3.5 Chapter Review   P. 149 1-20

1. a) $f(0) = -2$
b) $f(3) = 3$
c) $f(-3) =$ undefined
d) $\lim_{x \to -3^+} f(x) = \infty$
e) $\lim_{x \to -3^-} f(x) = -\infty$
f) $\lim_{x \to -3} f(x) =$ does not exist
g) $\lim_{x \to 2} f(x) = 1$
h) $\lim_{x \to 0^+} f(x) = -2$
i) $\lim_{x \to 0^-} f(x) = 0$
j) $\lim_{x \to 0} f(x) =$ does not exist
k) $\lim_{x \to 3} f(x) = 1$
l) $\lim_{x \to 5} f(x) = 0$
m) $\lim_{x \to -\infty} f(x) = 0$
n) $\lim_{x \to \infty} f(x) = -\infty$

2. a) $x = -3$ → $3^+ \to \infty$ $3^- \to -\infty$ f(b) does not exist (infinite)
b) $x = 0$ → $0^+ \to -2$ $0^- \to 0$ lim f(b) approaches different values (jum
c) $x = 3$ → $3 = 3$ $3^+ + 3^- = 1$ $\lim_{x \to 3} f(x) \neq f(3)$ (removable)

3. a) $f(x) = \dfrac{\sin\left(\frac{\pi x^2}{2}\right)}{x^2 - 1}$

$x = 1$

$= \dfrac{\sin \frac{\pi}{2}}{0}$

$= \dfrac{1}{0}$

vertical asymptote line
infinite discontinuity

b) $f(x) = \dfrac{x-1}{x^2-1}$

$= \dfrac{0}{0}$

indeterminate

$\lim_{x \to 1} \dfrac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} = \dfrac{1}{2}$

∴ removable discontinuity

c) $f(x) = \dfrac{|x-1|}{x-1}$

$\lim_{x \to 1^+} f(x) = \dfrac{(x-1)}{x-1}$

$= 1$

$\lim_{x \to 1^-} f(x) = \dfrac{-(x-1)}{x-1}$

$= -1$

∴ jump discontinuity

4. $f(x) = \dfrac{x^2 - 4x + 3}{x - 3}$

$\lim_{x \to 3} \dfrac{\cancel{(x-3)}(x-1)}{\cancel{(x-3)}}$

$= 3 - 1 = 2$

5. $f(x) = \dfrac{x\,|x-4|}{x^3 - 2x^2 - 8x}$

$= \dfrac{x\,|x-4|}{x(x-4)(x+2)}$

① At $x = 0$ $f(0) = \frac{0}{0}$ $\lim_{x \to 0} f(x) = \frac{-1}{2}$
removable discontinuity

② At $x = -2$ since $f(-2) = \frac{12}{0}$
infinite discontinuity

③ At $x = 4$ $\lim_{x \to 4^+} f(x) = \frac{1}{6}$ $\lim_{x \to 4^-} f(x) = -\frac{1}{6}$
jump discontinuity

$2x^2-7x-4$
$2x^2-8x+1x-4$
$2x(x-4)+1(x-4)$

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6 a) $\lim_{x\to 5} \frac{x^2-x-12}{x+5}$

$= \frac{(5)^2-(5)-12}{5+5}$

$= \frac{8}{10} = \frac{4}{5}$

b) $\lim_{x\to -4} \frac{2x^2-7x-4}{4-x}$

$\lim_{x\to -4} \frac{(2x+1)(x-4)}{(4-x)}$

$\lim_{x\to -4} -1(2x+1)$

$= -1(2(-4)+1)$

$= 7$

c) $\lim_{x\to -3} \frac{x^2-x-12}{x+3}$

$\lim_{x\to -3} \frac{\cancel{(x+3)}(x-4)}{\cancel{(x+3)}}$

$-3-4 = -7$

d) $\lim_{x\to 1} \frac{x^3-1}{x^4-1}$

$= \lim_{x\to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x^3+x^2+x+1)}$

$= \frac{1+1+1}{1+1+1+1} = \frac{3}{4}$

e) $\lim_{x\to 0} \frac{(5+x)^2-3(5+x)-10}{x}$

$\lim_{x\to 0} \frac{25+10x+x^2-15-3x-10}{x}$

$\lim_{x\to 0} \frac{x^2+7x}{x}$

$\lim_{x\to 0} \frac{\cancel{x}(x+7)}{\cancel{x}}$

$0+7 = 7$

f) $\lim_{x\to 0} \frac{(x-1)^2-1}{3x}$

$\lim_{x\to 0} \frac{x^2-2x+1-1}{3x}$

$\lim_{x\to 0} \frac{x-2}{3}$

$\frac{0-2}{3} = \frac{-2}{3}$

g) $\lim_{x\to 0} \frac{1-\sqrt{x+1}}{x} \cdot \frac{1+\sqrt{x+1}}{1+\sqrt{x+1}}$

$\lim_{x\to 0} \frac{1-(x+1)}{x(1+\sqrt{x+1})}$

$\lim_{x\to 0} \frac{-\cancel{x}}{\cancel{x}(1+\sqrt{x+1})}$

$\frac{-1}{1+\sqrt{0+1}} = \frac{-1}{2}$

h) $\lim_{x\to 3} \frac{3-x}{\sqrt{4-x}-\sqrt{x-2}} \cdot \frac{\sqrt{4-x}+\sqrt{x-2}}{\sqrt{4-x}+\sqrt{x-2}}$

$\lim_{x\to 3} \frac{(3-x)(\sqrt{4-x}+\sqrt{x-2})}{(4-x)-(x-2)}$

$\lim_{x\to 3} \frac{(3-x)(\sqrt{4-x}+\sqrt{x-2})}{6-2x}$

$\lim_{x\to 3} \frac{\cancel{(3-x)}(\sqrt{4-x}+\sqrt{x-2})}{2\cancel{(3-x)}} = \frac{\sqrt{4-3}+\sqrt{3-2}}{2}$

$= \frac{1+1}{2} = 1$

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6. i) $\lim\limits_{x\to\infty} \dfrac{3x^2-x-2}{5x^2+4x+1}$

$= \lim\limits_{x\to\infty} \dfrac{\frac{3x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}}{\frac{5x^2}{x^2}+\frac{4x}{x^2}+\frac{1}{x^2}}$

$= \dfrac{3-0-0}{5+0+0}$

$= \dfrac{3}{5}$

j) $\lim\limits_{x\to\infty} \dfrac{5x^2+1}{x^3+x^2}$

$\lim\limits_{x\to\infty} \dfrac{\frac{5x^2}{x^3}+\frac{1}{x^3}}{\frac{x^3}{x^3}+\frac{x^2}{x^3}}$

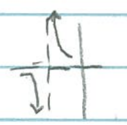
$= \dfrac{0+0}{1+0}$

$= \dfrac{0}{1} = 0$

k) $\lim\limits_{x\to 2^+} \dfrac{1+x^2}{x^2+3x-10}$

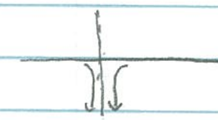
$\lim\limits_{x\to 2^+} \dfrac{1+x^2}{(x-2)(x+5)}$

$= \infty$



l) $\lim\limits_{x\to 0} \dfrac{x-1}{x^2(x+3)}$

$= -\infty$



m) $\lim\limits_{x\to 10^-} \dfrac{|x-10|}{x-10}$

$= \lim\limits_{x\to 10^-} \dfrac{-(x-10)}{x-10}$

$= -1$

n) $\lim\limits_{x\to 3} \dfrac{|x^2-9|}{x-3}$

$= \lim\limits_{x\to 3^+} \dfrac{(x^2-9)}{x-3}$ | $\lim\limits_{x\to 3^-} \dfrac{-(x^2-9)}{x-3}$

$= \lim\limits_{x\to 3^+} x+3 = 6$ | $= \lim\limits_{x\to 3^-} -(x+3) = -6$

No limit exists

o) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{x^2+6x}}{2x-4}$

$= \lim\limits_{x\to -\infty} \dfrac{\sqrt{x^2\left(1+\frac{6x}{x^2}\right)}}{x\left(2-\frac{4}{x}\right)}$

$= \lim\limits_{x\to -\infty} \dfrac{|x|\sqrt{1+\frac{6x}{x^2}}}{x\left(2-\frac{4}{x}\right)}$

$= \lim\limits_{x\to -\infty} \dfrac{-(x)\sqrt{1+\frac{6x}{x^2}}}{x\left(2-\frac{4}{x}\right)}$

$= \dfrac{-1\sqrt{1+0}}{(2-0)} = \dfrac{-1}{2}$

7. $f(x) = \begin{cases} cx+9, & x<4 \\ x^2+3x+1, & x\geq 4 \end{cases}$ is to be continuous

$4c+9 = (4)^2+3(4)+1$

$4c+9 = 29$

$4c = 20$

$c = 5$

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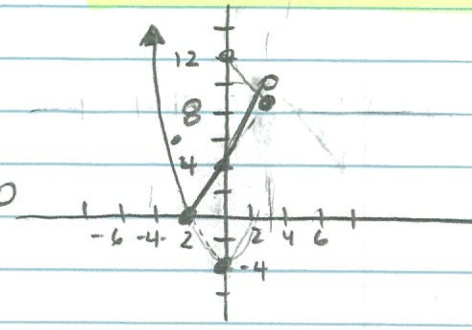
8. $f(x) = \begin{cases} x^2-4, & x<-2 \\ 2x+4, & -2 \le x < 3 \\ 12-x, & x \ge 3 \end{cases}$ (-2,0) (-2,0) (3,10) (3,9)



a) $\lim_{x\to -2^-} f(x) = 0$

b) $\lim_{x\to -2^+} f(x) = 0$

c) $\lim_{x\to -2} f(x) = 0$

d) $\lim_{x\to 3^-} f(x) = 10$

e) $\lim_{x\to 3^+} f(x) = 9$

f) $\lim_{x\to 3} f(x) =$ does not exist

9. $\lim_{x\to 0} \dfrac{(x-1)^2-1}{4x}$

$\lim_{x\to 0} \dfrac{x^2-2x+1-1}{4x}$

$\lim_{x\to 0} \dfrac{x^2-2x}{4x}$

$\lim_{x\to 0} \dfrac{x-2}{4}$

$= \dfrac{0-2}{4} = \dfrac{-1}{2}$ ← A

10. $\lim_{x\to 9} \dfrac{3-\sqrt{x}}{x^2-81} \cdot \dfrac{3+\sqrt{x}}{3+\sqrt{x}}$

$= \lim_{x\to 9} \dfrac{(9-x)}{(x+9)(x-9)(3+\sqrt{x})}$

$= \lim_{x\to 9} \dfrac{-1}{(x+9)(3+\sqrt{x})}$

$= \dfrac{-1}{(18)(6)} = \dfrac{-1}{108}$ ← B

11. $\lim_{x\to\infty} \dfrac{\log_2 2^x}{5x}$

$\lim_{x\to\infty} \dfrac{x}{5x}$

$\lim_{x\to\infty} \dfrac{1}{5}$

$= \dfrac{1}{5}$ ← A

12. $\lim_{x\to -\infty} \dfrac{\sqrt{x^2-3x}}{3x+2}$

$= \lim_{x\to -\infty} \dfrac{\sqrt{x^2\left(\frac{x^2}{x^2}-\frac{3}{x^2}\right)}}{x\left(3+\frac{2}{x}\right)}$

$= \lim_{x\to -\infty} \dfrac{|x|\sqrt{1-\frac{3}{x^2}}}{x\left(3+\frac{2}{x}\right)}$

$= \lim_{x\to -\infty} \dfrac{-(x)\left(1-\frac{3}{x^2}\right)}{x\left(3+\frac{2}{x}\right)}$

$= \dfrac{-1(1-0)}{(3+0)} = \dfrac{-1}{3}$ ← B

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13. $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+4}-2}{x^2} \cdot \dfrac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}$

$\lim\limits_{x\to 0} \dfrac{(x^2+4)-4}{x^2(\sqrt{x^2+4}+2)}$

$= \lim\limits_{x\to 0} \dfrac{1}{\sqrt{x^2+4}+2}$

$\dfrac{1}{\sqrt{0+4}+2} = \dfrac{1}{4}$ ← C

14. $\lim\limits_{x\to -3} \dfrac{x^2-x-12}{x^2+2x-3}$

$\lim\limits_{x\to -3} \dfrac{(x-4)(x+3)}{(x+3)(x-1)}$

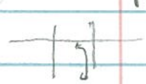
$= \dfrac{-3-4}{-3-1} = \dfrac{-7}{-4} = \dfrac{7}{4}$ ↗ E

$f(x) = \begin{cases} \dfrac{x^3-5x^2+6x}{(x-3)(x-4)}, & \text{if } x \neq 3,\ x\neq 4 \\ W, & \text{if } x=3 \\ 5, & \text{if } x=4 \end{cases}$

$\dfrac{x\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(x-4)} = \dfrac{3(3-2)}{3-4} = \dfrac{3}{-1} = -$

15. $f(x)$ is continuous at $x=3$ if $W = -3$ A

16. $\lim\limits_{x\to 4^-} f(x) = -\infty$ A

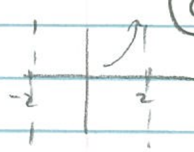


17. $\lim\limits_{x\to 2^-} \left(\dfrac{2}{x^2-4} - \dfrac{1}{x-2}\right)$

$= \lim\limits_{x\to 2^-} \dfrac{2-(x+2)}{x^2-4}$ G

$= \lim\limits_{x\to 2^-} \dfrac{-x}{x^2-4}$

$= \infty$



18. $\lim\limits_{x\to -3} \dfrac{x^2-9}{x^3+27}$

$\lim\limits_{x\to -3} \dfrac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x^2-3x+9)}$

$\dfrac{-3-3}{(9)-3(-3)+9} = \dfrac{-6}{27} = \dfrac{-2}{9}$ B

19. $f(x) = \dfrac{x^2+2x}{|x|(x^2-4)} = \dfrac{x\cancel{(x+2)}}{|x|\cancel{(x+2)}(x-2)}$

= discontinuous (not defined) at $x = 0, -2, 2$

G

20. $\lim\limits_{x\to 3} \dfrac{\frac{1}{x}-\frac{1}{3}}{x-3} = \lim\limits_{x\to 3} \dfrac{-1}{3x}$

$\lim\limits_{x\to 3} \dfrac{\frac{3-x}{3x}}{x-3}$ $= -\dfrac{1}{9}$ E