4.3 Mixed and Entire Radicals

Lesson Focus

Express an entire radical as a mixed radical, and vice versa

Radicals of the form $\sqrt[n]{x}$ such as $\sqrt{80}$, $\sqrt[3]{144}$, and $\sqrt[4]{162}$ are **entire radicals.**

Radicals of the form $a\sqrt[n]{x}$ such as $4\sqrt{5}$, $2\sqrt[3]{18}$, and $3\sqrt[4]{2}$ are **mixed radicals.**

Explore

- *Just as with fractions, equivalent expressions for any number have the same value*

$$\sqrt{144} \quad = 12$$

$$= \sqrt{16 \times 9}$$

$$= \sqrt{16} \times \sqrt{9}$$

$$= 4 \times 3$$

$$= 12$$

Multiplication Property of Radicals

$\sqrt{5 \cdot 3} = \sqrt{5} \cdot \sqrt{3}$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$,
where n is a natural number, and a and b are real numbers

- *Simplify the following radicals:*

$\sqrt{24}$

$= \sqrt{\cancel{4} \cdot 6}$

$= 2\sqrt{6}$

$\sqrt[3]{24}$

$= \sqrt[3]{\cancel{8} \cdot 3}$

$= 2\sqrt[3]{3}$

Example

Simplify each radical.

a) $\sqrt{80}$ **b)** $\sqrt[3]{144}$ **c)** $\sqrt[4]{162}$

$= \sqrt{16 \cdot 5}$

$= 4\sqrt{5}$

$= \sqrt[3]{8 \cdot 18}$

$= 2\sqrt[3]{18}$

$= \sqrt[4]{81 \cdot 2}$

$= 3\sqrt[4]{2}$

Example

Simplify each radical.

a) $\sqrt{63}$ $= \sqrt{9 \cdot 7} = 3\sqrt{7}$

b) $\sqrt[3]{108}$ $= \sqrt[3]{27 \cdot 4} = 3\sqrt[3]{4}$

c) $\sqrt[4]{128}$ $= \sqrt[4]{16 \cdot 8}$

$= 2\sqrt[4]{8}$

Simplest Form

Some numbers, such as 200, have more than one perfect square factor.
The factors of 200 are: 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
Since 4, 25, and 100 are perfect squares, we can simplify $\sqrt{200}$ in these ways.

$$\sqrt{200} = \sqrt{4 \cdot 50} = \sqrt{4} \cdot \sqrt{50} = 2\sqrt{50}$$

$$= 2\sqrt{25 \cdot 2}$$

$$= 10\sqrt{2}$$

$$\sqrt{200} = \sqrt{25 \cdot 8} = \sqrt{25} \cdot \sqrt{8} = 5\sqrt{8}$$

$$\sqrt{200} = \sqrt{100 \cdot 2} = \sqrt{100} \cdot \sqrt{2} = 10\sqrt{2}$$

$10\sqrt{2}$ is in simplest form because the radical contains no perfect square factors other than 1.

Example

Write each radical in simplest form, if possible.

a) $\sqrt[3]{40}$ **b)** $\sqrt{26}$ **c)** $\sqrt[4]{32}$

Going in Reverse

- *Any number can be written as the square root of its square*

Ex $2 = \sqrt{2 \times 2} = \sqrt{4}$

Ex $9 = \sqrt{9 \times 9} = \sqrt{81}$

Ex $10 =$

- *Then* $10\sqrt{2} = \sqrt{10 \times 10 \times 2}$

Example

Write each mixed radical as an entire radical.

a) $4\sqrt{3}$

$= \sqrt{3 \cdot 4^2}$

$= \sqrt{3 \cdot 16}$

$= \sqrt{48}$

b) $3\sqrt[3]{2}$

$= \sqrt[3]{2 \cdot 3^3}$

$= \sqrt[3]{2 \cdot 27}$

$= \sqrt[3]{54}$

c) $2\sqrt[5]{2}$

$= \sqrt[5]{2 \cdot 2^5}$

$= \sqrt[5]{2 \cdot 32}$

$= \sqrt[5]{64}$

Example

Write each mixed radical as an entire radical.

a) $7\sqrt{3}$ $= \sqrt{3 \cdot 7^2} = \sqrt{147}$

b) $2\sqrt[3]{4}$ $= \sqrt[3]{4 \cdot 2^3} = \sqrt[3]{32}$

c) $2\sqrt[5]{3}$ $= \sqrt[5]{3 \cdot 2^5} = \sqrt[5]{96}$

Homework

P. 217-219

9, 10, 11, 12, 14, 15, 17 a,d, 18 a,d, 20, 21