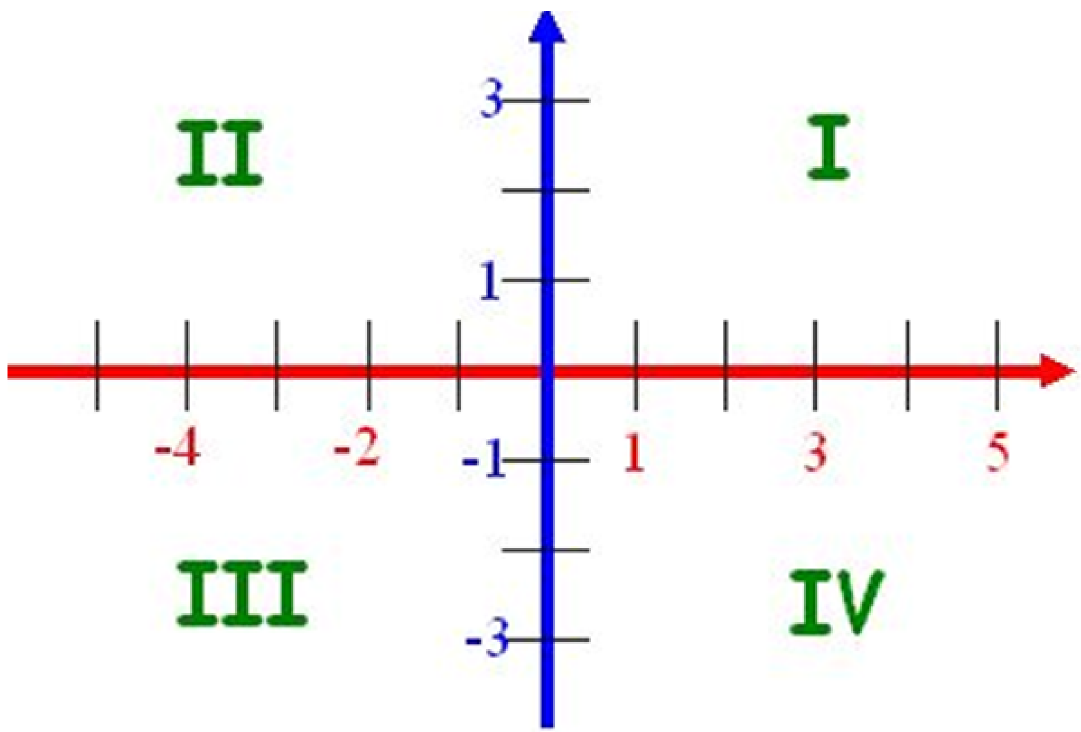
Chapter 4
Trigonometry and the Unit Circle

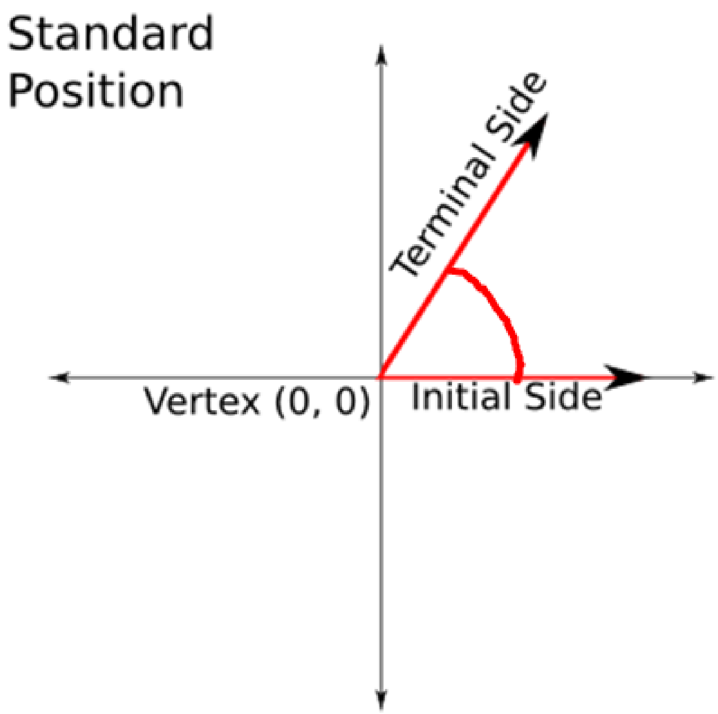
4.1 Angles and Angle Measure

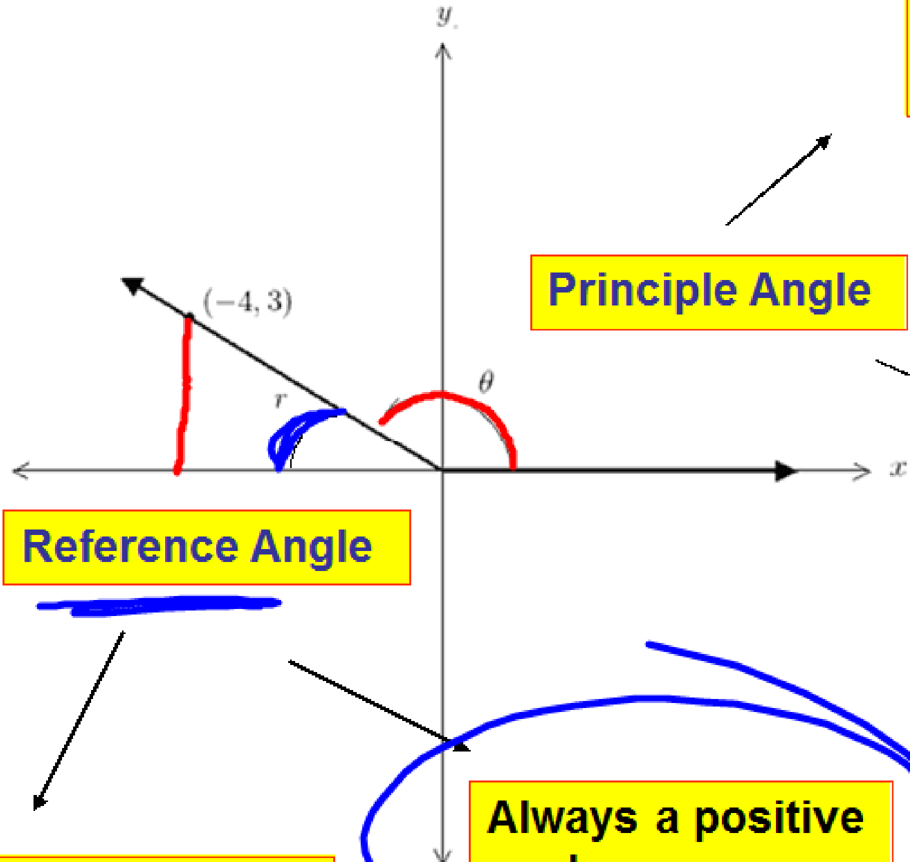
Quadrant Numbers



Angles

An **angle** is formed by one ray revolving about a stationary ray.





Principle Angle

The angle formed between the initial side and the terminal side in standard position.

Can be a positive or negative angle.

+ , −

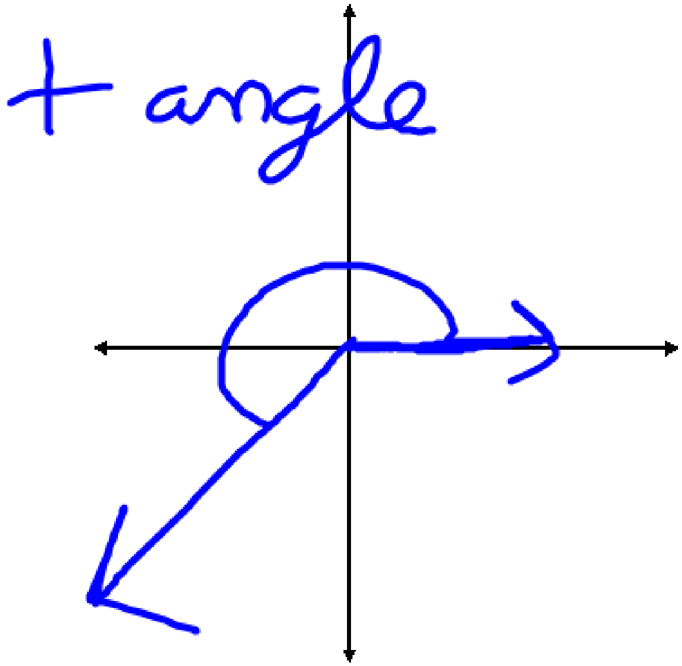
Reference Angle

The angle formed between the terminal side and the x-axis.

Always a positive angle.

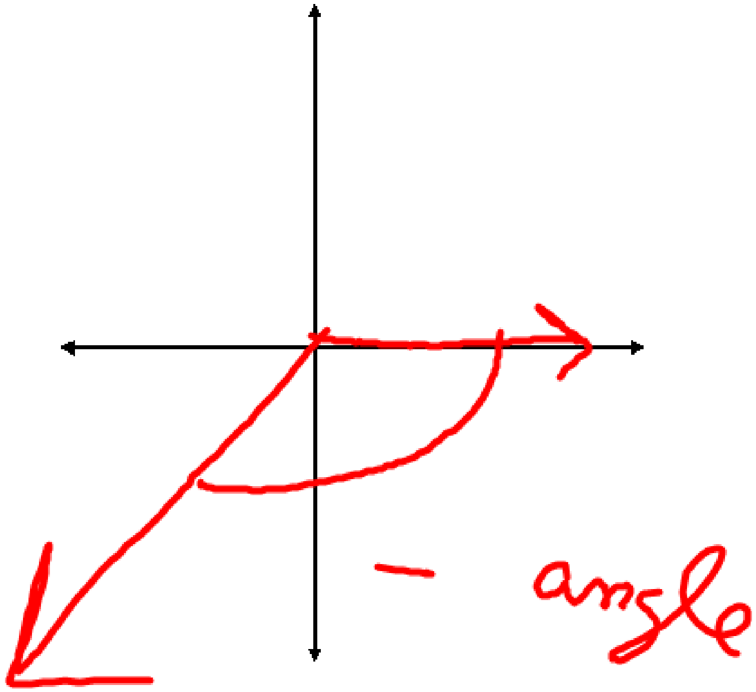
Positive Angles

Counterclockwise rotation



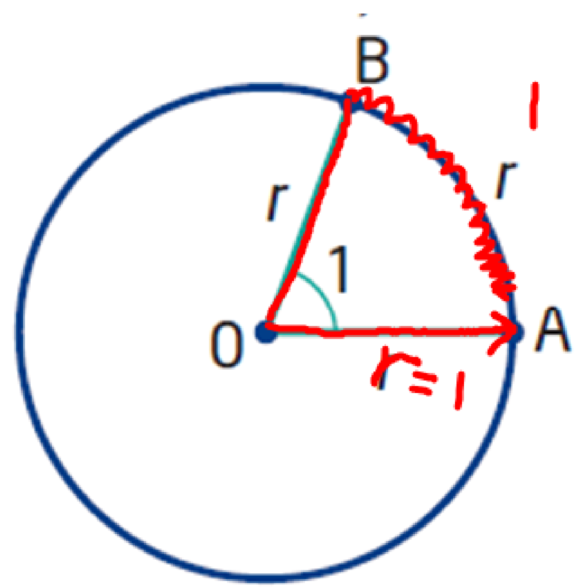
Negative Angles

Clockwise rotation



Thirty years ago, everyone measured temperature in Fahrenheit, today we use Celsius, both accomplish the same thing through different methods. A **radian** is just another way to measure the distance around a circle.

one radian is the measure of the central angle subtended in a circle by an arc equal in length to the radius of the circle



$\text{One revolution of a circle is } 360^\circ.$

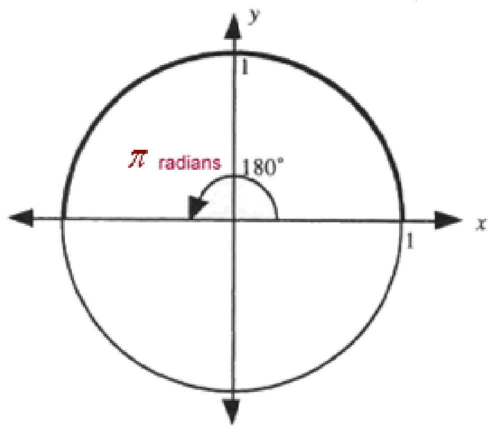
$\text{The circumference of a cirlce is } 2\pi r.$

$\text{On a unit circle the radius is } 1.$

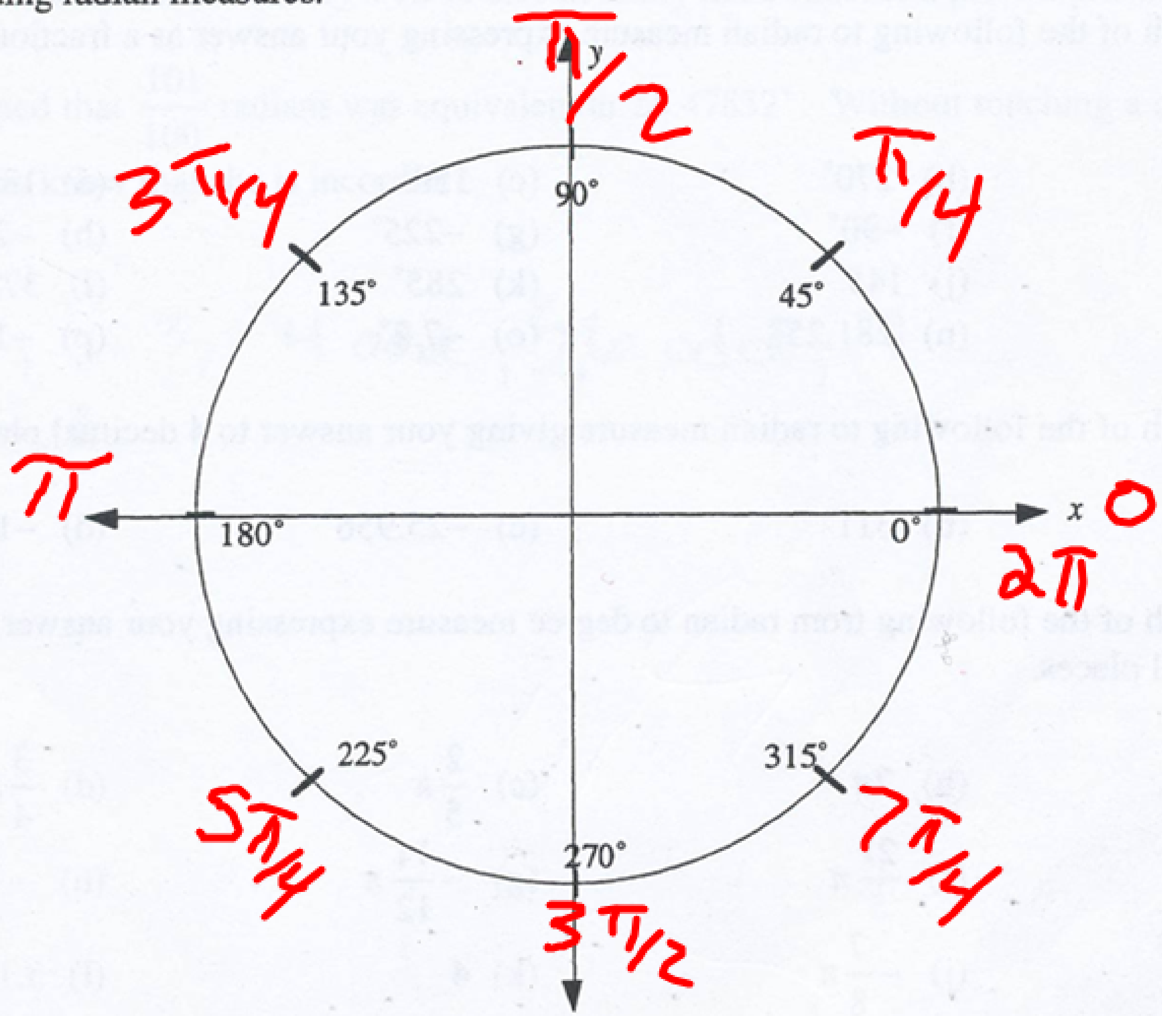
$\text{Therefore the circumference is } 2\pi \text{ radians.}$

$\text{Therefore } 360^\circ = 2\pi \text{ radians}$

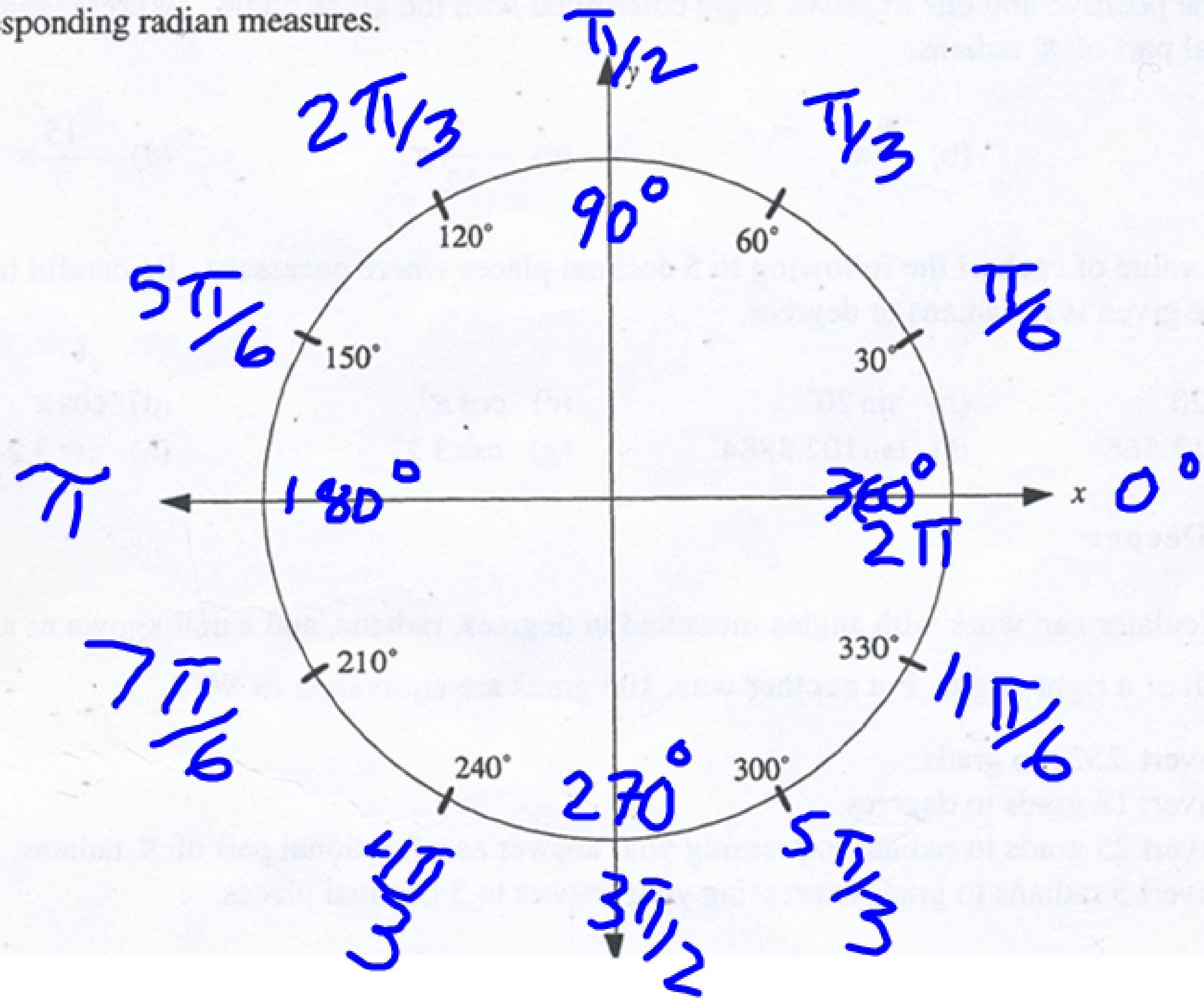
$\text{Therefore } 180^\circ = \pi \text{ radians.}$



1. The figure shows several angles marked in degrees. Without using a written proportion, find the corresponding radian measures.



2. The figure shows several angles marked in degrees. Without using a written proportion, find the corresponding radian measures.

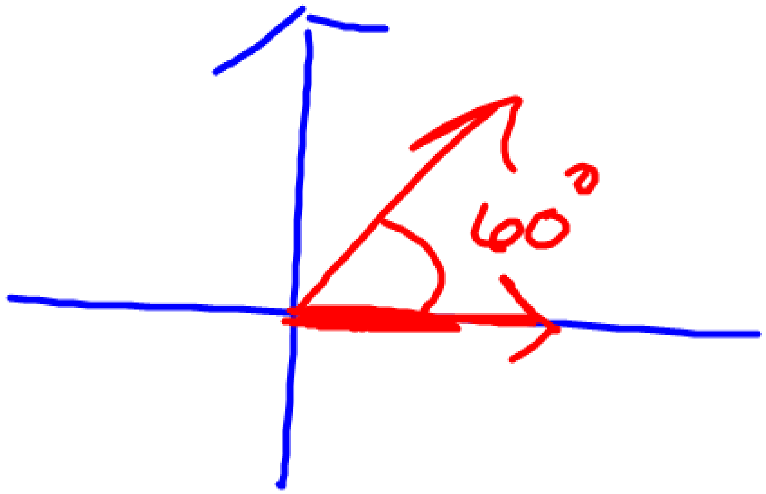


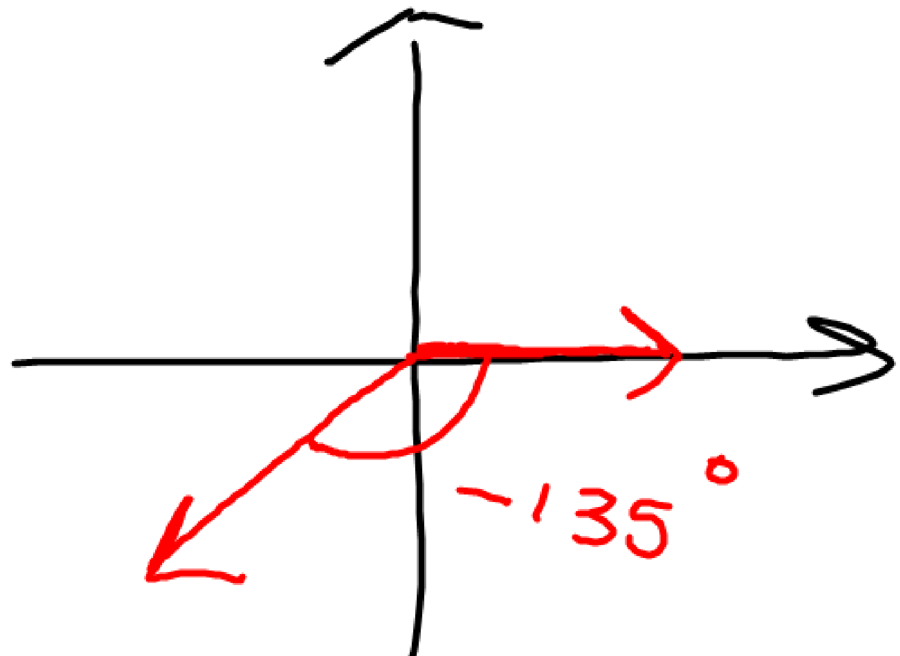
$\frac{\pi}{180^\circ}$ $\frac{180^\circ}{\pi}$

Converting from degrees to radians and radians to degrees.

Draw each angle in standard position then convert to **radian measure.**

a) 60° b) -135°





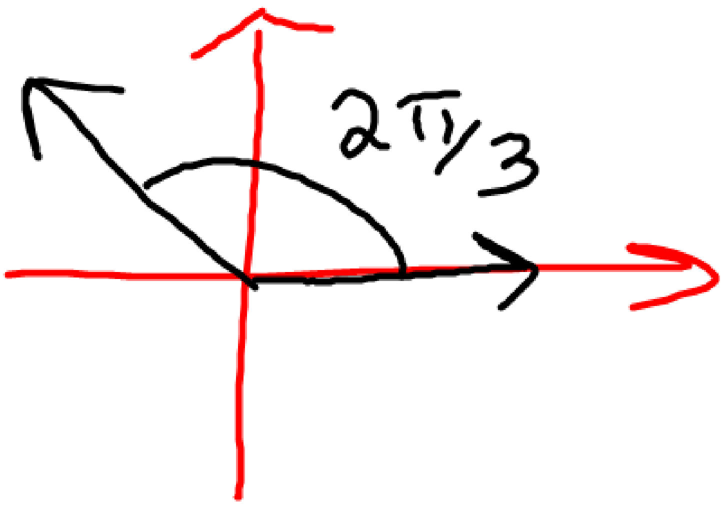
$$60^\circ \cdot \frac{\pi}{180^\circ} = \frac{60\pi}{180} = \frac{\pi}{3}$$

$$-135^\circ \cdot \frac{\pi}{180^\circ} = -\frac{3\pi}{4}$$

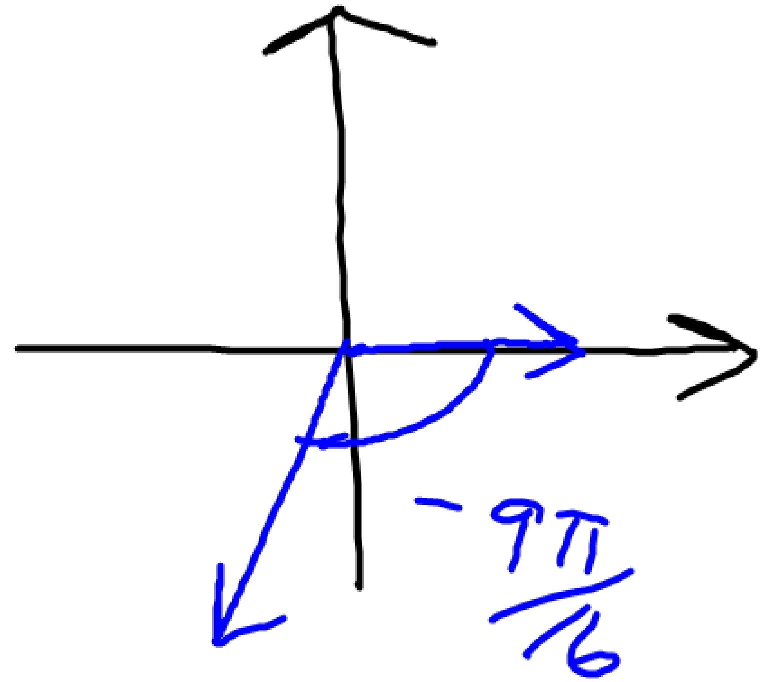
Draw in standard position then convert to **degree measure.**

a) $\frac{2\pi}{3}$

b) $\frac{-9\pi}{16}$



$$\frac{2\pi}{3}\cdot\frac{180^\circ}{\pi} = 120^\circ$$

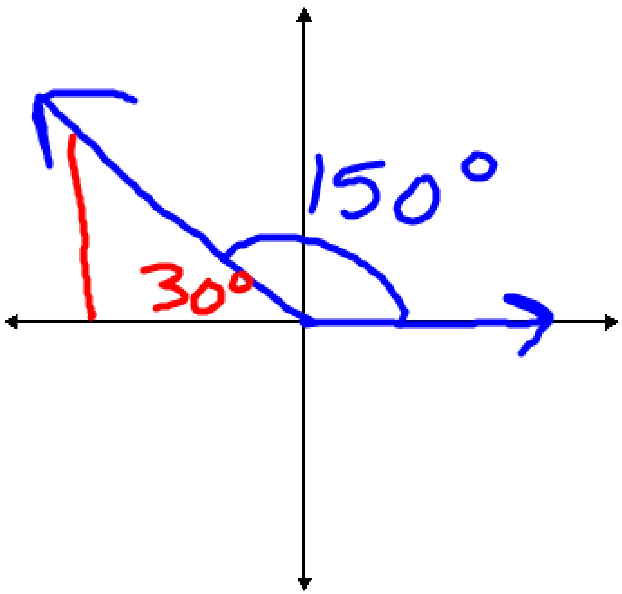


$$\frac{-9\pi}{16}\cdot\frac{180^\circ}{\pi}$$

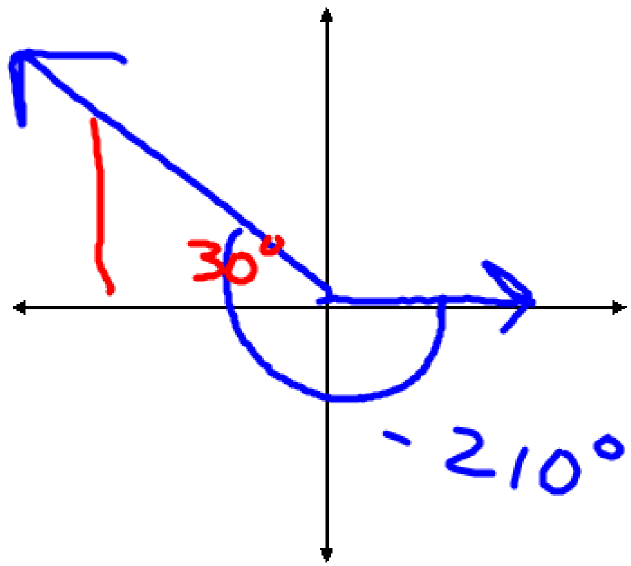
$$= -101.25^\circ$$

Coterminal Angles

- Draw 150 º



Draw -210º



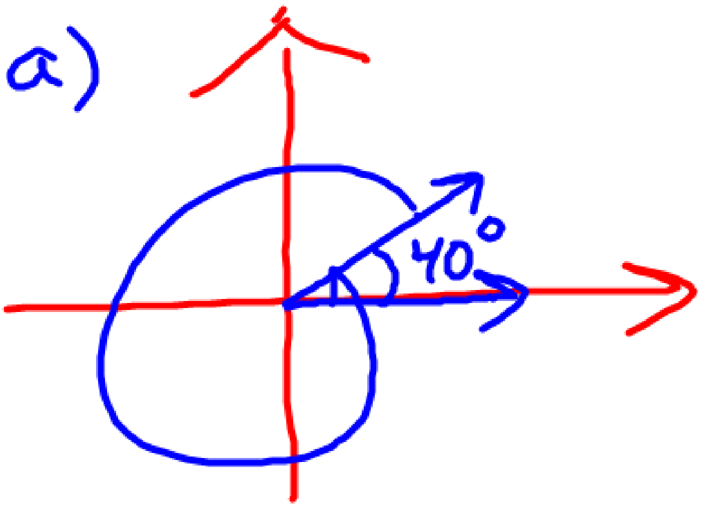
Definition: Two angles are said to be **coterminal** if they have the same terminal arm but different principal angles.

Your Turn

Determine one positive and one negative angle measure that is coterminal to

a) 40° b) -430° c) $\frac{5\pi}{4}$

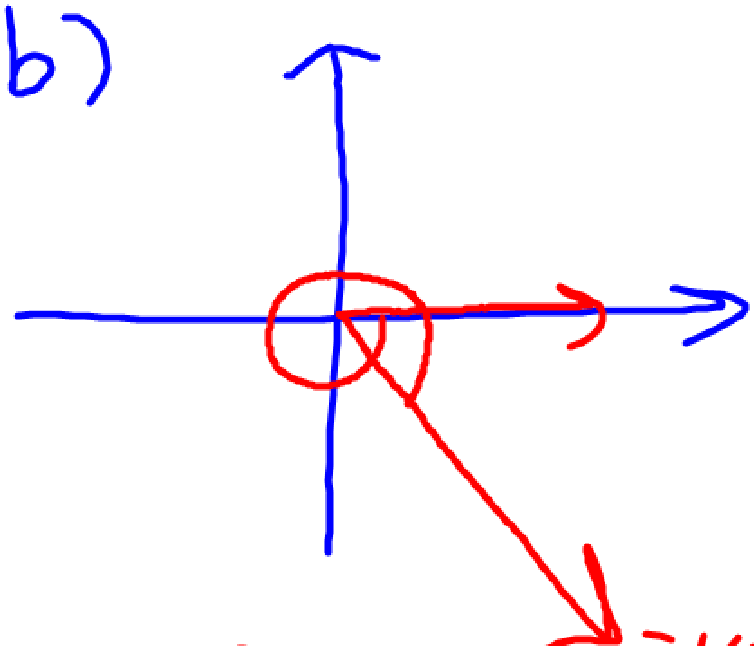
a)



$40^{\circ} + 360^{\circ} = 400^{\circ}$

$40^{\circ} - 360^{\circ} = -320^{\circ}$

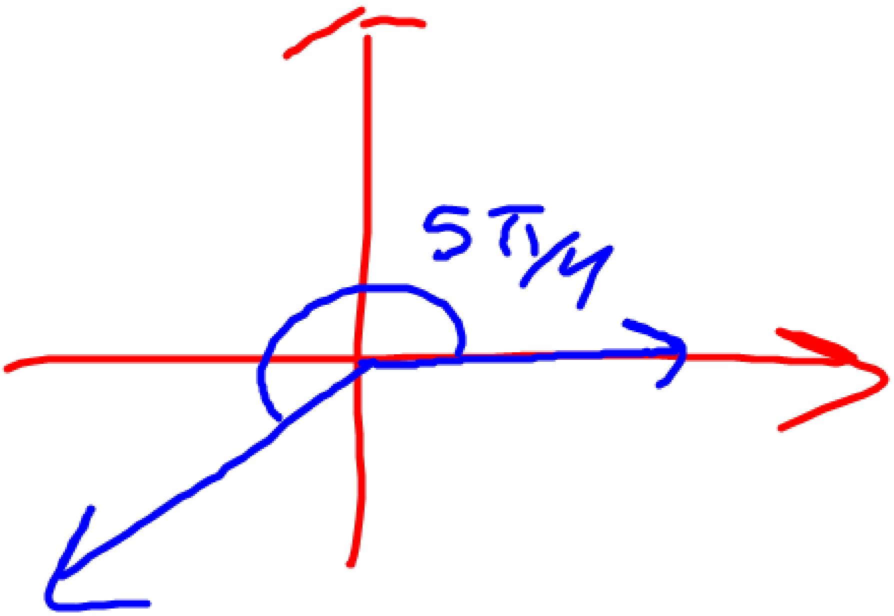
b)



$-430^{\circ} + 360^{\circ} = -70^{\circ}$

$-70^{\circ} + 360^{\circ} = 290^{\circ}$

c) $\frac{5\pi}{4}$



$$\frac{5\pi}{4} + 2\pi$$

$$\frac{5\pi}{4} + \frac{8\pi}{4} = 13\pi/4$$

$$\frac{5\pi}{4} - \frac{8\pi}{4} = -3\pi/4$$

Coterminal Angles in General Form

For 40° coterminal angles could be found by:

Positive	Negative
40 +360(1)=400	40-360(1)=-320
40+360(2)=760	40-360(2)=-680

$40^{\circ} \pm 360^{\circ} n, \; n \in N$

In general terms it would be:

$40 \pm 360(n), n \text{ is any natural number}$

$40^{\circ} + 360^{\circ} n \quad n \in I, n \neq 0$

For $\frac{2\pi}{3}$ radians coterminal angles could be found by adding or subtracting 2π

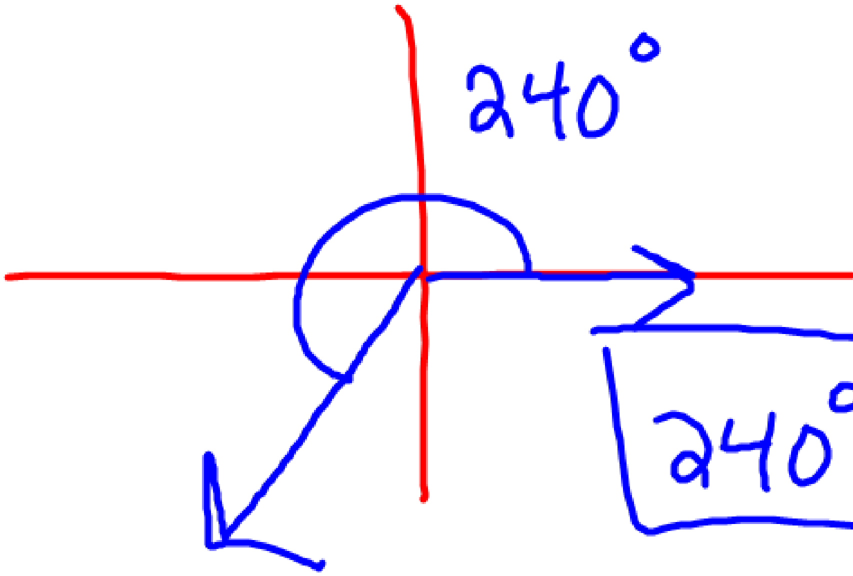
Therefore the coterminal angles in general form can be written as:

$$\frac{2\pi}{3} \pm 2\pi n \ \textit{ where n is any natural number}$$

$$\frac{2\pi}{3} \pm 2\pi n, \; n \in N$$

Example 1:

Express the angles coterminal with 240° in general form, then identify the angles coterminal with 240° that satisfy the domain $-720 \leq \theta \leq 720$.



$$240^{\circ} \pm 360^{\circ}n,\ n \in N$$

$-480^{\circ}, -120^{\circ}, 240^{\circ}, 600^{\circ}$

$6\pi/3$

Example 2:

Express the angles coterminal with $\frac{2\pi}{3}$ in general form. Identify the angles coterminal with $\frac{2\pi}{3}$ in the domain $-2\pi \leq \theta \leq 6\pi$.

$$\frac{2\pi}{3} \pm 2\pi n, \; n \in N$$

$$-\frac{4\pi}{3}, \; \boxed{\frac{2\pi}{3}}, \; \frac{8\pi}{3}, \; \frac{14\pi}{3}$$

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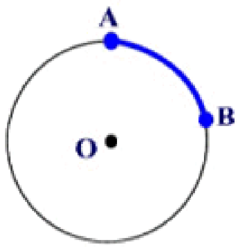
2a,c,e,3a,c,e,4a,c,e,5a,c,e,7,11b,c,e,g

Arc Length

An **arc** of a circle is a "portion" of the circumference of the circle.



The **length of an arc** is simply the length of its "portion" of the circumference.
The arc length is actually **proportional to the radius.**

To determine the arc length of a circle we use the following formula:

$$a = \theta r$$

where a = arc length

θ = measure of the central angle in radians

r = radius.

Be sure that *a* and *r* have the same units.

Arc Length Demo

http://www.mathopenref.com/arclength.html

Example 3:

Determine the arc length subtended by a central angle of $\frac{2\pi}{3}$ and a radius of 20 cm.

Example 4:

Determine the arc length subtended by a central angle of 195° and a radius of 15 mm

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#'s 12,13,14a