4.2 Derivatives of Reciprocal Trigonometric Functions

Recall:

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$y = \sin(3x)$

$y' = (\cos(3x)) \cdot 3 = 3\cos 3x$

Let's develop the reciprocal trigonometric derivatives

$y = \csc x$

$y = \frac{1}{\sin x}$

$y = (\sin x)^{-1}$

$y' = -(\sin x)^{-2} \cdot \cos x$

$= -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{(\sin x)(\sin x)} = -\csc x \cot x$

$$y = \sec x$$

$$y = \cot x$$

Derivatives of Reciprocal Trig Functions

$$\frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}\sec u = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}\cot u = -\csc^2 u \frac{du}{dx}$$

Ex. 1 Find the derivative of the following:

a) $y = \csc(x^2 + 2)$

$y' = (-\csc(x^2+2)\cot(x^2+2))\,2x$

$y' = -2x\csc(x^2+2)\cot(x^2+2)$

$$b)\ y = \sec^2(2x)$$

$y = (\sec 2x)^2$

$y' = 2(\sec 2x)^1 \cdot (\sec 2x \tan 2x)\, 2$

$y' = 4 \sec^2 2x \tan 2x$

c) $y = \cot(3x^2 - 4x)$

$y' = (-\csc^2(3x^2 - 4x))(6x - 4)$

$y' = -2(3x - 2)\csc^2(3x^2 - 4x)$

d) $y = e^{3x}\csc 5x$

$y' = e^{3x}(-\csc 5x \cot 5x)\cdot 5 + (\csc 5x)e^{3x}\cdot 3$

$= -5e^{3x}\csc 5x \cot 5x + 3e^{3x}\csc 5x$

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$= e^{3x}\csc 5x\,(-5\cot 5x + 3)$

e) $y = \dfrac{2x}{\cot 3x}$

$$y' = \frac{(\cot 3x)(2) - 2x(-\csc^2 3x)\cdot 3}{\cot^2 3x}$$

$$= \frac{2\cot 3x + 6x\csc^2 3x}{\cot^2 3x}$$

f) $x^3 + 2y = \sec(4y^3) + 9$

$$3x^2 + 2\frac{dy}{dx} = \sec(4y^3)\tan(4y^3) \cdot 12y^2 \frac{dy}{dx}$$

$$3x^2 = 12y^2 \sec(4y^3)\tan(4y^3)\frac{dy}{dx} - 2\frac{dy}{dx}$$

$$3x^2 = \frac{dy}{dx}\left(12y^2\sec(4y^3)\tan(4y^3) - 2\right)$$

$$\frac{3x^2}{12y^2\sec(4y^3)\tan(4y^3) - 2} = \frac{dy}{dx}$$

Assignment
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