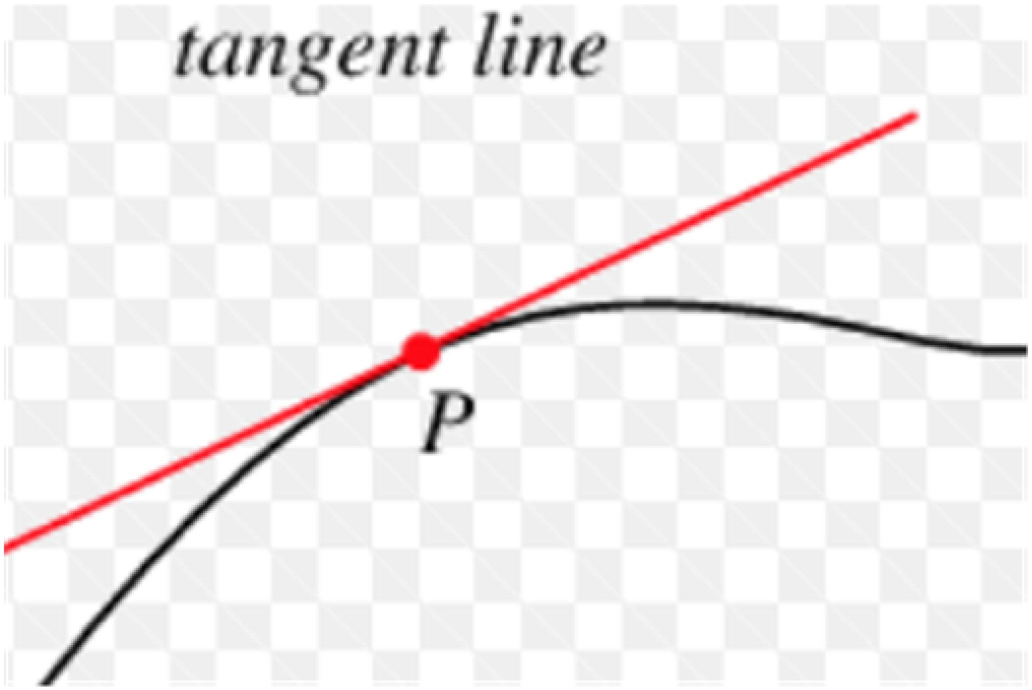
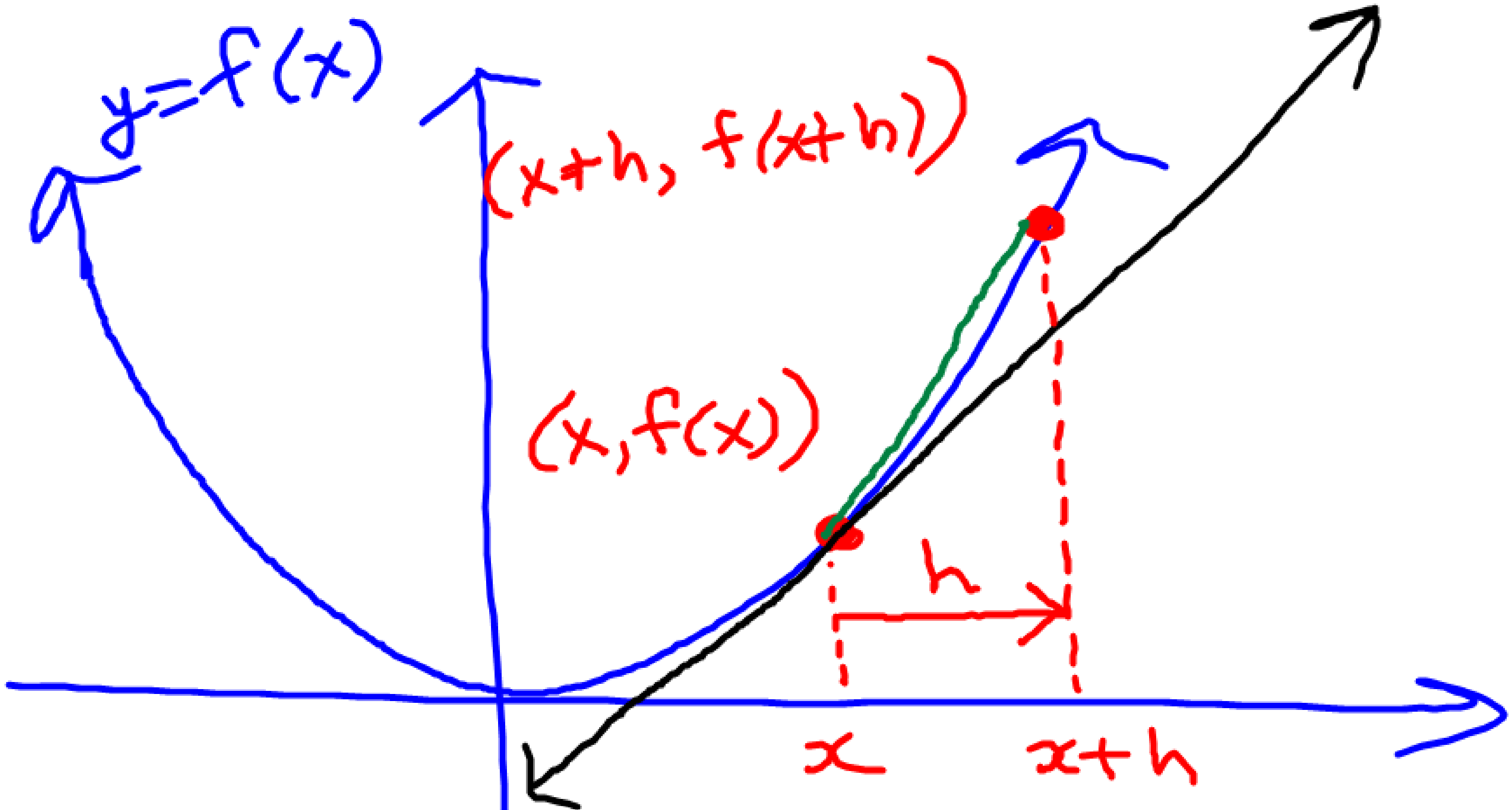
Oct 22/18

Chapter 4 Differentiation

4.3/4.4 Determining Slopes of Tangent Lines at the General and Specific Points

Today we are going to learn how to find the slope of a tangent line!





$$m_{sec} = \frac{f(x+h) - f(x)}{x+h-x}$$

$$m_{sec} = \frac{f(x+h) - f(x)}{h}$$

$$\boxed{m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Therefore to find the instantaneous rate change or the slope of a tangent line we have:

$$m_{\tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

f'

$\frac{dy}{dx}$ $\quad f'(x)$

New Notation

The slope of the tangent line can also be represented by the following:

$$f'(x) = \frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example 1

a) Find the **general slope** of the tangent line to the curve $f(x) = 2x^2$

b) Find the slope of the tangent line at the point (1,f(1)). $(1,2)$

c) Use this slope to find the **equation of the tangent** line at the point (1,f(1)).

$f(x) = 2x^2$

a) $$f' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4x \cdot h + 2h^2 \cancel{- 2x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h)}{\cancel{h}} = 4x + 2(0) = 4x$$

$$\boxed{f' = 4x}$$

b) $f'(1) = 4(1) = 4$

slope tangent
when $x = 1$

$(1, 2)$

c) Point/Slope Formula

$$y - y_1 = m(x - x_1)$$

$$y-2=4(x-1)$$

1) Slope Intercept $\quad y=mx+b$

$y-2=4x-4$

$$\boxed{y=4x-2}$$

2) Standard Form $\quad Ax+By=C$

No fractions

$Ax>0$

$2=4x-y$

3) General Form $\quad Ax+By+C=0$

No Fractions $\quad Ax>0$

$0=4x-y-2$

Example 2

Consider the function

$f(x) = 3x^2 - 4x - 2$.

~~(a) Find $f'(x)$ using~~

$$\lim_{h\to 0} \cancel{\frac{f(x+h)-f(x)}{h}}.$$

~~(b) Find the slope of the tangent line at the point $(1, f(1))$.~~

(c) Find the equation of the tangent line ~~in part (b)~~ at point $(1, -3)$

$f(x) = 3x^2 - 4x - 2$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{3(1+h)^2 - 4(1+h) - 2 - (-3)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3} + 6h + 3h^2 \cancel{- 4} - 4h \cancel{- 2} + \cancel{3}}{h}$$

$$= \lim_{h \to 0} \frac{3h^2 + 2h}{h} = \lim_{h \to 0} \frac{\cancel{h}(3h+2)}{\cancel{h}} = 2$$

Equation tangent line

$$y - y_1 = m(x - x_1)$$

$$m = 2$$
$$(1, -3)$$

$$y + 3 = 2(x - 1)$$

$$\boxed{y = 2x - 5}$$

Ex3. Find the **equation of the tangent** line to the curve $y = \dfrac{1}{x}$ at x = -2.

$\left(-2, -\frac{1}{2}\right)$

$$m = \lim_{h \to 0} \frac{f(-2+h) - f(-2)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{-2+h} - \left(-\frac{1}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{h-2} + \frac{1}{2}}{h} \quad \frac{2(h-2)}{2(h-2)}$$

$$= \lim_{h \to 0} \frac{2 + (h-2)}{2h(h-2)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\, 1}{2\cancel{h}(h-2)} = \frac{1}{2(0-2)} = -\frac{1}{4}$$

$$(-2, -1/2)$$

$$y - y_1 = m(x - x_1)$$

$$y + \frac{1}{2} = -\frac{1}{4}(x+2)$$

$$\left(y + \frac{1}{2} = -\frac{1}{4}x - \frac{1}{2}\right)4$$

$4y + 2 = -x - 2$

$$\boxed{x + 4y = -4}$$

Ex.4 Find the equation of the tangent line to the curve $y=\sqrt{x-2}$ at the point (6,2).

$$m=\lim_{h\to 0}\frac{f(6+h)-f(6)}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{6+h-2}-2}{h}$$

$$=\lim_{h\to 0}\frac{(\sqrt{4+h}-2)}{h}\cdot\frac{(\sqrt{4+h}+2)}{(\sqrt{4+h}+2)}$$

$$=\lim_{h\to 0}\frac{\cancel{4}+\cancel{h}-\cancel{4}\;1}{\cancel{h}(\sqrt{4+h}+2)}=\frac{1}{4}$$

$y - y_1 = m(x - x_1)$

$\boxed{y - 2 = \frac{1}{4}(x - 6)}$

$\left(y - 2 = \frac{1}{4}x - \frac{6}{4}\right)4$

$4y - 8 = x - 6$

$-2 = x - 4y$

$y - 2 = \frac{1}{4}x - \frac{3}{2}$

$y = \frac{1}{4}x + \frac{1}{2}$

This special slope formula we have been using also has another name. It is known as:

The Definition Of The Derivative

$\dfrac{dy}{dx} = f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ is known as the definition of the derivative.

Assignment

~~Page 176 #8 top of page do together.~~

Page 176

#'s 1 a, c, f, 2, 3,

Page 167

#'s 1, 3, 5 ,6, 10

Handout

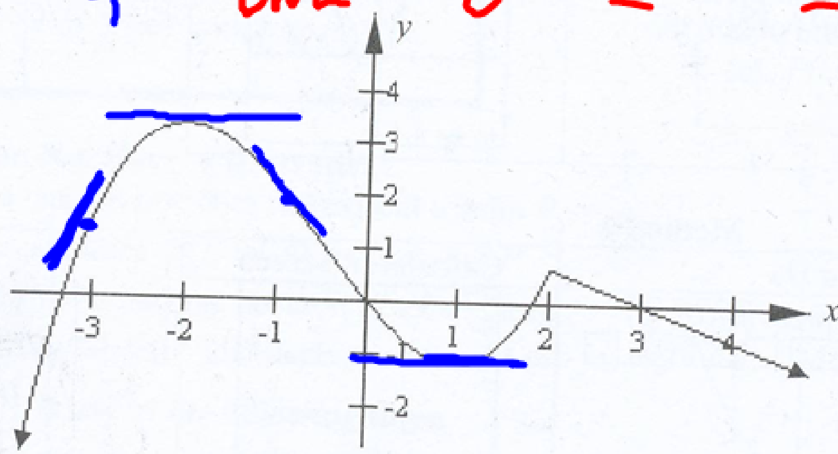
#7 a,b, #8, #9, #10

+, −, 0, DNE

8. Examine the graph of the function $f(x)$ below. Based on your study of the graph describe each of the following as being "zero", "positive", "negative", or "does not exist".

(a) $f(-3)$ + (b) $f'(-3)$ + (c) $f(-2)$ + (d) $f'(-2)$ 0 (e) $f(-1)$ + (f) $f'(-1)$ − (g) $f(0)$ 0 (h) $f'(0)$ −

(i) $f(1)$ − (j) $f'(1)$ 0 (k) $f(2)$ + (l) $f'(2)$ DNE (m) $f(3)$ 0 (n) $f'(3)$ − (o) $f(4)$ − (p) $f'(4)$ −



$f(x)$

1a) $y = 4 - x^2 \qquad (-2, 0)$

$$m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4 - (x+h)^2 - (4 - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{4} \cancel{- x^2} - 2xh - h^2 \cancel{- 4} \cancel{+ x^2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{-2xh - h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-2x - h)}{\cancel{h}} = -2x - 0 = -2x$$

$$f'(x) = -2x$$ Slope of tangent

$$f'(-2) = -2(-2) = 4 \qquad (-2, 0)$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = +4(x+2)$$

$$y = +4x + 8$$