(2,10)

3.4 Continuity of Functions P. 146 1 - 10

1. jump discontinuity at $x=-6$ (no limit exists)
infinite discontinuity at $x=-4$ (asymptote line)
removable discontinuity at $x=-2$ (hole)
removable discontinuity at $x=1$ $(g(1) \neq \lim_{x\to 1} g(x))$ ← Closed circle at other point
jump discontinuity at $x=4$ ($g(4)$ doesn't exist)

2. a) $f(x) = \frac{12}{x+3}$ at $x=-3$ ← infinite discontinuity (asymptote line)

b) $g(x) = \frac{x-\pi}{x^2-\pi^2}$ at $x=\pi$

$g(\pi) = \frac{\pi-\pi}{\pi^2-\pi^2}$ at $x=\pi$

$= \frac{0}{0}$ ← indeterminate ← removable discontinuity →

$\lim_{x\to\pi} g(x)$

$\lim_{x\to\pi} \frac{x-\pi}{x^2-\pi^2}$

$\lim_{x\to\pi} \frac{\cancel{x-\pi}}{(\cancel{x-\pi})(x+\pi)}$

$\lim_{x\to\pi} \frac{1}{x+\pi} = \frac{1}{2\pi}$

c) $h(x) = \begin{cases} x^2+2x, & x \le 3 \\ 4x+5, & x>3 \end{cases}$, at $x=3$

JUMP discontinuity $\lim_{x\to 3^+} h(x) = 17$, $\lim_{x\to 3^-} h(x) = 15$ (not same #)

d) $i = \begin{cases} \frac{x^2+5x+4}{x^3+1}, & x \neq -1 \\ 2, & x=-1 \end{cases}$, at $x=-1$

$\frac{(\cancel{x+1})(x+4)}{(\cancel{x+1})(x^2-x+1)} = \frac{-1+4}{1+1+1} = \frac{3}{3}$

removable discontinuity at $x=-1$ $(-1,1)$

e) $j(x) = \frac{|x+4|}{x^2+3x-4}$ at $x=-4$

$= \frac{|x+4|}{(x+4)(x-1)}$

$\lim_{x\to -4^+} \frac{(x+4)}{(x+4)(x-1)} = \lim_{x\to -4^+} \frac{1}{x-1} = \frac{1}{-4-1} = \frac{1}{-5}$

$\lim_{x\to -4^-} \frac{-(x+4)}{(x+4)(x-1)} = \lim_{x\to -4^-} \frac{-1}{x-1} = \frac{-1}{-4-1} = \frac{1}{5}$

jump discontinuity at $x=-4$

3.4 - Continued

3. a) $f(x) = \dfrac{2x-6}{x^2-4x+3}$ at $x=3$

$= \dfrac{2\cancel{(x-3)}}{\cancel{(x-3)}(x-1)}$

$f(3) = \dfrac{2}{3-1} = \dfrac{2}{2} = 1$

b) $f(x) = \begin{cases} \dfrac{x^2-9}{x^3-27} & x \neq 3 \\ 1, & x=3 \end{cases}$

$\dfrac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x^2+3x+9)}$

$f(3) = \dfrac{3+3}{9+9+9} = \dfrac{6}{27} = \dfrac{2}{9}$

c) $f(x) = \dfrac{\sqrt{x}-\sqrt{3}}{x-3} \cdot \dfrac{\sqrt{x}+\sqrt{3}}{\sqrt{x}+\sqrt{3}}$

$= \dfrac{\cancel{(x-3)}}{\cancel{(x-3)}(\sqrt{x}+\sqrt{3})}$

$f(3) = \dfrac{1}{\sqrt{3}+\sqrt{3}} = \dfrac{1}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{6}$

4. $f(x) = \begin{cases} x^2+5x+2, & x<1 \\ cx-3, & x \geq 1 \end{cases}$

$(1)^2+5(1)+2$

$1+5+2=8$

$8 = c(1)-3$

$11 = c$

5. $g(x) = \begin{cases} 2x^2-dx+3, & x<-3 \\ x^2+x+9, & x \geq -3 \end{cases}$

$(-3)^2+(-3)+9$

$= 9-3+9 = 15$

$15 = 2(-3)^2 - d(-3)+3$

$15 = 18+3d+3$

$-6 = 3d$

$-2 = d$

6. $h(x) = \begin{cases} 4x+5, & x \leq -1 \\ \dfrac{ax+b}{x-1}, & -1<x<2 \\ x^2-1, & x \geq 2 \end{cases}$

$4x+5 = \dfrac{ax+b}{x-1}$

$4(-1)+5 = \dfrac{a(-1)+b}{-1-1}$

$-2 = -a+b$

$b = a-2$

$\dfrac{ax+b}{x-1} = x^2-1$

$\dfrac{a(2)+b}{2-1} = (2)^2-1$

$2a+b=3$

$2a+(a-2)=3$

$3a=5$ $a=\dfrac{5}{3}$

$b = \dfrac{5}{3}-\dfrac{6}{3} = \dfrac{-1}{3}$

7. $f(x) = \begin{cases} \dfrac{x^4-x}{x-1}, & x \neq 1 \\ e, & x=1 \end{cases}$

$\dfrac{x(x^3-1)}{x-1} = \dfrac{x\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}}$

$1(1^2+1+1) = 3$

$e=3$

3.4 - Contined

8. a) $f(x) = \dfrac{3x - x^2}{x^2 + x}$ $\qquad \dfrac{3-0}{0+1} = \dfrac{3}{1} = 3 \leftarrow$ y-co-ordinate of hole

$= \dfrac{\cancel{x}(3-x)}{\cancel{x}(x+1)}$ Hole: $(0, 3)$

Vertical Asymptote: $x = -1$

b) $f(x) = \dfrac{x^3 + 3x^2 - 10x}{x^2 + x - 6}$ $\qquad \dfrac{2(2+5)}{(2+3)} = \dfrac{2(7)}{5} = \dfrac{14}{5} \leftarrow$ y co-ord of hole

$= \dfrac{x(x+5)\cancel{(x-2)}}{(x+3)\cancel{(x-2)}}$ Hole: $(2, \frac{14}{5})$

Vertical Asymptote: $x = -3$

9. $F(x) = (2^{x+5})\sqrt[3]{x^2 - x - 12}$

both have all real #'s as DOMAIN. All exponential & root functions are continuous $\therefore$ their product is continuous.

10. a) $f(x) = \sin x + \cos x$ - continuous

b) $f(x) = \dfrac{x-3}{x^2+4}$ - continuous

c) $f(x) = \dfrac{x+2}{x^2 - 2x - 3} = \dfrac{x+2}{(x-3)(x+1)}$ infinite discontinuitys at $x = -1, 3$

d) $F(x) = |x^2 - 6x|$

$= |x(x-6)|$ - continuous

e) $F(x) = \dfrac{|x-1|}{x^2 - x} = \dfrac{-(x-1)}{x(x-1)} \Big| \dfrac{(x-1)}{x(x-1)}$ hole $\rightarrow$ $(1, -1)$ $(1, 1)$ $\leftarrow$ jump discontinuity $\quad f(1)$ doesn't exist

asymptote $\rightarrow x = 0 \leftarrow$ infinite discontinuitys

f) $f(x) = \left|\dfrac{x+3}{x+3}\right| = |1|$ removable discontinuity at $x = -3$ (indeterminate)

g) $F(x) = \dfrac{x-4}{\sqrt{x} - 2} = \dfrac{(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}}{\cancel{(\sqrt{x}-2)}} = \sqrt{x} + 2$ hole: removable discontinuity at $x = 4$ $(x \neq \text{neg})$

$(4, 4)$

h) $f(x) = \frac{1}{2}x^2 - \frac{3}{4}x + \frac{2}{5}$ - continuous

i) $F(x) = \dfrac{x+5}{\log_2(x^2+1)}$ $\qquad \log_2(0^2+1) = \log_2 1 = 0 \qquad 2^0 = 1$

infinite discontinuity at $x = 0$

j) $f(x) = \dfrac{x+6}{|x-6|}$ $\quad x - 6 = 0 \; x = 6$. infinite discontinuity at $x = 6$

3.4 - continued

k) $f(x) = \dfrac{x^2+x}{x^4-x} = \dfrac{x(x+1)}{x(x-1)(x^2+x+1)}$ removable discontinuity at $x=0$ (0, -

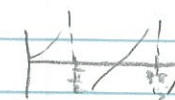
$x(x^3-1)$ always positive infinite discontinuity at $x=1$

l) $f(x) = \begin{cases} 2^x+1, & x \geq 2 \\ 2^{x+1}-3, & x < 2 \end{cases}$ continuous

$2^2+1=5$

$2^{2+1}-3=5$

m) $f(x) = \begin{cases} -4, & x < -3 \\ x-1, & -3 \leq x \leq 2 \\ x^2, & x > 2 \end{cases}$ jump discontinuity at $x=2$



n) $f(x) = \tan x$ infinite discontinuity at $x = \frac{\pi}{2} + n\pi$ where n is an integer

o) $f(x) = \sqrt[3]{9-x^2}$ continuous

m)