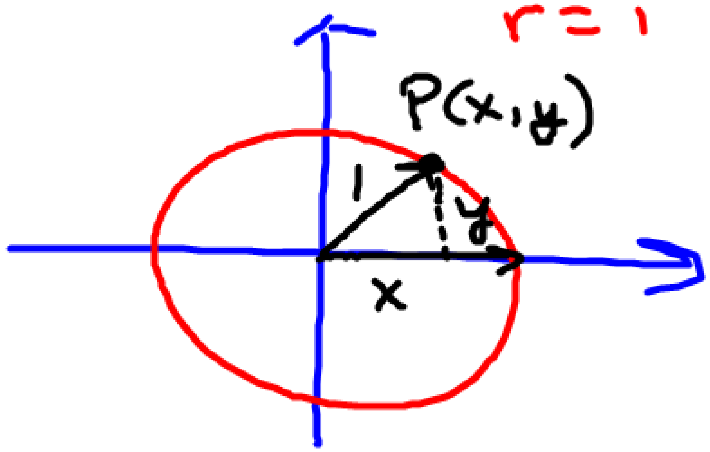
4.2 The Unit Circle

By definition, the **Unit Circle** is a circle that has a radius of 1 with the centre at the origin of the Cartesian plane.



$$x^2 + y^2 = (1)^2$$

$$\boxed{x^2 + y^2 = 1}$$

$C(0,0)$ $r = 1$

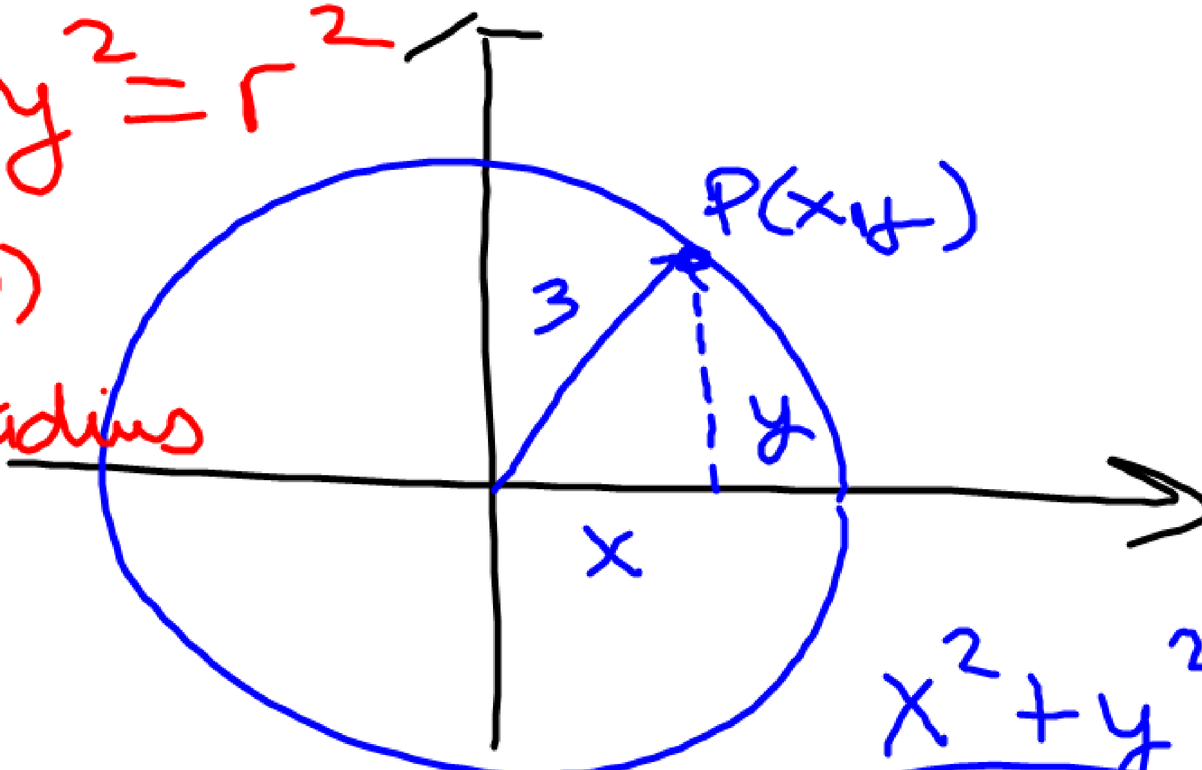
Your Turn:

Determine the equation for a circle with its centre at the origin and having a radius of 3.

$x^2 + y^2 = r^2$

$C(0,0)$

$r = \text{radius}$

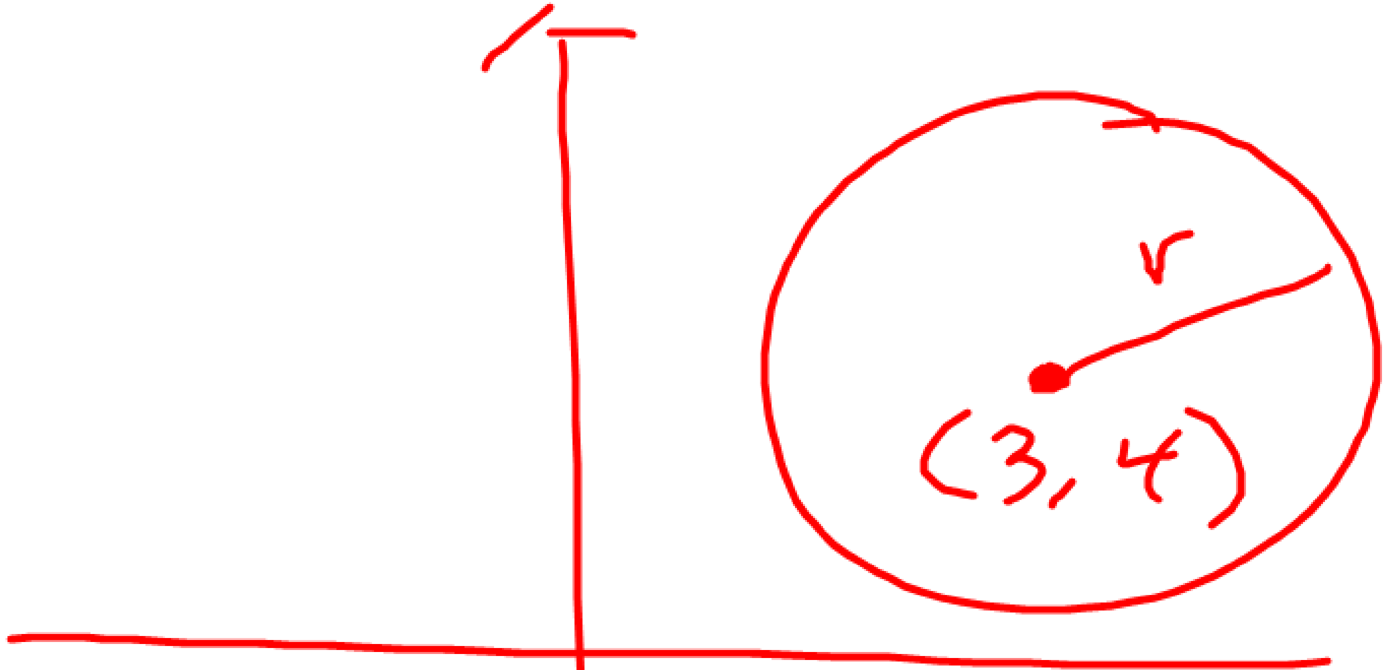


$C(0,0)$

$r = 3$

$x^2 + y^2 = (3)^2$

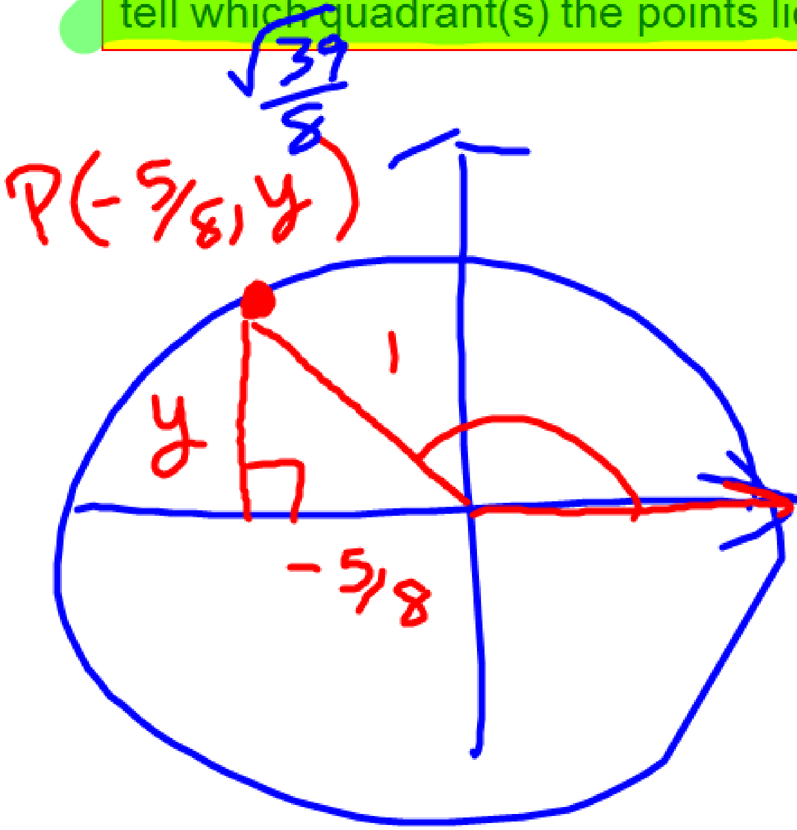
$x^2 + y^2 = 9$

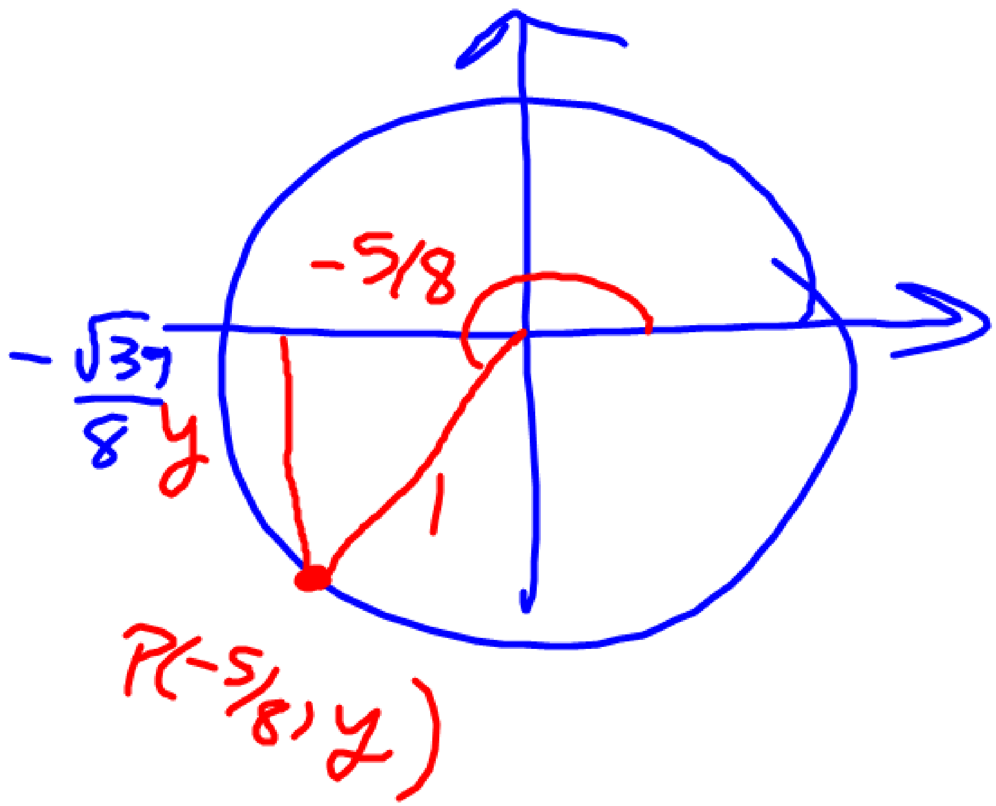


$$x^2 + y^2 = r^2$$

$$(x-h)^2 + (y-k)^2 = r^2$$

Example 1:
Determine the missing coordinate(s) for all points on the unit circle satisfying the given conditions: point $P\left(\frac{-5}{8}, y\right)$. Draw a diagram and tell which quadrant(s) the points lie.





$$x^2 + y^2 = r^2$$

$$\left(-\frac{5}{8}\right)^2 + y^2 = (1)^2$$

$$\frac{25}{64} + y^2 = 1$$

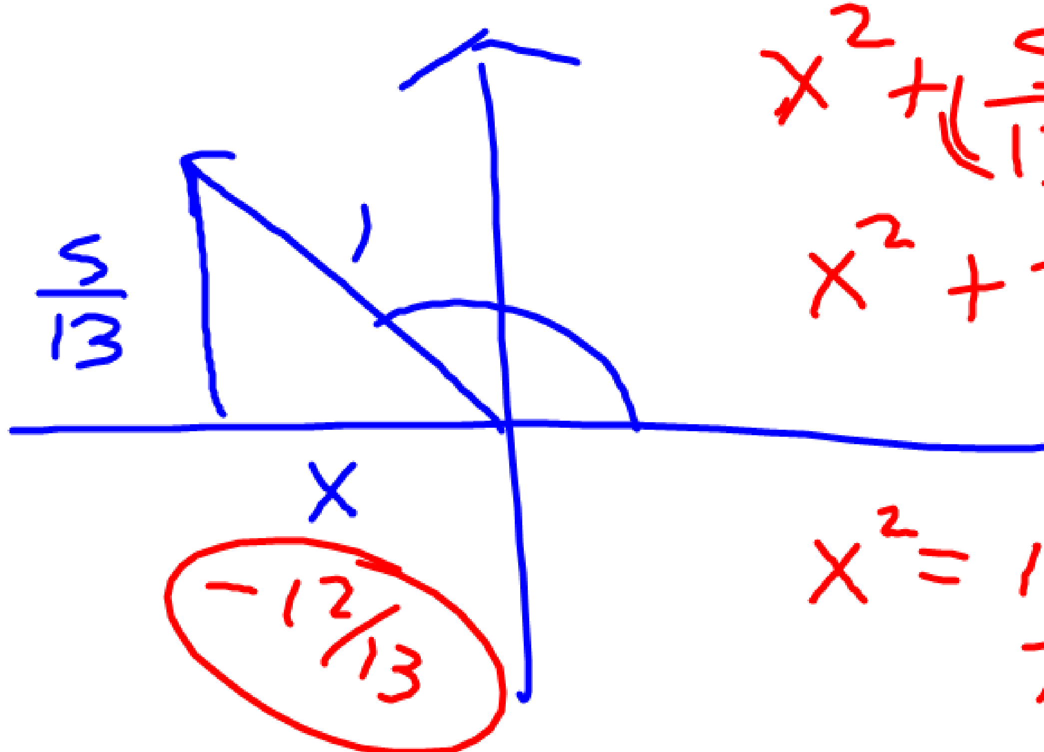
$$y = \frac{\pm\sqrt{39}}{8}$$

$$y^2 = \frac{64}{64} - \frac{25}{64}$$

$$y^2 = \frac{39}{64}$$

Your Turn:

Determine the missing coordinate(s) for all points on the unit circle satisfying the given conditions: point $P\left(x, \frac{5}{13}\right)$ where the point P is in quadrant 2. Draw a diagram.



$$x^2 + \left(\frac{5}{13}\right)^2 = 1^2$$

$$x^2 + \frac{25}{169} = 1$$

$$x^2 = \frac{144}{169}$$

$$x = \pm \frac{12}{13}$$

Relating Arc Length and Angle Measure in Radians

Recall arc length formula from section 4.1

$$a = \theta r$$

where a = arc length

θ = measure of the central angle in radians

r = radius.

What does this formula become in the unit circle?

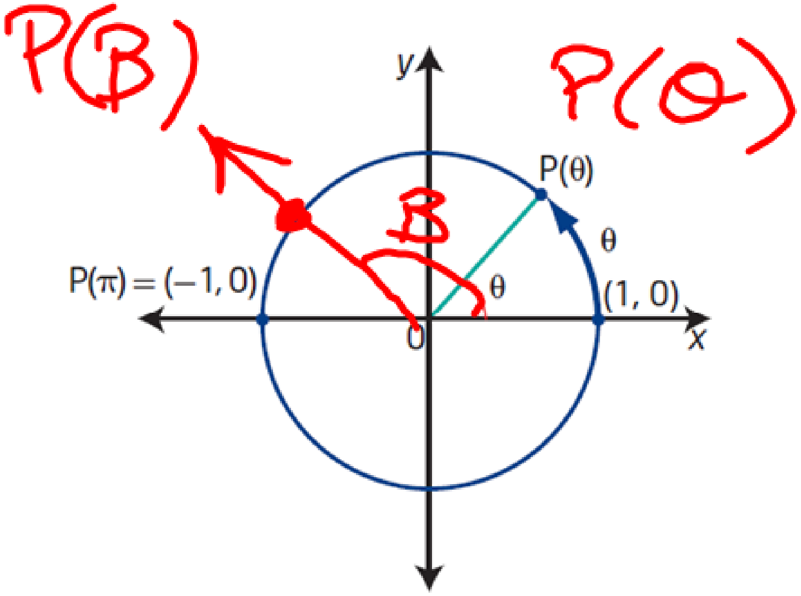
$a = \theta(1)$

$a = \theta$

This means that a **central angle in radians** and its **subtended arc** on the unit circle have the same numerical value!

We can use the function $P(\theta) = (x, y)$ to link the arc length θ, of a central angle on the unit circle to the coordinates (x, y), of the point of intersection of the terminal arm and the unit circle.

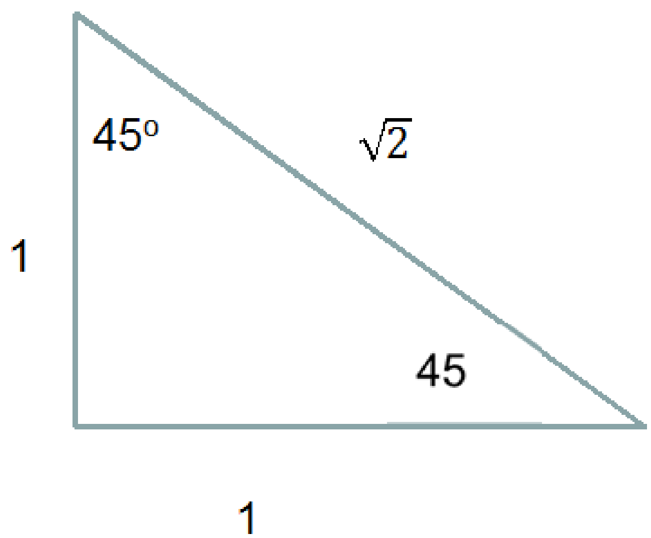
If you join $P(\theta)$ to the origin, you create an angle θ in standard position. Now, θ in radians is the central angle and the arc length is θ units!



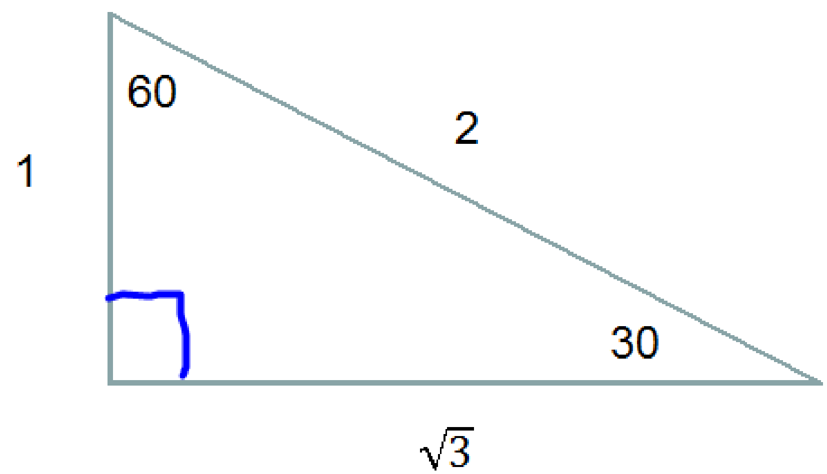
Let's develop the coordinates of the points on the unit circle that are associated with the special angles!

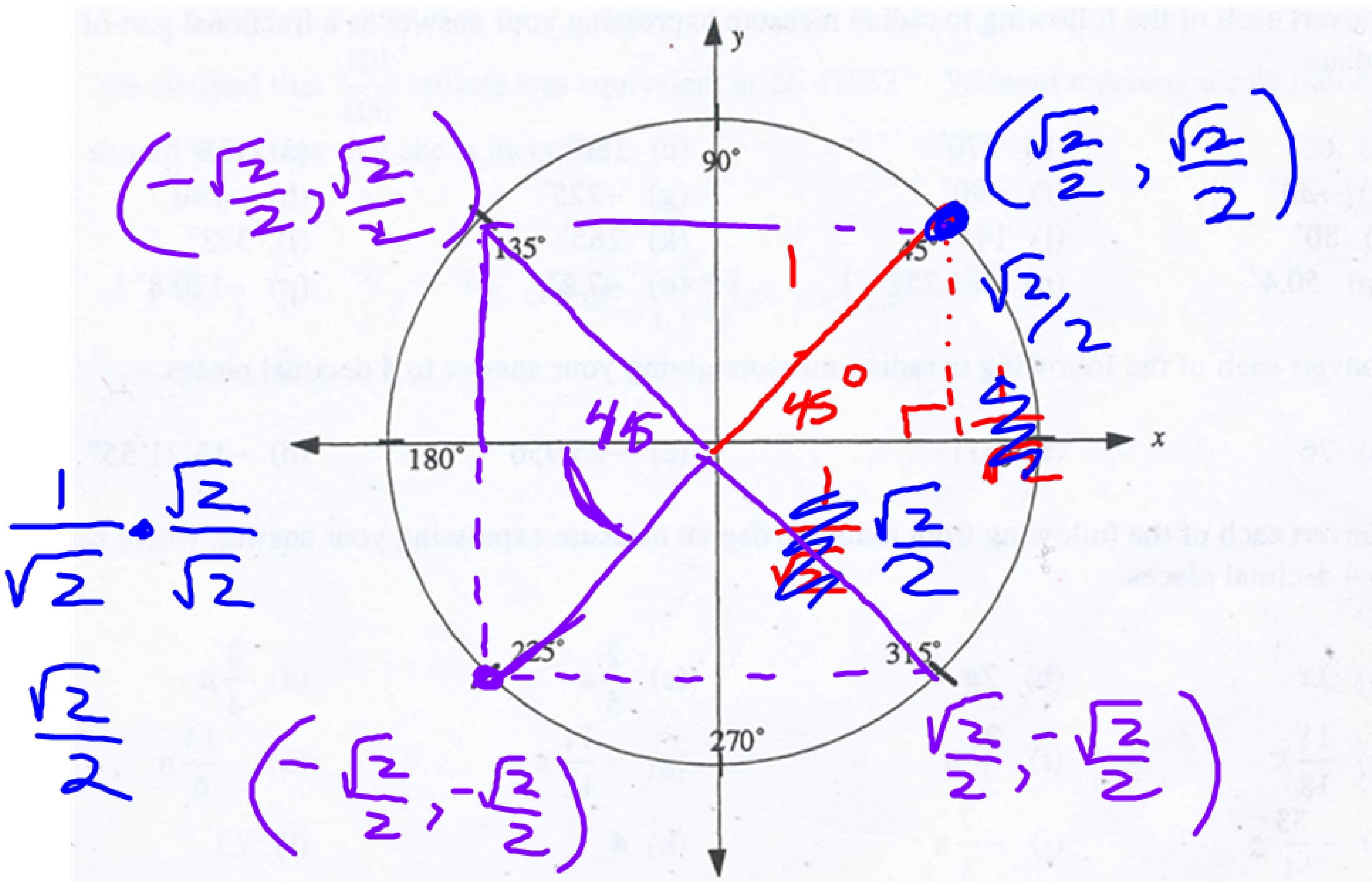
Special Triangles

45^0-45^0-90^0

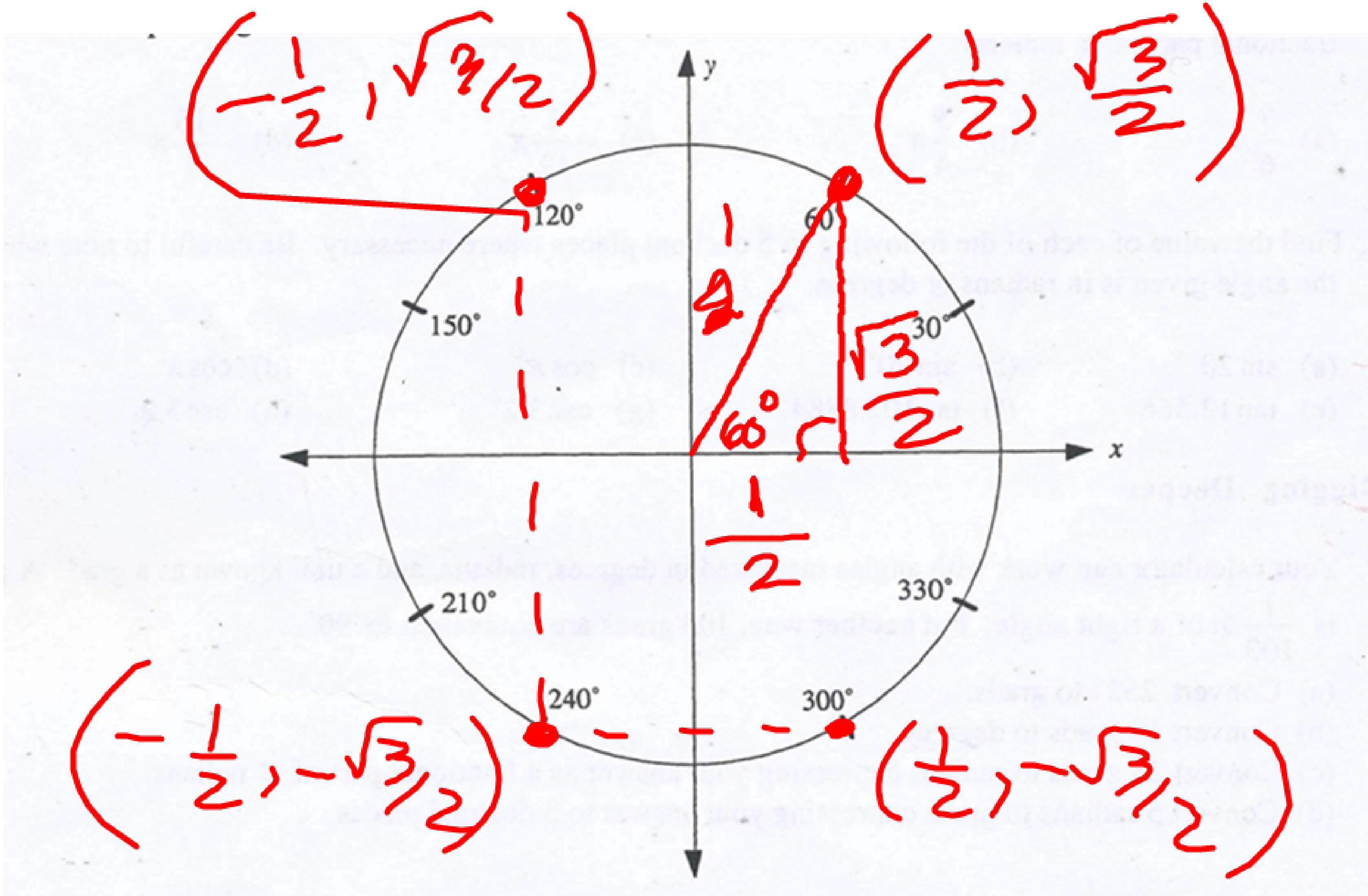


30^0-60^0-90^0

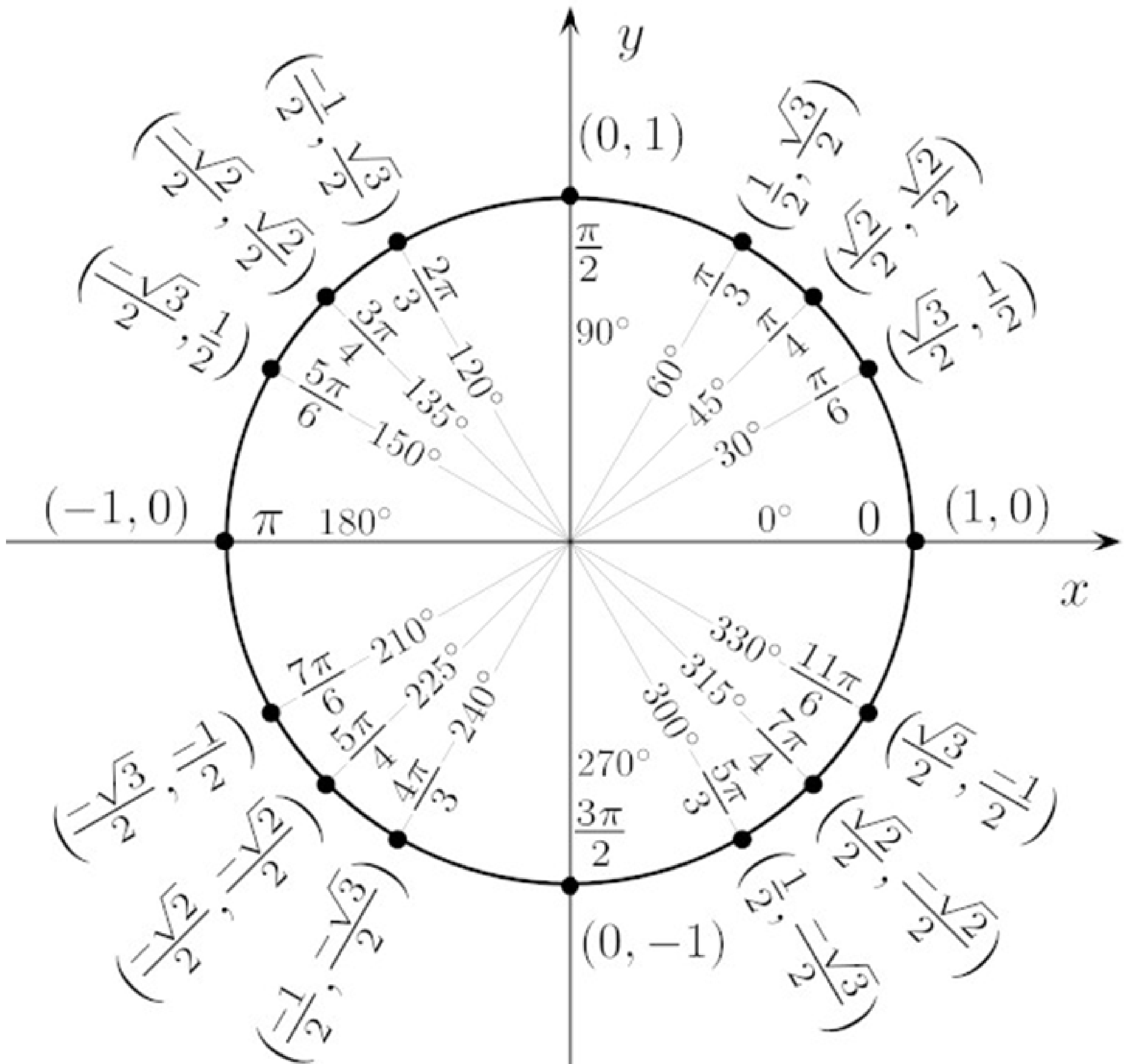




Using your special triangles, find the coordinates of the points on the unit circle at all angles in the above diagram.



Using your special triangles, find the coordinates of the points on the unit circle at all angles in the above diagram.



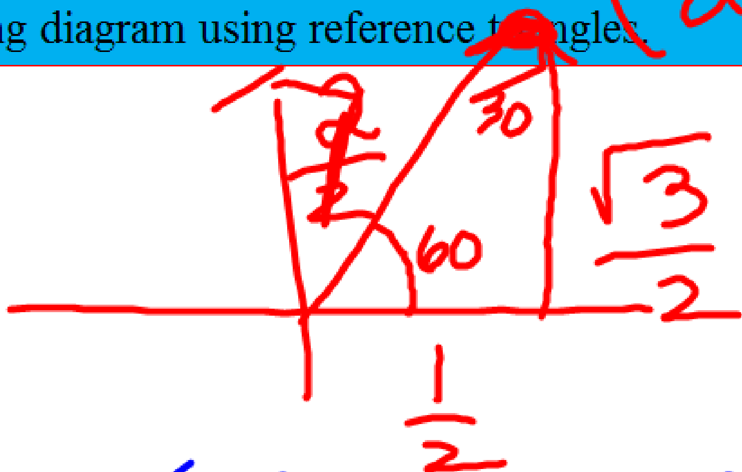
y
x
$(0,1)$
$(1,0)$
$(-1,0)$
$(0,-1)$
$\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
$\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
$\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$
$\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$
0 $0°$
$\frac{\pi}{6}$ $30°$
$\frac{\pi}{4}$ $45°$
$\frac{\pi}{3}$ $60°$
$\frac{\pi}{2}$ $90°$
$\frac{2\pi}{3}$ $120°$
$\frac{3\pi}{4}$ $135°$
$\frac{5\pi}{6}$ $150°$
π $180°$
$\frac{7\pi}{6}$ $210°$
$\frac{5\pi}{4}$ $225°$
$\frac{4\pi}{3}$ $240°$
$\frac{3\pi}{2}$ $270°$
$\frac{5\pi}{3}$ $300°$
$\frac{7\pi}{4}$ $315°$
$\frac{11\pi}{6}$ $330°$

Example 1:
Finding the coordinates on the end of a terminal arm on a unit circle for a given central angle.

$\left(\frac{1}{2}, \sqrt{3}/2\right)$

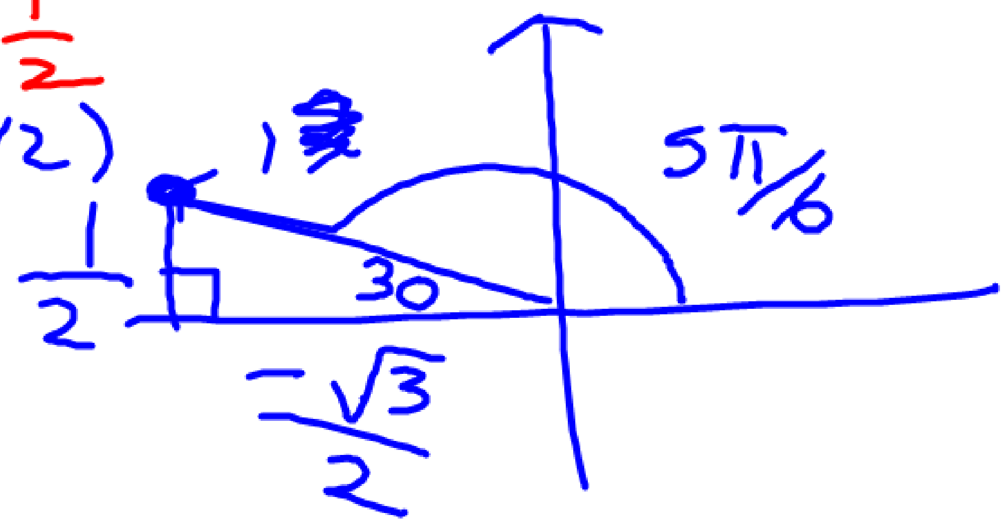
Show both options to solve, using unit circle and drawing diagram using reference triangles.

a) $\frac{\pi}{3}$



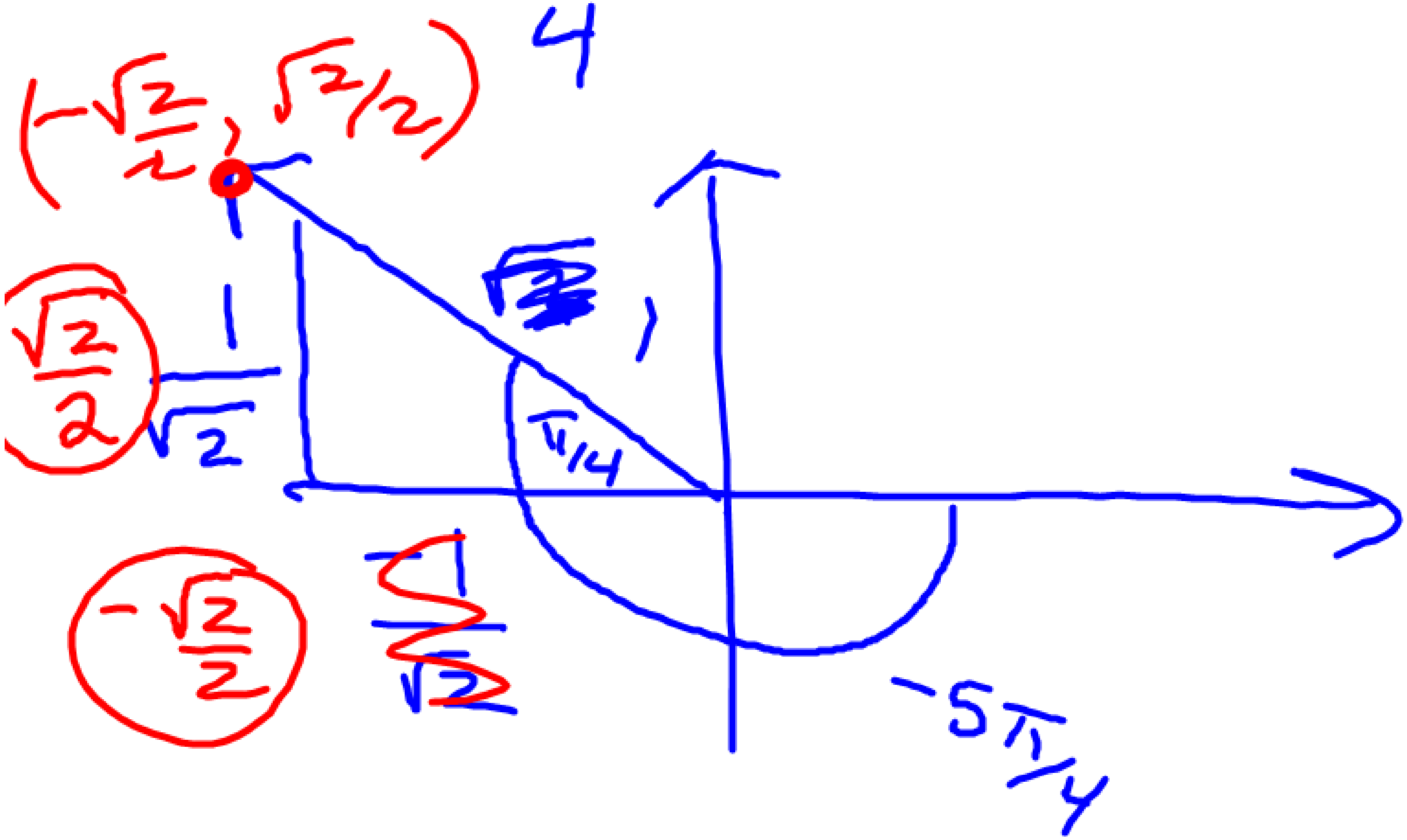
b) $\frac{5\pi}{6}$

$\left(-\sqrt{3}/2, 1/2\right)$



c) $\frac{-5\pi}{4}$

c) $-\frac{5\pi}{4}$


$\left(-\frac{\sqrt{2}}{2}, \sqrt{2}/2\right)$
$\frac{1}{\sqrt{2}}$
$\frac{\sqrt{2}}{2}$
$\frac{-1}{\sqrt{2}}$
$-\frac{\sqrt{2}}{2}$
$\pi/4$
$-5\pi/4$

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1, 2a,c,e, 3,4,5