

## 4.10 Implicit Differentiation

$$y = \sqrt{x^2 - 4x} \quad \text{explicit}$$

$$x^2 y - 4y^3 + 3xy = 7 \quad \text{implicit}$$

## 4.10 Implicit Differentiation

### Learning Targets:

1. SWBAT recognize when implicit differentiation is used.
2. SWBAT find the derivative using implicit differentiation..



So far we have described functions by expressing one variable **explicitly** in terms of another variable.

$$y = x^2 \text{ or } y = \frac{\sqrt{4 - x^2}}{x + 1}$$

Sometimes relations and/or functions are defined **implicitly**.

$$x^2 + y^2 = 25$$

We can still find the slope of a tangent line to this relation using **implicit differentiation**.

Ex.1 If  $x^2 + y^2 = 25$  then :

a) Find  $\frac{dy}{dx}$   $\rightarrow x$  wrt  $t$

b) Find the equation of the tangent line to the circle at the point  $(-4,3)$ .

$$x^2 + y^2 = 25$$

$$\frac{dy}{dy} \cdot \frac{dy}{dx}$$

$$2x + 2y \boxed{\frac{dy}{dx}} = 0$$

$$\frac{\cancel{2y} \frac{dy}{dx}}{\cancel{2y}} = - \frac{\cancel{2x}}{\cancel{2y}}$$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

$$b) \frac{dy}{dx} = -\frac{x}{y}$$

$$(-4, 3)$$

↑  
slope

$$\frac{dy}{dx} = \frac{-(-4)}{3} = \frac{4}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{4}{3}(x + 4)$$

$$y - 3 = \frac{4}{3}x + \frac{16}{3}$$

$$3y - 9 = 4x + 16$$

$$-25 = 4x + 3y$$

# Implicit Differentiation Process

1. Differentiate each term of the equation with respect to  $x$ .
2. Collect the terms with  $dy/dx$  on one side of the equation.
3. Factor out the  $dy/dx$ .
4. Solve for  $dy/dx$ .



Ex.2 Use implicit differentiation to find  $dy/dx$  for the following:

$$2x^5 + x^4y + y^5 = 36$$

$$10x^4 + x^4 \frac{dy}{dx} + y(4x^3) + 5y^4 \frac{dy}{dx} = 0$$

$$10x^4 + x^4 \frac{dy}{dx} + 4x^3y + 5y^4 \frac{dy}{dx} = 0$$
$$x^4 \frac{dy}{dx} + 5y^4 \frac{dy}{dx} = -10x^4 - 4x^3y$$

$$\frac{dy}{dx} \left( \cancel{x^4 + 5y^4} \right) = \frac{-10x^4 - 4x^3y}{x^4 + 5y^4}$$

Ex.3 Find  $dy/dx$  for the following:

$$x^3 - 4x^2y^4 + 3y = 9$$

①

$$3x^2 - 4x^2 \left( 4y^3 \frac{dy}{dx} \right) + y^4 (-8x) + 3 \frac{dy}{dx} = 0$$

$$3x^2 - 16x^2y^3 \frac{dy}{dx} - 8xy^4 + 3 \frac{dy}{dx} = 0$$

②

$$3 \frac{dy}{dx} - 16x^2y^3 \frac{dy}{dx} = 8xy^4 - 3x^2$$

③

$$\frac{dy}{dx} \left( \cancel{3 - 16x^2y^3} \right) = \frac{8xy^4 - 3x^2}{\textcircled{4} 3 - 16x^2y^3}$$

# Assignment

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#'s 3, 5, 8, 9, 11, 14, 16, 17, 20  
22, 23, 27, 30, 32