

1-9 plug in 40-76 ✓
 10-18 sign analysis 48-51 ✓
 19-23 factor 52-60 //
 24-29 simplify 61-63 plug in

3.3 Strategies for Evaluating Limits P.138 1-63

1. $\lim_{x \rightarrow -1} x^3 - x^2 - x$

$= (-1)^3 - (-1)^2 - (-1)$
 $= -1 - 1 + 1$
 $= (-1) \checkmark$

2. $\lim_{x \rightarrow 6} \frac{x^2 + 36}{x + 3}$

$= \frac{(6)^2 + 36}{6 + 3}$
 $= \frac{72}{9} = (8) \checkmark$

3. $\lim_{x \rightarrow -2} \frac{x^2 - 2x - 8}{2x + 1}$

$= \frac{(-2)^2 - 2(-2) - 8}{2(-2) + 1}$
 $= \frac{4 + 4 - 8}{-3} = \frac{0}{-3} = (0) \checkmark$

4. $\lim_{x \rightarrow 3} \frac{2^{x-4}}{x-1}$

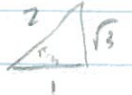
$= \frac{2^{3-4}}{3-1}$
 $= \frac{2^{-1}}{2} = (\frac{1}{4}) \checkmark$

5. $\lim_{x \rightarrow 5} x$

$= (5) \checkmark$

6. $\lim_{x \rightarrow -4} 8$

$= (8) \checkmark$



7. $\lim_{x \rightarrow \frac{\pi}{4}} \sin^2 x$

$= (\sin \frac{\pi}{4})^2$
 $= (\frac{1}{\sqrt{2}})^2 = (\frac{1}{2}) \checkmark$

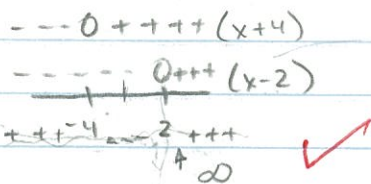
8. $\lim_{x \rightarrow 8} [(\log_2 x)(2^{10-x})]$

$= (\log_2 8)(2^{10-8})$
 $= (3)(4)$
 $= (12) \checkmark$

9. $\lim_{x \rightarrow \frac{\pi}{3}} \left[\frac{3x}{\pi} (\tan^4 x) \right]$

$= \frac{3(\frac{\pi}{3})}{\pi} \left[(\tan \frac{\pi}{3})^4 \right]$
 $= 1 \left(\frac{\sqrt{3}}{1} \right)^4 = (9) \checkmark$

10. $\lim_{x \rightarrow 2^+} \frac{x+4}{x-2} = \infty$



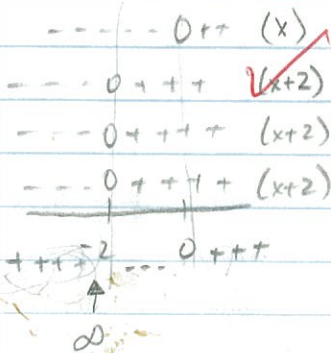
11. $\lim_{x \rightarrow 2^-} \frac{x+4}{x-2} = -\infty$



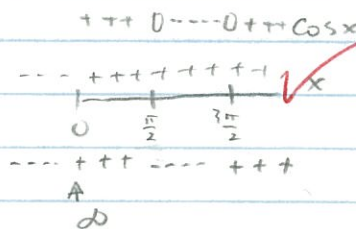
12. $\lim_{x \rightarrow 2} \frac{x+4}{x-2}$

does not exist.

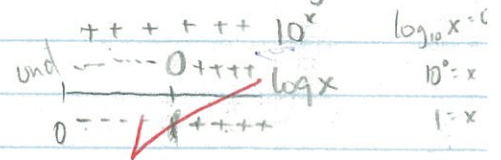
13. $\lim_{x \rightarrow -2^-} \frac{x}{(x+2)^3} = -\infty$



14. $\lim_{x \rightarrow 0^+} \frac{\cos x}{x} = \infty$



15. $\lim_{x \rightarrow 1^-} \frac{10^x}{\log x} = -\infty$



$\log 1$
 $= \log_{10} 10^0$
 $= 0$

Hilbert

$$-3 - \frac{(1+2h+h^2)(1+h)}{1+2h+h^2+h+2h^2+h^3}$$

3.3 continued.

27. $\lim_{x \rightarrow 1} \frac{x^7 - 1}{x - 1}$

$$\lim_{x \rightarrow 1} \frac{(x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)}{x-1}$$

$$= (1)^6 + (1)^5 + (1)^4 + (1)^3 + (1)^2 + 1 + 1$$

$$= \boxed{7}$$

28. $\lim_{x \rightarrow 0} \frac{(x+1)^2 - 1}{x}$

$$\lim_{x \rightarrow 0} \frac{(x+1-1)(x+1+1)}{x}$$

$$\lim_{x \rightarrow 0} \frac{(x)(x+2)}{x}$$

$$0+2 = \boxed{2}$$

29. $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 2(3+h) - 3}{h}$

$$\lim_{h \rightarrow 0} \frac{9 + 6h + h^2 - 6 - 2h - 3}{h}$$

$$\lim_{h \rightarrow 0} \frac{h^2 + 4h}{h}$$

$$\lim_{h \rightarrow 0} h + 4$$

$$= 0 + 4 = \boxed{4}$$

30. $\lim_{h \rightarrow 0} \frac{(1+h)^3 - 2(1+h) + 1}{h}$

$$\lim_{h \rightarrow 0} \frac{1 + 3h + 3h^2 + h^3 - 2 - 2h + 1}{h}$$

$$\lim_{h \rightarrow 0} \frac{h + 3h^2 + h^3}{h}$$

$$\lim_{h \rightarrow 0} 1 + 3h + h^2$$

$$= 1 + 3(0) + 0 = \boxed{1}$$

31. $\lim_{x \rightarrow 4} \frac{\frac{1}{x-2} - \frac{1}{2}}{x-4}$

$$\lim_{x \rightarrow 4} \frac{2 - x + 2}{2(x-2)} \cdot \left(\frac{1}{x-4}\right)$$

$$\lim_{x \rightarrow 4} \frac{4 - x}{2(x-2)(x-4)}$$

$$\frac{-1}{2(4-2)} = \boxed{\frac{-1}{4}}$$

32. $\lim_{x \rightarrow 1} \frac{\frac{1}{2x+1} - \frac{1}{3}}{x-1}$

$$\lim_{x \rightarrow 1} \frac{3 - 2x - 1}{3(2x+1)} \left(\frac{1}{x-1}\right)$$

$$\lim_{x \rightarrow 1} \frac{-2x + 2}{3(2x+1)} \left(\frac{1}{x-1}\right)$$

$$\lim_{x \rightarrow 1} \frac{-2(x-1)}{3(2x+1)} \cdot \left(\frac{1}{x-1}\right)$$

$$\frac{-2}{3(2(1)+1)} = \boxed{\frac{-2}{9}}$$

33. $\lim_{a \rightarrow 0} \frac{\frac{1}{(a+2)^2} - \frac{1}{4}}{a}$

$$\lim_{a \rightarrow 0} \frac{4 - a^2 - 4a - 4}{4(a+2)^2} \cdot \frac{1}{a}$$

$$\lim_{a \rightarrow 0} \frac{-a(a+4)}{4(a+2)^2} \cdot \frac{1}{a}$$

$$= \frac{-(0+4)}{4(0+2)^2} = \frac{-4}{16} = \boxed{\frac{-1}{4}}$$

34. $\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3}$

$$\lim_{x \rightarrow 9} \frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(\sqrt{x}-3)}$$

$$= \sqrt{9} + 3$$

$$= 3 + 3$$

$$= \boxed{6}$$

3.3 continued

$$35. \lim_{y \rightarrow 0} \frac{\sqrt{y+2} - \sqrt{2}}{y} \cdot \frac{\sqrt{y+2} + \sqrt{2}}{\sqrt{y+2} + \sqrt{2}}$$

$$\lim_{y \rightarrow 0} \frac{y+2 - 2}{y(\sqrt{y+2} + \sqrt{2})}$$

$$\lim_{y \rightarrow 0} \frac{y}{y(\sqrt{y+2} + \sqrt{2})}$$

$$\frac{1}{\sqrt{0+2} + \sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \left(\frac{\sqrt{2}}{4}\right)$$

$$36. \lim_{x \rightarrow 1} \frac{2 - \sqrt{5-x}}{1-x} \cdot \frac{2 + \sqrt{5-x}}{2 + \sqrt{5-x}}$$

$$= \lim_{x \rightarrow 1} \frac{4 - (5-x)}{(1-x)(2 + \sqrt{5-x})}$$

$$= \lim_{x \rightarrow 1} \frac{-1(1-x)}{(1-x)(2 + \sqrt{5-x})}$$

$$= \frac{-1}{2 + \sqrt{5-1}} = \left(\frac{-1}{4}\right)$$

$$37. \lim_{x \rightarrow 5} \frac{x-5}{\sqrt{x+4} - 3} \cdot \frac{\sqrt{x+4} + 3}{\sqrt{x+4} + 3}$$

$$\lim_{x \rightarrow 5} \frac{(x-5)(\sqrt{x+4} + 3)}{(x+4) - 9}$$

$$\lim_{x \rightarrow 5} \frac{(x-5)(\sqrt{x+4} + 3)}{(x-5)}$$

$$= \sqrt{5+4} + 3 = (6)$$

$$38. \lim_{q \rightarrow 0} \frac{\frac{1}{\sqrt{4+q}} - \frac{1}{2}}{q}$$

$$= \lim_{q \rightarrow 0} \frac{2 - \sqrt{4+q}}{2\sqrt{4+q}} \cdot \frac{1}{q} \cdot \frac{2 + \sqrt{4+q}}{2 + \sqrt{4+q}}$$

$$= \lim_{q \rightarrow 0} \frac{4 - (4+q)}{2q\sqrt{4+q}(2 + \sqrt{4+q})}$$

$$= \lim_{q \rightarrow 0} \frac{-q}{2q\sqrt{4+q}(2 + \sqrt{4+q})}$$

$$= \frac{-1}{2\sqrt{4+0}(2 + \sqrt{4+0})} = \left(\frac{-1}{16}\right)$$

$$39. \lim_{x \rightarrow 4} \frac{8\sqrt{x} - x^2}{2 - \sqrt{x}} \left(\frac{8\sqrt{x+x^2}}{8\sqrt{x+x^2}}\right) \left(\frac{2 + \sqrt{x}}{2 + \sqrt{x}}\right)$$

$$\lim_{x \rightarrow 4} \frac{(64x - x^4)(2 + \sqrt{x})}{(4-x)(8\sqrt{x+x^2})}$$

$$\lim_{x \rightarrow 4} \frac{x(4-x)(16+4x+x^2)(2 + \sqrt{x})}{(4-x)(8\sqrt{x+x^2})}$$

$$= \frac{(4)(16+4(4)+4^2)(2 + \sqrt{4})}{8\sqrt{4+4^2}}$$

$$= \frac{(4)(48)(4)}{32} = (24)$$

32

$$41. \lim_{x \rightarrow \infty} \frac{2x+5}{x+1}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{2x}{x} + \frac{5}{x}}{\frac{x}{x} + \frac{1}{x}}$$

$$\lim_{x \rightarrow \infty} \frac{2 + \frac{5}{x}}{1 + \frac{1}{x}}$$

$$= \frac{2+0}{1+0} = (2)$$

$$40. \lim_{x \rightarrow \infty} \frac{6}{3x-2}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{6}{x}}{\frac{3x}{x} - \frac{2}{x}}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{6}{x}}{3 - \frac{2}{x}}$$

$$\frac{0}{3-0} = (0)$$

$$3-0$$

$$42. \lim_{x \rightarrow \infty} \frac{6x^2 - 1}{2x^2 + 3x} = \lim_{x \rightarrow \infty} \frac{6 - \frac{1}{x^2}}{2 + \frac{3}{x}}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{6x^2}{x^2} - \frac{1}{x^2}}{\frac{2x^2}{x^2} + \frac{3x}{x^2}}$$

$$\frac{6-0}{2+0} = (3)$$

$$= (3)$$

3.3 - Continued

$$43. \lim_{x \rightarrow -\infty} \frac{-4x^3}{x^3 - 2x^2}$$

$$\lim_{x \rightarrow \infty} \frac{-4x^3}{x^3 - 2x^2}$$

$$\lim_{x \rightarrow -\infty} \frac{-4}{1 - \frac{2}{x}}$$

$$= \frac{-4}{1-0} = (-4)$$

$$44. \lim_{x \rightarrow \infty} \frac{(x+2)(2x-1)}{x^2+4x+1}$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2+3x-2}{x^2+4x+1}$$

$$\lim_{x \rightarrow -\infty} \frac{\frac{2x^2}{x^2} + \frac{3x}{x^2} - \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{4x}{x^2} + \frac{1}{x^2}}$$

$$\lim_{x \rightarrow -\infty} \frac{2 + \frac{3}{x} - \frac{2}{x^2}}{1 + \frac{4}{x} + \frac{1}{x^2}}$$

$$\frac{2+0-0}{1+0+0} = (2)$$

$$45. \lim_{x \rightarrow \infty} \frac{2x^2}{x-1}$$

$$\lim_{x \rightarrow \infty} \frac{2x^2}{\frac{x}{x} - \frac{1}{x}}$$

$$\lim_{x \rightarrow \infty} \frac{2x}{1 - \frac{1}{x}}$$

$$= \frac{2x}{1-0} = (\infty)$$

$$46. \lim_{x \rightarrow -\infty} \frac{2x^2}{x-1}$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2}{\frac{x}{x} - \frac{1}{x}}$$

$$\lim_{x \rightarrow -\infty} \frac{2x}{1 - \frac{1}{x}}$$

$$= \frac{2x}{1-0} = (-\infty)$$

$$47. \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2-x}}{x-2}$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2(4-\frac{1}{x})}}{x(1-\frac{2}{x})}$$

$$\lim_{x \rightarrow -\infty} \frac{|x| \sqrt{4-\frac{1}{x}}}{x(1-\frac{2}{x})}$$

Because it approaches $-\infty$

$$\lim_{x \rightarrow -\infty} \frac{-x \sqrt{4-\frac{1}{x}}}{x(1-\frac{2}{x})}$$

$$\frac{-\sqrt{4-0}}{1-0} = (-2)$$

$$48. \lim_{g \rightarrow \infty} \frac{2g+5}{\sqrt{g^2+6g}}$$

$$\lim_{g \rightarrow \infty} \frac{g(2+\frac{5}{g})}{\sqrt{g^2(1+\frac{6}{g})}}$$

$$\lim_{g \rightarrow \infty} \frac{g(2+\frac{5}{g})}{|g| \sqrt{1+\frac{6}{g}}}$$

$$\lim_{g \rightarrow \infty} \frac{g(2+\frac{5}{g})}{g \sqrt{1+\frac{6}{g}}}$$

Because it approaches $+\infty$

$$\frac{2+0}{\sqrt{1+0}} = (2)$$

$\sqrt{x^2} = |x|$

3.3 continued

$$49. \lim_{x \rightarrow -\infty} \sqrt{x^2 - 4x + 4}$$

$$\lim_{x \rightarrow -\infty} \frac{5-x}{\sqrt{x^2 \left(1 - \frac{4}{x} + \frac{4}{x^2}\right)}}$$

$$\lim_{x \rightarrow -\infty} \frac{|x| \sqrt{1 - \frac{4}{x} + \frac{4}{x^2}}}{x \left(\frac{5}{x} - 1\right)}$$

$$\lim_{x \rightarrow -\infty} \frac{-x \sqrt{1 - \frac{4}{x} + \frac{4}{x^2}}}{x \left(\frac{5}{x} - 1\right)}$$

$$\frac{-1 \sqrt{1-0+0}}{0-1}$$

$$= \textcircled{1}$$

$$50. \lim_{x \rightarrow \infty} \sqrt{x^2 - 2x}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 \left(1 - \frac{2}{x}\right)}}{x^2}$$

$$\lim_{x \rightarrow \infty} \frac{|x| \sqrt{1 - \frac{2}{x}}}{x^2}$$

$$\lim_{x \rightarrow \infty} \frac{x \sqrt{1 - \frac{2}{x}}}{x^2} = \sqrt{1 - \frac{2}{x}} \cdot \frac{1}{x}$$

$$\sqrt{1-0} \cdot 0 = \textcircled{0}$$

$$= \textcircled{0}$$

$$51. \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 4x} - x}{1} \cdot \frac{\sqrt{x^2 + 4x} + x}{\sqrt{x^2 + 4x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{x^2 + 4x - x^2}{\sqrt{x^2 + 4x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{4x}{\sqrt{x^2 \left(1 + \frac{4}{x}\right)} + x} = \lim_{x \rightarrow \infty} \frac{4x}{x \sqrt{1 + \frac{4}{x}} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{4}{\sqrt{1 + \frac{4}{x}} + 1}$$

$$= \frac{4}{\sqrt{1+0} + 1} = \textcircled{2}$$

$$52. \lim_{x \rightarrow 0^+} \frac{|x|}{x}$$

$$= \lim_{x \rightarrow 0^+} \frac{(x)}{x}$$

$$= \lim_{x \rightarrow 0^+} 1 = \textcircled{1}$$

$$53. \lim_{x \rightarrow 0^-} \frac{|x|}{x}$$

$$\lim_{x \rightarrow 0^-} \frac{-(x)}{x}$$

$$\lim_{x \rightarrow 0^-} -1$$

$$= \textcircled{-1}$$

$$54. \lim_{x \rightarrow 0} \frac{|x|}{x}$$

does not exist.

3.3 continued

55. $\lim_{x \rightarrow 2^+} \frac{|x-2|}{4-x^2}$

$$\lim_{x \rightarrow 2^+} \frac{(x-2)^{-1}}{(2-x)(2+x)}$$

$$\frac{-1}{2+2} = \left(\frac{-1}{4}\right)$$

56. $\lim_{x \rightarrow -1^-} \frac{|x^2-x-2|}{x+1}$

$$= \lim_{x \rightarrow -1^-} \frac{-(x+1)(x-2)}{x+1}$$

$$= \lim_{x \rightarrow -1^-} -|x-2|$$

$$= -|-1-2| = -|-3| = -(3) = \boxed{-3}$$

57. $\lim_{x \rightarrow 1^-} \frac{|x^5-1|}{x-1}$

$$\lim_{x \rightarrow 1^-} \frac{-(x-1)(x^4+x^3+x^2+x+1)}{(x-1)}$$

$$\lim_{x \rightarrow 1^-} -(x^4+x^3+x^2+x+1)$$

$$-(1+1+1+1+1) = \boxed{-5}$$

58. $\lim_{x \rightarrow 3^-} \frac{x^3-27}{|x-3|}$

$$= \lim_{x \rightarrow 3^-} \frac{(x-3)(x^2+3x+9)}{-(x-3)}$$

$$= \lim_{x \rightarrow 3^-} -1(x^2+3x+9)$$

$$= -1(9+9+9) = \boxed{-27}$$

59. $\lim_{x \rightarrow 3^+} \frac{x^3-27}{|x-3|}$

$$= \lim_{x \rightarrow 3^+} \frac{(x-3)(x^2+3x+9)}{(x-3)}$$

$$= \lim_{x \rightarrow 3^+} x^2+3x+9$$

$$9+9+9 = \boxed{27}$$

60. $\lim_{x \rightarrow 3} \frac{x^3-27}{|x-3|}$

= does not exist

61. $f(x) = \begin{cases} (x+1)2^{x-1}, & \text{if } x \geq 0 \\ 2^{1-x}, & \text{if } x < 0 \end{cases}$

a) $\lim_{x \rightarrow 3} f(x) =$

$$= (3+1)2^{3-1}$$

$$= 4(4)$$

$$= \boxed{16}$$

b) $\lim_{x \rightarrow -3} f(x) =$

$$= 2^{1-(-3)}$$

$$= 2^4$$

$$= \boxed{16}$$

c) $\lim_{x \rightarrow 0^+} f(x) =$

$$= (0+1)2^{0-1}$$

$$= 1(2^{-1})$$

$$= \boxed{\frac{1}{2}}$$

d) $\lim_{x \rightarrow 0^-} f(x) =$

$$= 2^{1-0}$$

$$= 2^1$$

$$= \boxed{2}$$

e) $\lim_{x \rightarrow 0} f(x)$
does not exist

3.3 continued

$$62 \quad g(x) = \begin{cases} \frac{x^2-4}{x+2}, & \text{if } x \neq -2 \\ -5, & \text{if } x = -2 \end{cases}$$

$$a) \lim_{x \rightarrow 3} g(x)$$

$$= \frac{3^2-4}{3+2} = \frac{5}{5} = \textcircled{1}$$

$$b) \lim_{x \rightarrow -2} f(x)$$

$$= \textcircled{-5} \quad \leftarrow \text{different than book}$$

$$\frac{x^2-4}{x+2} = \frac{(x-2)(x+2)}{(x+2)}$$

$$= -2-2 = \textcircled{-4}$$

$$63 \quad h(x) = \begin{cases} 2, & \text{if } x < 0 \\ x+2, & \text{if } 0 \leq x \leq 4 \\ x^2-11, & \text{if } x > 4 \end{cases}$$

$$a) \lim_{x \rightarrow -6} h(x)$$

$$= \textcircled{2}$$

$$b) \lim_{x \rightarrow 0^-} h(x)$$

$$= \textcircled{2}$$

$$c) \lim_{x \rightarrow 0^+} h(x)$$

$$= 0+2 = \textcircled{2}$$

$$d) \lim_{x \rightarrow 0} h(x)$$

$$= \textcircled{2}$$

$$e) \lim_{x \rightarrow 3} h(x)$$

$$= 3+2 = \textcircled{5}$$

$$f) \lim_{x \rightarrow 4^-} h(x)$$

$$= 4+2 = \textcircled{6}$$

$$g) \lim_{x \rightarrow 4^+} h(x)$$

$$= (4)^2-11 = \textcircled{5}$$

$$h) \lim_{x \rightarrow 4} h(x)$$

= does not exist

$$i) \lim_{x \rightarrow 5} h(x)$$

$$(5)^2-11 = \textcircled{14}$$