4.2 Cosine and Sine Law for Obtuse Triangles

Reminder from 4.1

- For any angle θ,

30° 150°

- $\sin\theta = \sin(180° - \theta)$

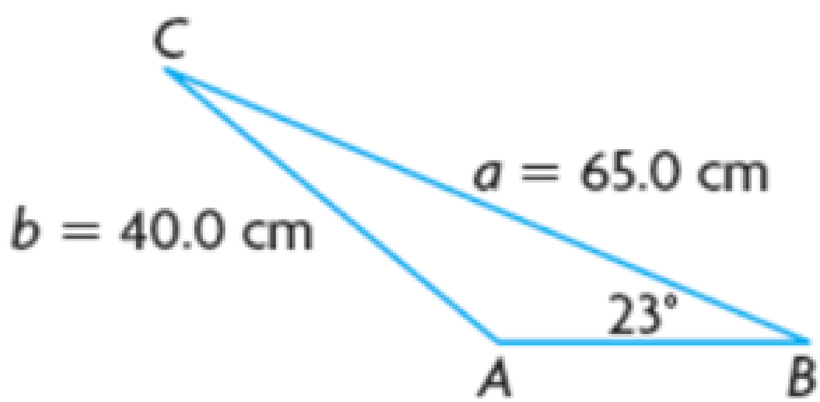
cos 30° ~ cos 150°

- $\cos\theta = -\cos(180° - \theta)$

- $\tan\theta = -\tan(180° - \theta)$

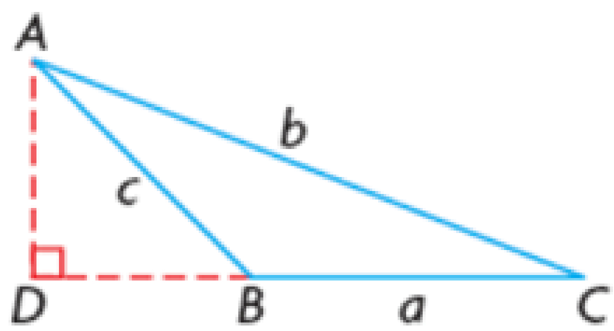
Example

- Draw ΔABC such that <B = 23°, a = 65.0cm and b = 40.0cm.
- Solve for <A
- *Did you get <A = 140.58°???*
- *Did your picture look like this?*



Proof – Sin Law obtuse triangles

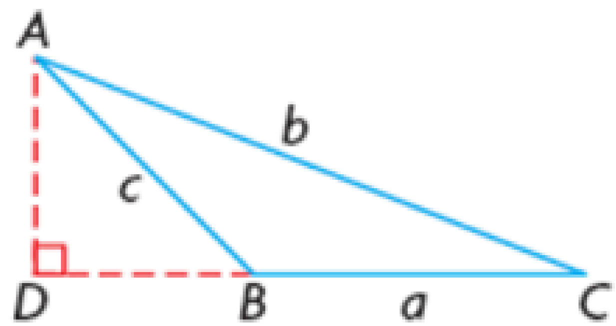
Here we have obtuse triangle ΔABC
We've extended the line BC to form right triangle ΔADC



Now we have two right triangles
ΔADC
ΔABD
Write an equation for each triangle using sine (remember SOH CAH TOA)

Proof – Sin Law obtuse triangles cont'd

ΔADC



ΔABD

- $\sin ACD = \frac{opp}{hyp}$
- $\sin ACD = \frac{AD}{b}$
- $AD = (\sin ACD)(b)$

- $\sin ABD = \frac{opp}{hyp}$
- $\sin ABD = \frac{AD}{c}$
- $AD = (\sin ABD)(c)$

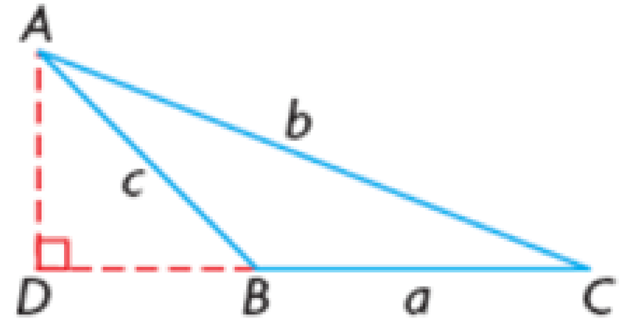
$$(\sin ACD)(b) = (\sin ABD)(c)$$

$$\boldsymbol{\frac{\sin ACD}{c} = \frac{\sin ABD}{b}}$$

Proof – Sin Law obtuse triangles cont'd

- ***KEY IDEA!!!*** *is angle <ABD part of the triangle ΔABC that we're trying to verify the sin law for???*
- *NO, but we know that* $\sin x = \sin(180 - x)$
- *Therefore:*
  - $\sin ABD = \sin(180 - ABD)$
  - $\sin ABD = \sin(ABC)$

$$\frac{\sin ACB}{c} = \frac{\sin ABC}{b}$$

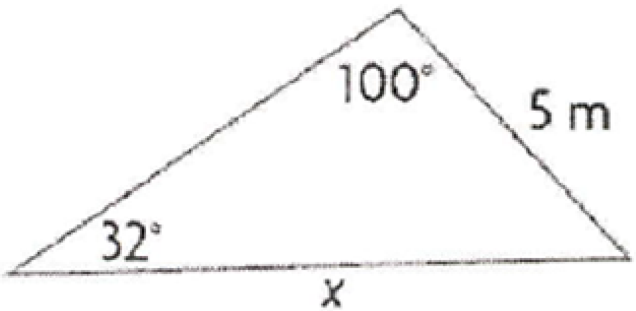


Moving Forward

Now that we know the Sine Law holds true for obtuse triangles, lets apply it

Example

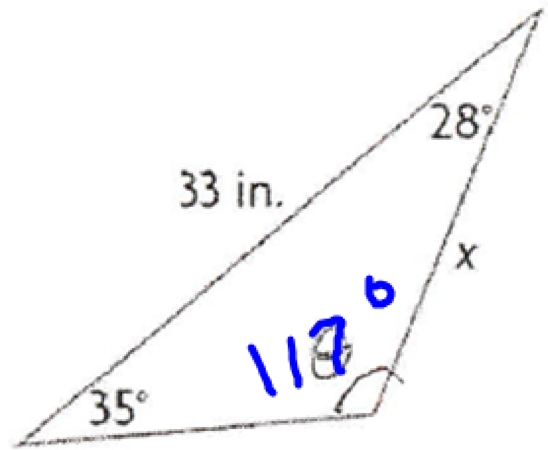
Solve for x and theta in the two diagrams below.



$$\frac{x}{\sin 100^\circ} = \frac{5}{\sin 32^\circ}$$

$$x = \frac{5(\sin 100^\circ)}{\sin 32^\circ}$$

$$x = 9.3\text{m}$$



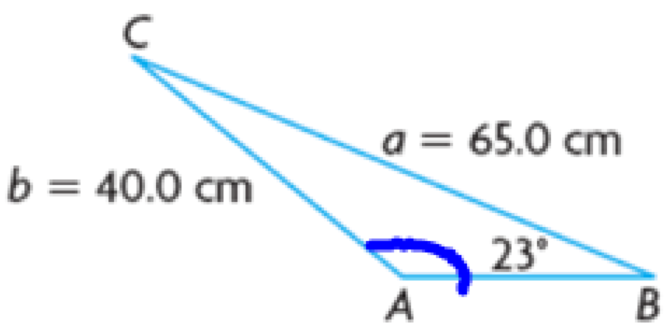
$$\frac{33}{\sin 117^\circ} = \frac{x}{\sin 35^\circ}$$

$$\frac{(\sin 35^\circ)33}{\sin 117^\circ} = x$$

$$21.2\text{ in} = x$$

Example 1 (p. 165)

* In an obtuse triangle, $\angle B$ measures 23.0° and its opposite side, b, has a length of 40.0 cm. Side a is the longest side of the triangle, with a length of 65.0 cm. Determine the measure of $\angle A$ to the nearest tenth of a degree.



$$\frac{\sin 23^\circ}{40} = \frac{\sin A}{65}$$

$$\frac{65(\sin 23^\circ)}{40} = \sin A$$

$$0.6349 = \sin A$$

$$\sin^{-1}(0.6349) = A$$

$A = 39.4^\circ$

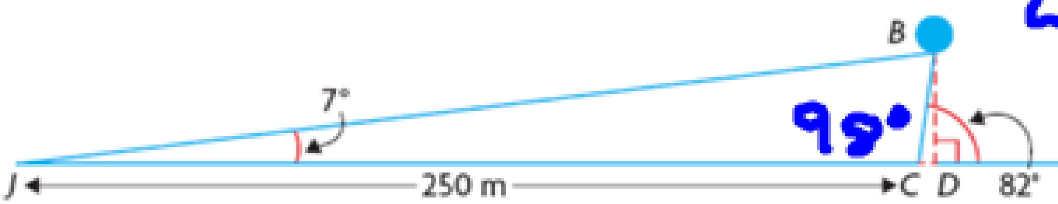
$A = 180 - 39.4^\circ$

$A = 140.6^\circ$

Example 2 (p. 166)

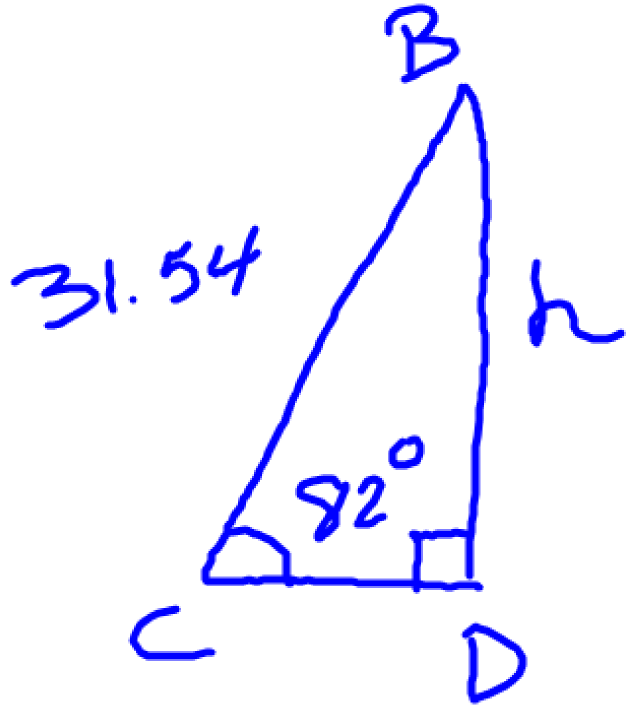
Colleen and Juan observed a tethered balloon advertising the opening of a new fitness centre. They were 250 m apart, joined by a line that passed directly below the balloon, and were on the same side of the balloon. Juan observed the balloon at an angle of elevation of 7° while Colleen observed the balloon at an angle of elevation of 82°. Determine the height of the balloon to the nearest metre.

$\angle B = 75^\circ$



$$\frac{BC}{\sin 7^\circ} = \frac{250}{\sin 75^\circ}$$

$$BC = \frac{250 \sin 7^\circ}{\sin 75^\circ} = 31.54\text{m}$$



$$\sin 82^\circ = \frac{h}{31.54}$$

$$(31.54)\sin 82^\circ = h$$

$$31.2\text{m} = h$$

Homework

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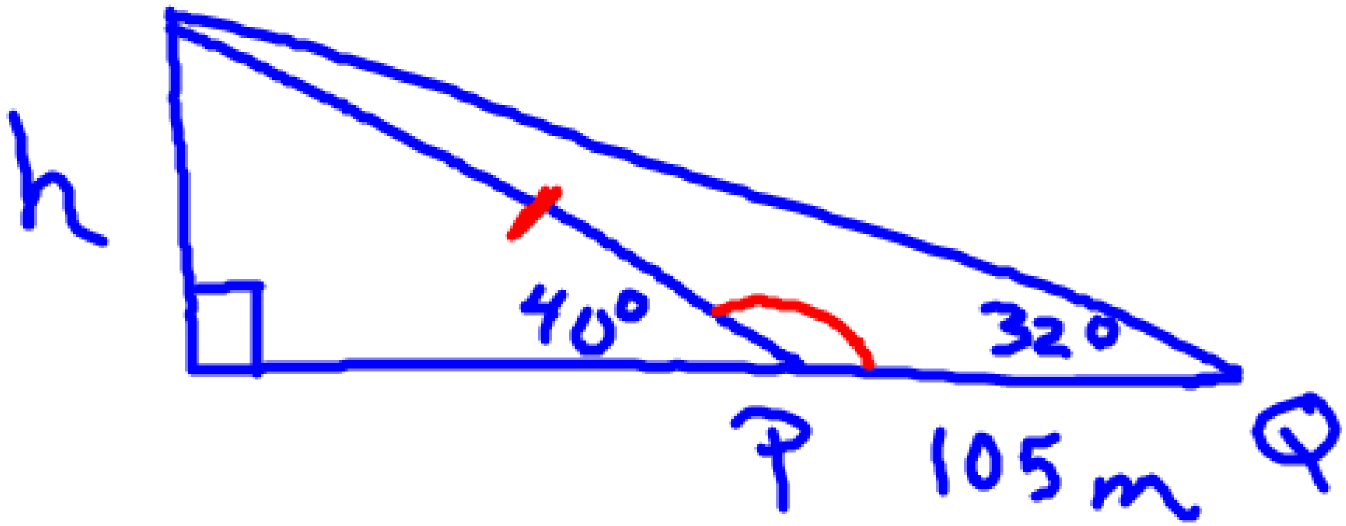


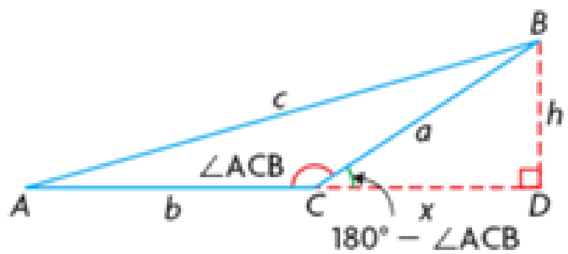
Going Deeper

We've proven the Sine Law for obtuse triangles, now lets prove the Cosine Law

Proof – Cosine Law obtuse triangles

Here we have obtuse triangle ΔABC

We've extended the line AC to form right triangle ΔADC



Now we have two right triangles

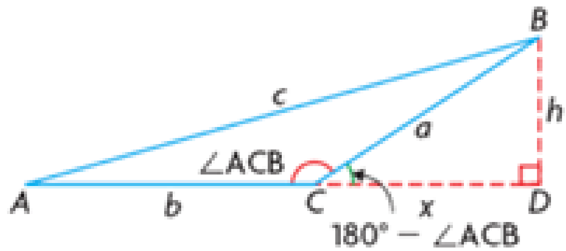
ΔABD

ΔCBD

Write an equation for each triangle using Pythagorean theorem

Proof – Cosine Law obtuse triangles cont'd

ΔABD **ΔCBD**



- $c^2 = h^2 + (b + x)^2$
- $h^2 = c^2 - (b + x)^2$

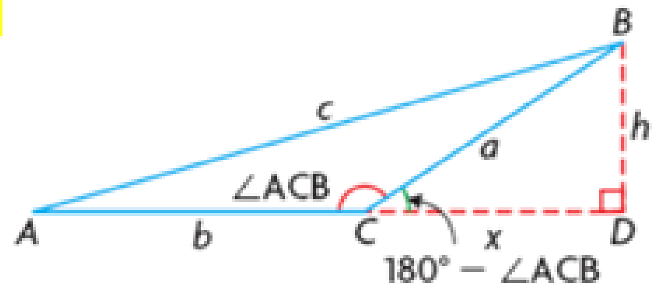
- $a^2 = h^2 + x^2$
- $h^2 = a^2 - x^2$

$$c^2 - (b + x)^2 = a^2 - x^2$$

$$c^2 = a^2 + b^2 + 2bx$$

Proof – Cosine Law obtuse triangles cont'd

- $c^2 = a^2 + b^2 + 2bx$

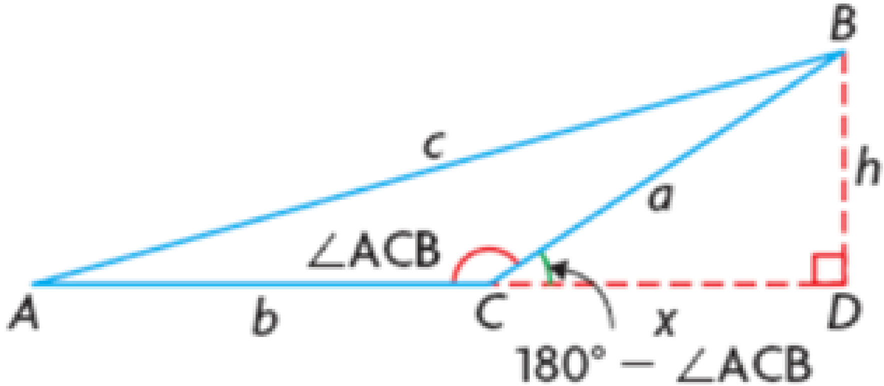


- *x is not part of ΔABC so we need to replace it*
- $\cos(180 - < ACB) = \frac{x}{a}$
- $x = (\cos(180 -< ACB))(a)$

  $-\cos(180 -< ACB) = -\cos ACB$

- $x = -(\cos ACB)(a)$

Proof – Cosine Law obtuse triangles cont'd



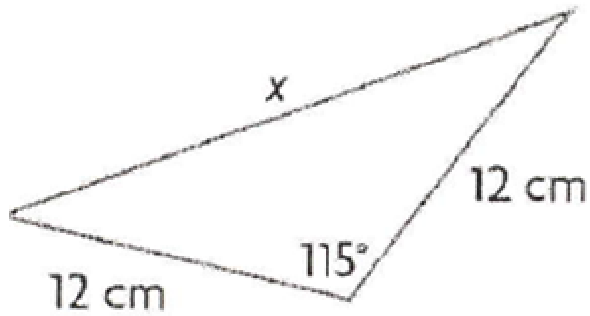
- $c^2 = a^2 + b^2 + 2bx$
- $x = -(\cos ACB)(a)$

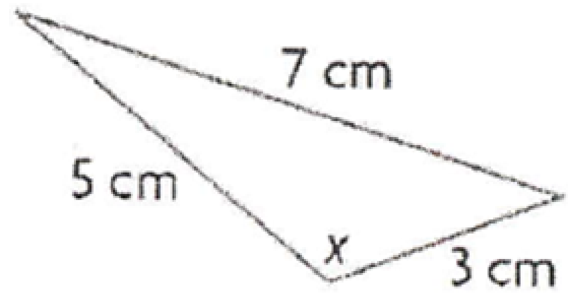
- $\boldsymbol{c^2 = a^2 + b^2 - 2ba(\cos C)}$

Moving Forward

Now that we know the Cosine Law holds true for obtuse triangles, lets apply it

Solve for x





$$x^2 = (12)^2 + (12)^2 - 2(12)(12)(\cos 115^\circ)$$

$$x^2 = 144 + 144 - (-121.714)$$

$$x^2 = 144 + 144 + 121.714$$

$$x^2 = 409.714$$

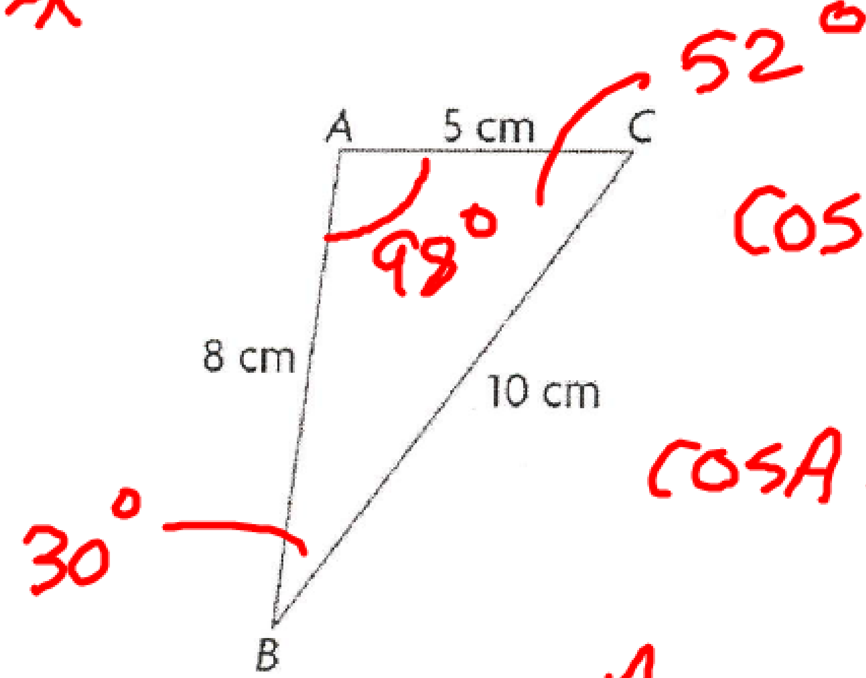
$$x = 20.2$$

$$\cos X = \frac{(5)^2 + (3)^2 - (7)^2}{2(5)(3)}$$

$$\cos X = -0.5$$

$$x = 120^\circ$$

* Solve for all missing angles



$$\cos A = \frac{(8)^2 + (5)^2 - (10)^2}{2(8)(5)}$$

$$\cos A = \frac{-11}{80}$$

$$A = \cos^{-1}\left(-11/80\right)$$

$$A = 98°$$

* ***If given SSS situation, solve largest angle first***

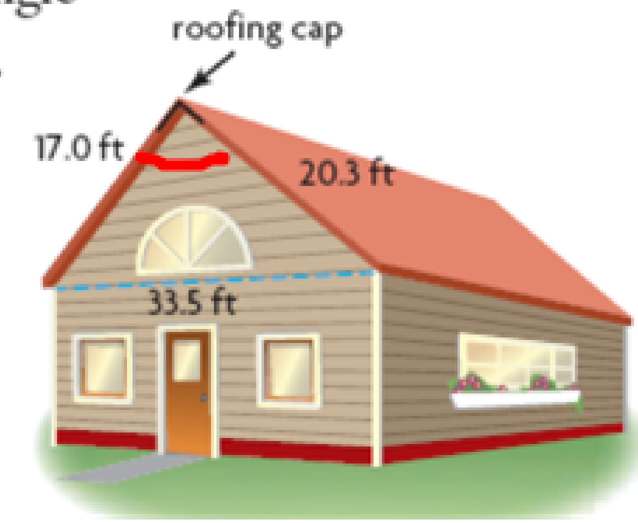
$$\frac{\sin C}{8} = \frac{\sin 98^\circ}{10}$$

$$\sin C = 0.7922$$

$$C = 52^\circ$$

Example 4 (p. 168)

The roof of a house consists of two slanted sections, as shown. A roofing cap is being made to fit the crown of the roof, where the two slanted sections meet. Determine the measure of the angle needed for the roofing cap, to the nearest tenth of a degree.

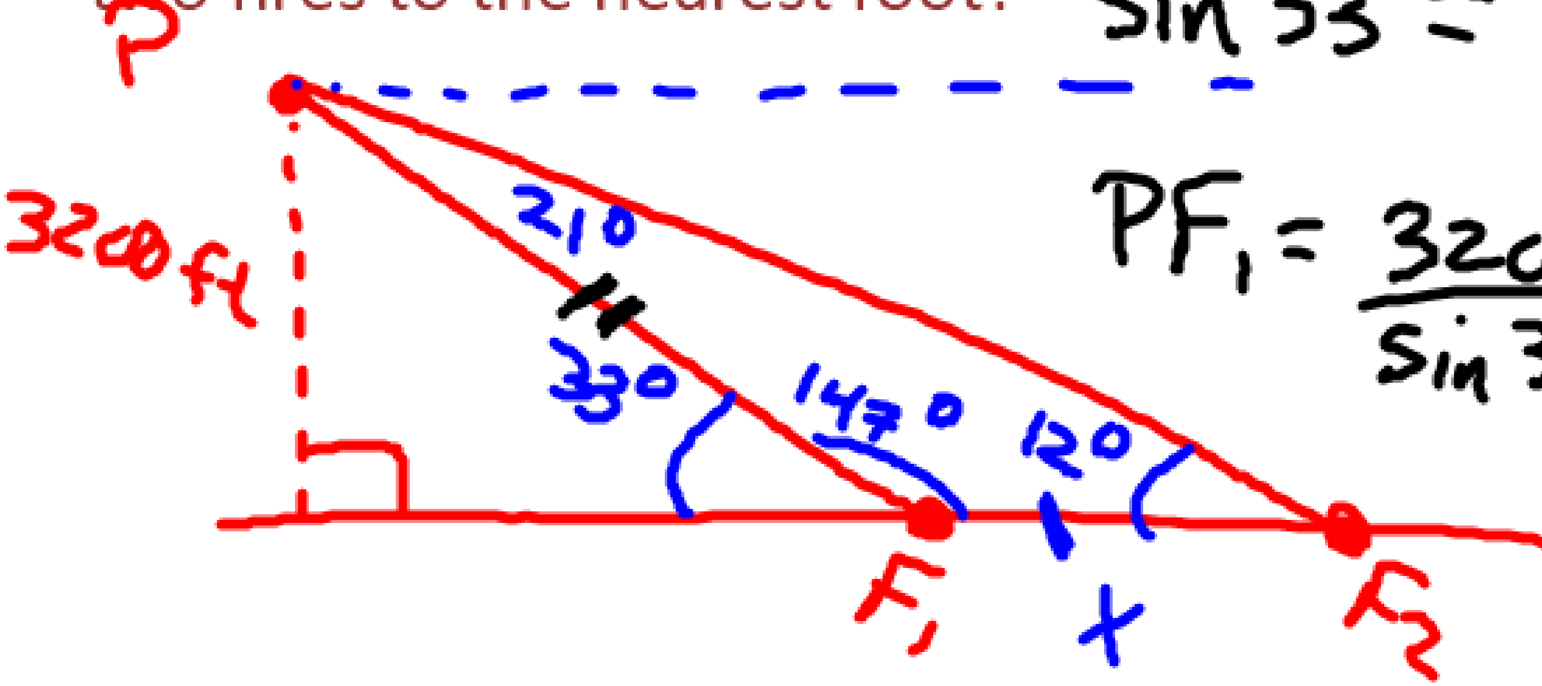


Example 4 (cont'd)

Your Turn: determine the angle of elevation for each roof section.

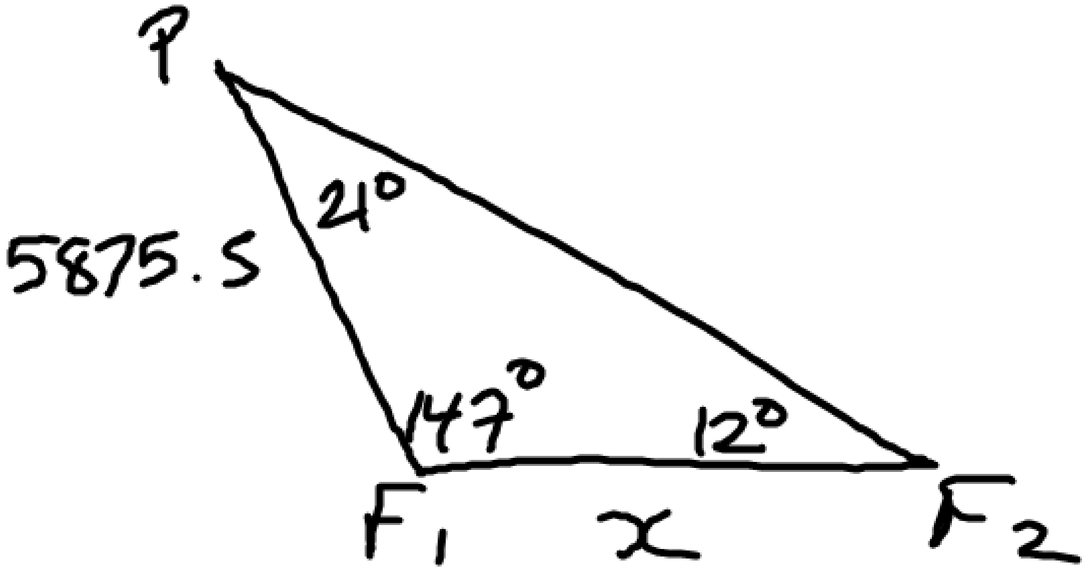
Example

An airplane is flying directly toward two forest fires. From the airplane, the angle of depression to one fire is 33° and 12° to the other fire. The airplane is flying at an altitude of 3200 ft. What is the distance between the two fires to the nearest foot?



$$\sin 33^\circ = \frac{3200}{PF_1}$$

$$PF_1 = \frac{3200}{\sin 33^\circ} = 5875.5$$



$$\frac{x}{\sin 21^\circ} = \frac{5875.5}{\sin 12^\circ}$$

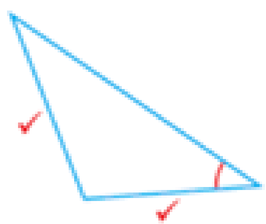
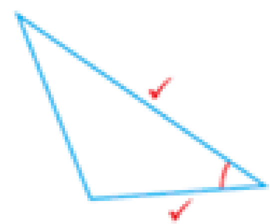
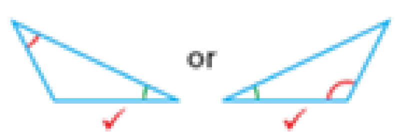
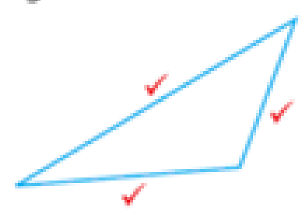
$x = 10127.4$ feet

Key Ideas

The sine law and cosine law can be used to determine unknown side lengths and angle measures in obtuse triangles

Need to Know

The sine law and cosine law are used with obtuse triangles in the same way they are used with acute triangles

Use the sine law when you know ...	Use the cosine law when you know ...
- the lengths of two sides and the measure of the angle that is opposite a known side 	- the lengths of two sides and the measure of the contained angle 
- the measures of two angles and the length of any side 	- the lengths of all three sides 

WARNING!!!

- *Be careful when using the sine law to determine the measure of an angle. The inverse sine of a ratio always gives an acute angle, but the supplementary angle has the same ratio. You must decide whether the acute angle, θ, or the obtuse angle, $(180- \theta)$, is the correct angle for your triangle.*

- ***Hint: the biggest angle is always opposite the longest side***

Homework

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 # 11, 13 (from before)

3 (a, b), 4 (a, b), 5, 6b, 9, 10,

 # 17 – won't be easy, give it a try!!!