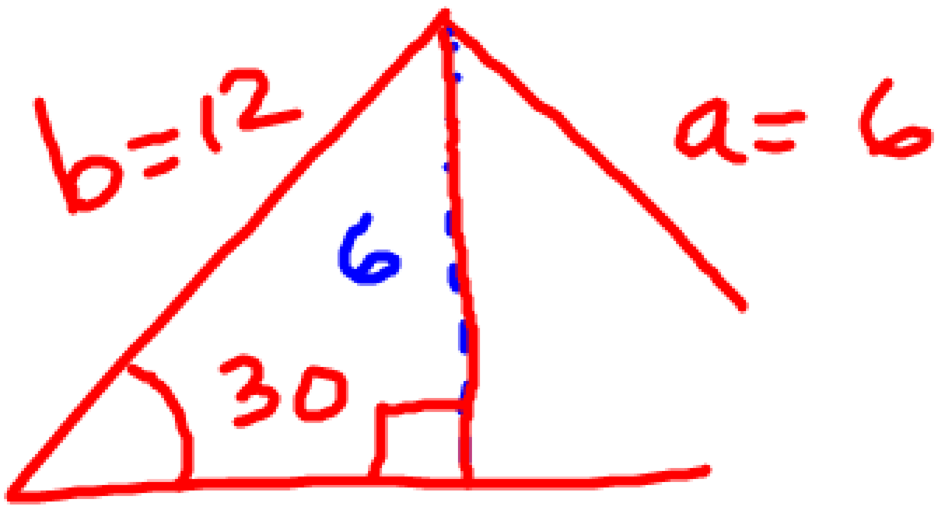
$h = 12 \sin 30°$

$h = 6$ m

NO △'s possible

Example 1 (p. 177) cont'd

b)



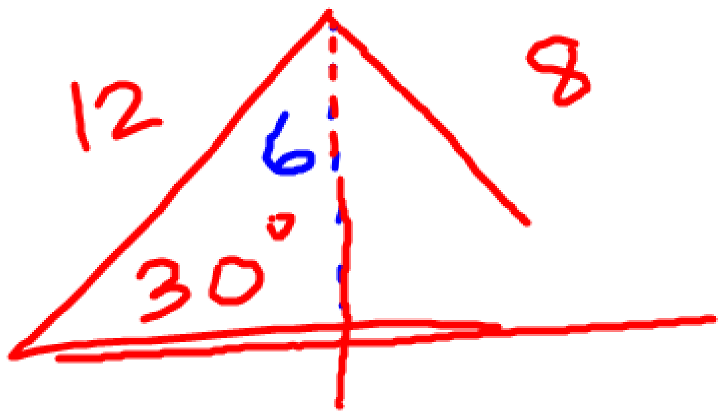
height

$12 \sin 30 = h$

$6 = h$

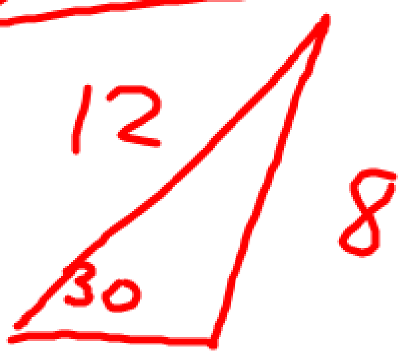
1 right △

c)



$h = (\sin 30)12$

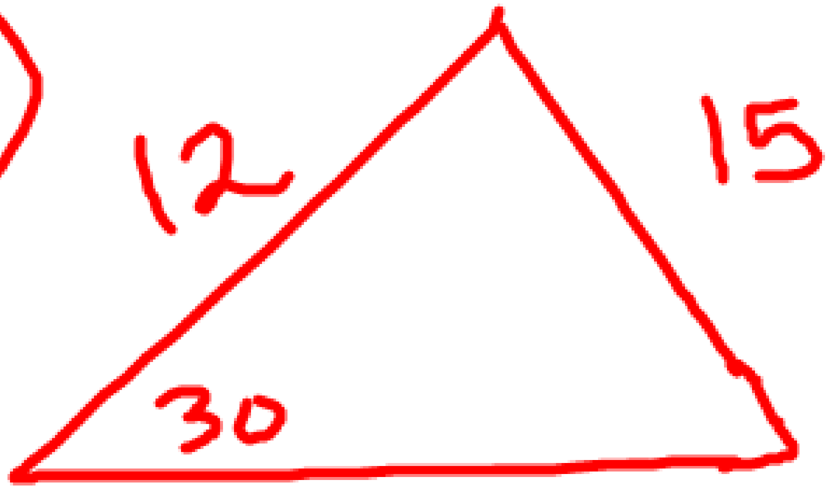
$h = 6$



2 △'s

Example 1 (p. 177) cont'd

d)



1 △

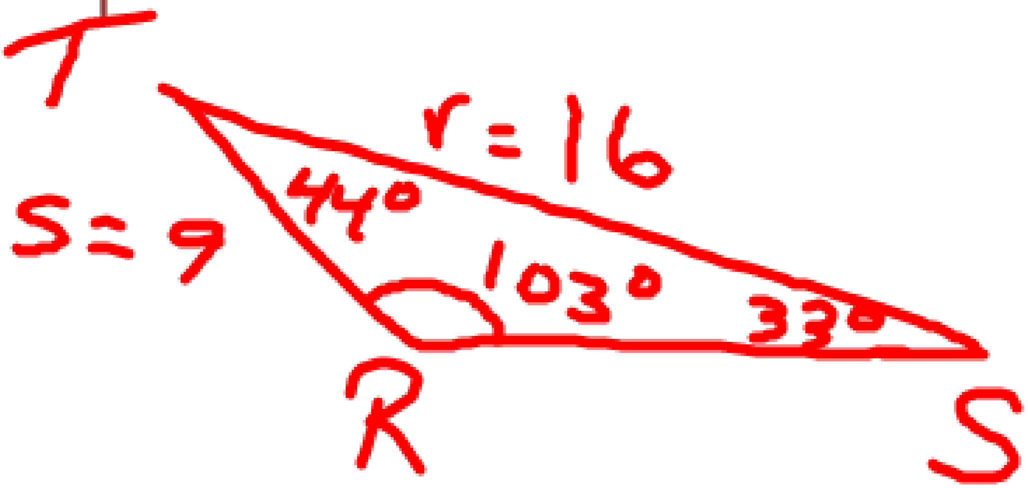
Steps to follow: Ambiguous Case

Flow Chart

Example

Determine how many triangles there are for *ΔRST*, <*R=103°*, *r=16* and *s=9*. Then solve for all possibilities

1 Δ.



$$\frac{\sin S}{9} = \frac{\sin 103°}{16}$$

$$\sin S = .5481$$

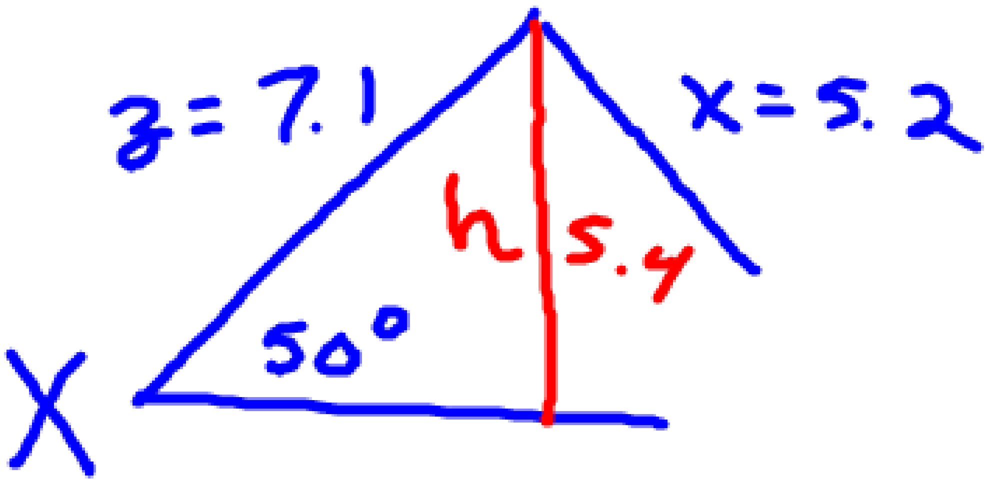
$$S = 33°$$

$$\frac{t}{\sin 44°} = \frac{9}{\sin 33°}$$

$$t = 11.5$$

Example

Determine how many triangles there are for *ΔXYZ*, *<X=50°*, *x=5.2* and *z=7.1*. Then solve for all possibilities



$h = 7.1 \sin 50°$

$h = 5.4$

0 Δ's

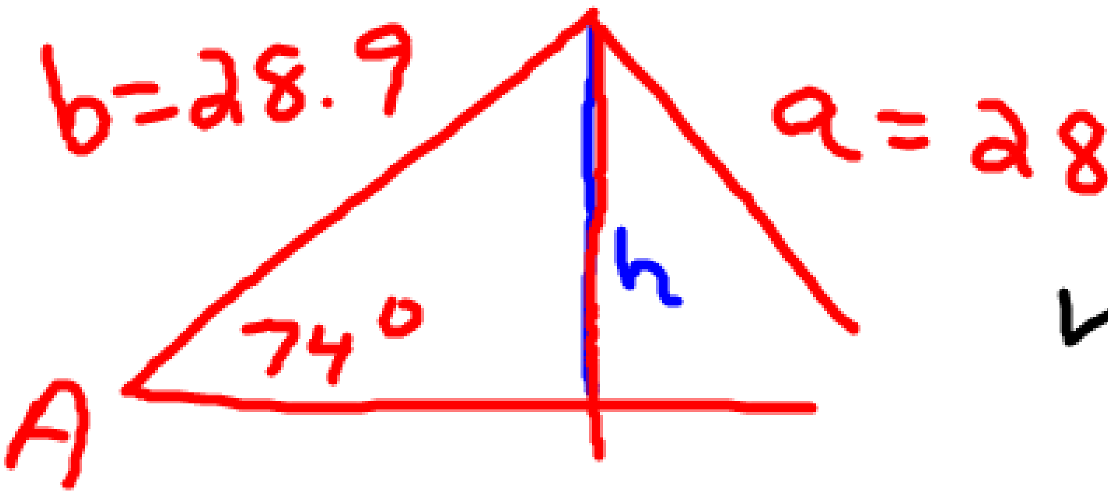
Example

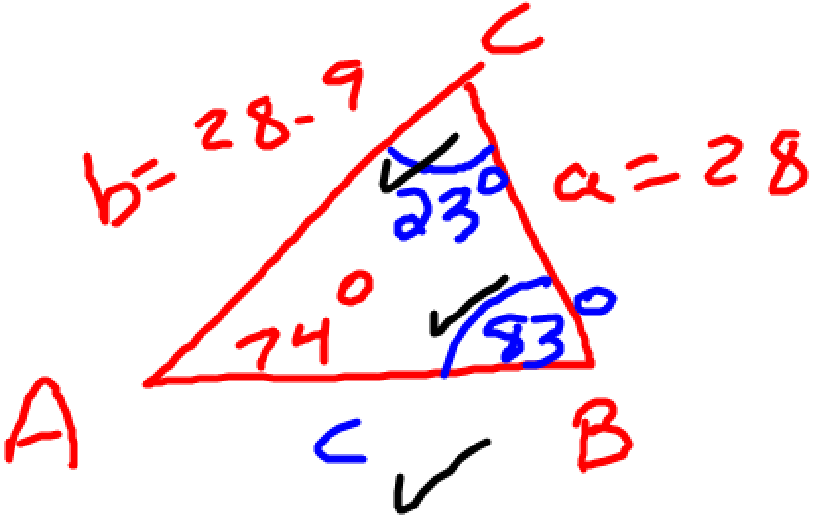
Determine how many triangles there are for ΔABC, $<A=74^{o}$, $a=28.0$ and $b=28.9$. Then solve for all possibilities

$h = 28.9 \sin 74^{\circ}$

$h = 27.7$

✓✓ 2 △s



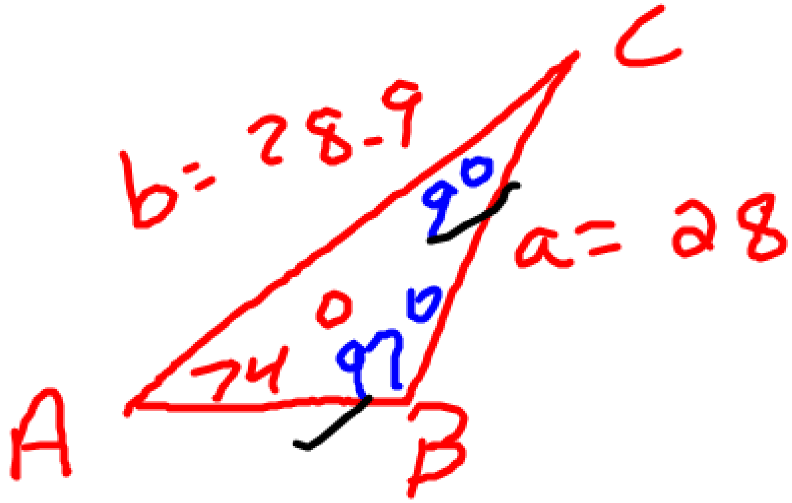


$$\frac{\sin B}{28.9} = \frac{\sin 74}{28}$$

$$B = 83°$$

$$\frac{c}{\sin 23°} = \frac{28}{\sin 74}$$

$$c = 17.4$$



$$\angle B = 180 - 83°$$

$$\frac{c}{\sin 9°} = \frac{28}{\sin 74°}$$

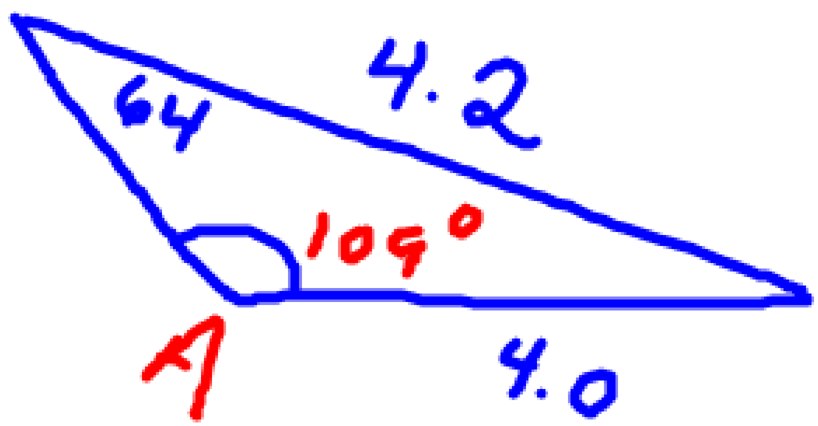
$$c = 4.6$$

Example 8 (p. 184)

8. An obtuse triangle has two known side lengths: 4.0 m and 4.2 m. The angle that is opposite the shorter side measures 64.0°.

a) Calculate the obtuse angle in the triangle, to the nearest tenth of a degree.

b) Is there only one possible answer? Explain. NO



$$\frac{\sin A}{4.2} = \frac{\sin 64°}{4.0}$$

$$\sin A = .9437$$

$$A \approx 71°$$

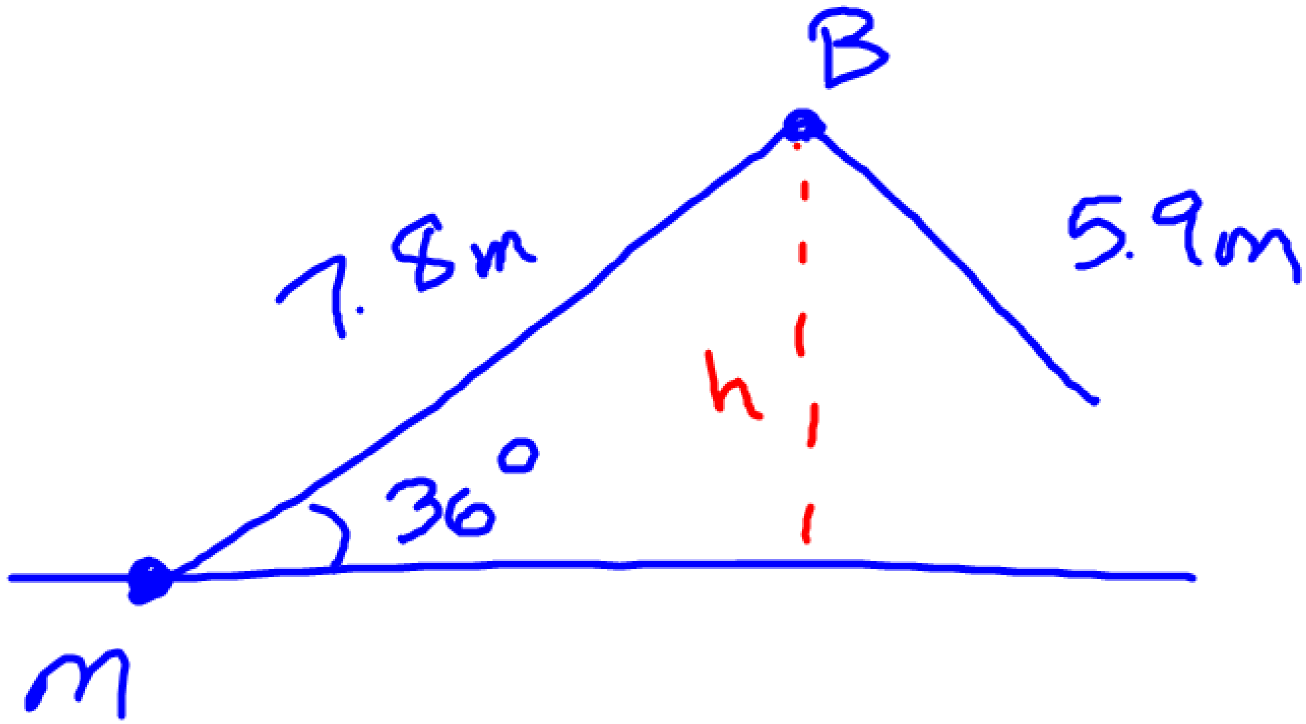
$$\angle A = 180 - 71° = 109°$$

Example 2 (p. 178)

Martina and Carl are part of a team that is studying weather patterns. The team is about to launch a weather balloon to collect data. Martina's rope is 7.8 m long and makes an angle of 36.0° with the ground. Carl's rope is 5.9 m long. Assuming that Martina and Carl form a triangle in a vertical plane with the weather balloon, what is the distance between Martina and Carl, to the nearest tenth of a metre?

Are they standing on the same side of the balloon? Or opposite sides of the balloon?

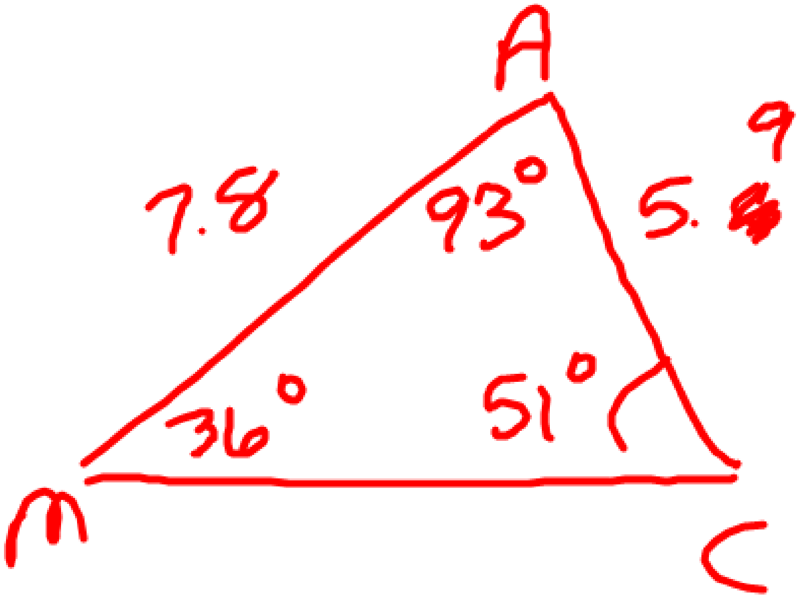
How many possible situations are there?

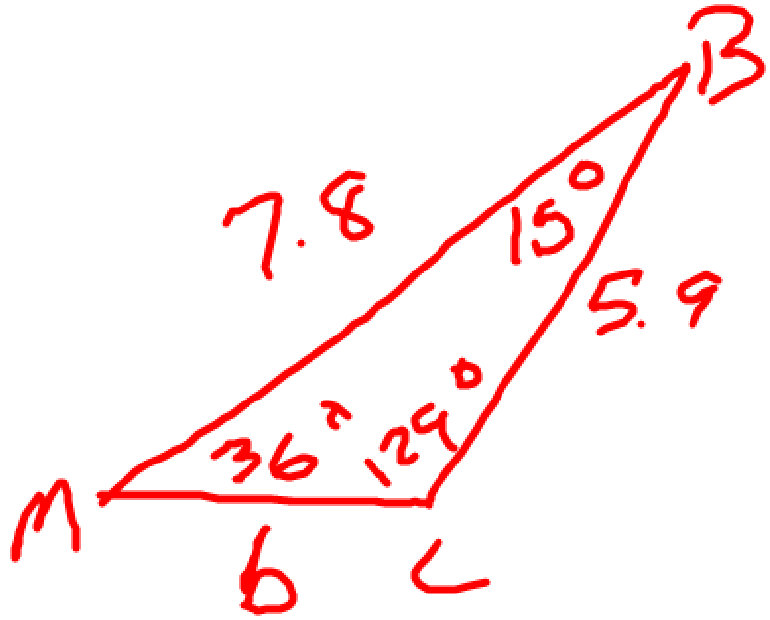


$h = 7.8(\sin 36°)$

$h = 4.6$

$\therefore$ 2 $\triangle$'s





$$\frac{\sin C}{7.8} = \frac{\sin 36^\circ}{5.9}$$

$$\sin C = .777$$

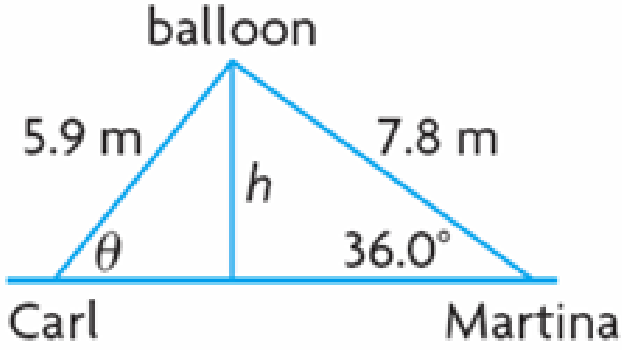
$$51^\circ = C$$

$$\frac{b}{\sin 15} = \frac{5.9}{\sin 36}$$

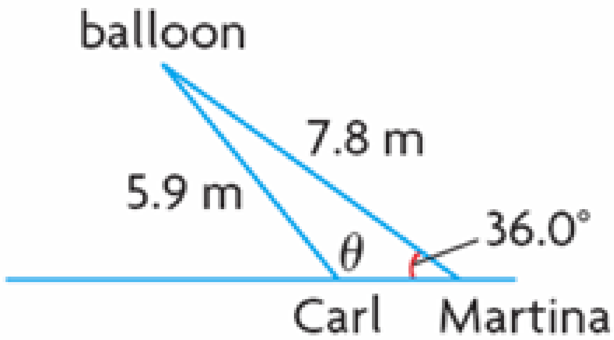
$$b = 2.6$$

Example 2 (p. 178) cont'd

Situation 1:



Situation 2:



Key Idea

The ambiguous case of the sine law may occur when you are given ***SSA*** *information*

Depending on the given information you may have to construct and solve ***zero, one, or two triangles***

Need to know

*When given SSA such that **<A is obtuse**, there are **two cases** to consider*

*When given SSA such that **<A is acute**, there are **four cases** to consider*

See above or p. 182 of textbook

Homework

P. 183-185

1, 4, 5, 6, 11, 13