1. $f(x) = e^{3x}$

$f'(x) = e^{3x} \cdot 3$

$= 3e^{3x}$

2. $f(x) = e^{2x-5}$

$f'(x) = e^{2x-5}(2)$

$= 2e^{2x-5}$

3. $f(x) = e^{-x^2-4x}$

$f'(x) = e^{-x^2-4x}(-2x-4)$

$= (-2x-4)(e^{-x^2-4x})$

$= -2(x+2)(e^{-x^2-4x})$

4. $f(x) = e^{-(6x-5)}$

$f(x) = e^{-6x+5}$

$f'(x) = e^{-6x+5}(-6)$

$= -6e^{-6x+5}$

5. $f(x) = 4e^{x^3}$

$f'(x) = 4e^{x^3} \cdot (3x^2)$

$f'(x) = 12x^2 e^{x^3}$

6. $f(x) = (e^{3x+1})^2$

$f'(x) = 2(e^{3x+1})^1 \cdot (e^{3x+1})(3)$

$= 6(e^{3x+1})^2$

7. $f(x) = e^{\frac{1}{x+7}}$

$f(x) = e^{(x+7)^{-1}}$

$f'(x) = e^{(x+7)^{-1}} \left(-(x+7)^{-2}\right)$

$= \dfrac{-e^{\frac{1}{x+7}}}{(x+7)^2}$

8. $f(x) = e^{(2x+1)^{10}}$

$f'(x) = e^{(2x+1)^{10}} \cdot \left(10(2x+1)^9 \cdot (2)\right)$

$= 20(2x+1)^9 e^{(2x+1)^{10}}$

9. $f(x) = e^{\frac{x}{x+1}}$

$f'(x) = e^{\frac{x}{x+1}} \cdot \left[\dfrac{x+1(1) - x(1)}{(x+1)^2}\right]$

$= e^{\frac{x}{x+1}} \left[\dfrac{1}{(x+1)^2}\right] = \dfrac{e^{\frac{x}{x+1}}}{(x+1)^2}$

10. $f(x) = x^2 e^x$

$f'(x) = x^2(e^x) + e^x(2x)$

$= xe^x[x+2]$

11. $f(x) = (2x+3)\,e^{x^2}$

$$f'(x) = (2x+3)(e^{x^2}\cdot 2x) + e^{x^2}(2)$$

$$= 2e^{x^2}\left[x(2x+3) + 1\right]$$

$$= 2e^{x^2}\left[2x^2+3x+1\right]$$

12. $f(x) = \dfrac{x}{e^x}$

$$f'(x) = \frac{\cancel{e^x}(1) - x(\cancel{e^x})}{(e^x)^2} = \frac{1-x}{e^x}$$

13. $f(x) = \dfrac{e^{x-2}}{x-2}$

$$f'(x) = \frac{(x-2)e^{x-2} - e^{x-2}(1)}{(x-2)^2}$$

$$= \frac{e^{x-2}(x-2-1)}{(x-2)^2}$$

$$= \frac{e^{x-2}(x-3)}{(x-2)^2}$$

14. $f(x) = \dfrac{x+1}{e^{x^2-1}}$

$$f'(x) = \frac{(\cancel{e^{x^2-1}})(1) - (x+1)(\cancel{e^{x^2-1}})(2x)}{\left(e^{x^2-1}\right)^2}$$

$$f'(x) = \frac{1 - 2x(x+1)}{e^{x^2-1}} = \frac{1-2x^2-2x}{e^{x^2-1}}$$

15 $\cancel{f(x) = \dfrac{x+1}{e^{x^2-1}}}$

$\cancel{f'(x) = (e^{x^2-1})(1) - (x+1)(e^{x^2-1})(2x)}$

15. $f(x) = \pi x\, e^{\pi x^2}$

$$f'(x) = \pi x\left(e^{\pi x^2} \cdot 2\pi x\right) + e^{\pi x^2}(\pi)$$

$$= 2\pi^2 x^2 e^{\pi x^2} + \pi e^{\pi x^2}$$

$$= \pi e^{\pi x^2}\left[2\pi x^2 + 1\right]$$

16. $f(x) = e^4$

$f'(x) = 0$

17. $f(x) = \left(e^{x^2-x-3}\right)\left(e^{5+x-x^2}\right)$

$f'(x) = \left(e^{x^2-x-3}\right)\left(e^{5+x-x^2}\right)(1-2x)$

$+ \left(e^{5+x-x^2}\right)\left(e^{x^2-x-3}\right)(2x-1)$

$f'(x) = \left(e^{x^2-x-3}\right)\left(e^{5+x-x^2}\right)\left[1-2x+2x-1\right]$

$f'(x) = 0$

18. $\ln[\ln(x^3+9x)]$

$f'(x) = \frac{1}{\ln(x^3+9x)} \cdot \frac{1}{(x^3+9x)} \cdot (3x^2+9)$

$$\frac{3x^2+9}{(x^3+9x)\ln(x^3+9x}$$

19. a) $f(x) = e^x$ $\qquad x = 0 \qquad y = 1$

$f'(x) = e^x$

$m = f'(0) = e^0 = 1$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 0)$

$y - 1 = x$

$y = x + 1$

b) $f(x) = xe^x$ $\qquad x = 1 \qquad y = e$

$f'(x) = xe^x + e^x 1$

$f'(x) = e^x(x+1)$

$m = f'(1) = e^1(1+1)$

$m = 2e$

$y - y_1 = m(x - x_1)$

$y - e = 2e(x - 1)$

$y - e = 2ex - 2e$

$y = 2ex - e$

c) $f(x) = \dfrac{e^{2x}}{x+1}$ $\quad x = 0 \quad y = 1$

$$f'(x) = \frac{(x+1)(e^{2x} \cdot 2) - e^{2x}(1)}{(x+1)^2}$$

$$f'(x) = \frac{(x+1)(2e^{2x}) - e^{2x}}{(x+1)^2}$$

$$m = f'(0) = \frac{(0+1)(2e^{2(0)}) - e^{2(0)}}{(0+1)^2}$$

$$= \frac{1(2) - 1}{1} = 1$$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 0)$

$y - 1 = x$

$y = x + 1$

20a) $y = 3^{4x^2}$

$y' = 3^{4x^2} \cdot 8x \cdot \ln 3$

b) $y = 7^{-4x + 3x^2}$

$y' = 7^{-4x + 3x^2} (-4 + 6x) \cdot \ln 7$

c) $y = 5^{3x^3} 7^{x^2}$

$y' = 5^{3x^3} \cdot (7^{x^2} \cdot 2x \cdot \ln 7) + 7^{x^2} (5^{3x^3} \cdot 9x^2 \cdot \ln 5)$

$y' = x (5^{3x^3} 7^{x^2}) [2 \ln 7 + 9x \ln 5)$

Logarithmic Differentiation

1. $f(x) = \ln(7x+3)$

$$f'(x) = \frac{1}{7x+3} \cdot 7 = \frac{7}{7x+3}$$

2. $f(x) = \ln(2x^3 - 5x + 1)$

$$f'(x) = \frac{1}{(2x^3 - 5x + 1)} \cdot (6x^2 - 5)$$

$$f'(x) = \frac{6x^2 - 5}{2x^3 - 5x + 1}$$

3. $f(x) = 6 \ln(15x+7)$

$$f'(x) = 6\left[\frac{1}{15x+7} \cdot 15\right]$$

$$f'(x) = \frac{90}{15x+7}$$

4. $f(x) = -3 \ln(2 - x^4)$

$$f'(x) = -3\left[\frac{1}{(2-x^4)} \cdot -4x^3\right]$$

$$f'(x) = \frac{12x^3}{(2-x^4)}$$

5. $y = \ln (x^2+3)^{1/2}$

$y' = \dfrac{1}{(x^2+3)^{1/2}} \cdot \frac{1}{2}(x^2+3)^{-1/2} \cdot 2x$

$y' = \dfrac{1}{\sqrt{x^2+3}} \cdot \dfrac{x}{\sqrt{x^2+3}} = \dfrac{x}{x^2+3}$

6. $y = \ln (1-6x)^{1/2}$

$y' = \dfrac{1}{(1-6x)^{1/2}} \cdot \frac{1}{2}(1-6x)^{-1/2}(-6)$

$= \dfrac{1}{\sqrt{1-6x}} \cdot \dfrac{-3}{\sqrt{1-6x}} = \dfrac{-3}{1-6x}$

7. $y = 6 \ln (x^2-3x)^{1/3}$

$y' = 6\left[\dfrac{1}{(x^2-3x)^{1/3}}\right]\left[\frac{1}{3}(x^2-3x)^{-2/3}\left(\frac{2x}{-3}\right)\right]$

$y' = \dfrac{2(2x-3)}{(x^2-3x)^{1/3}(x^2-3x)^{2/3}} = \cancel{\dfrac{x-6}{x^2-3x}} \quad \dfrac{4x-6}{x^2-3x}$

8. $y = \ln [(x-3)(x+2)]$

$y' = \dfrac{1}{(x-3)(x+2)}\left[(x-3)(1) + (x+2)(1)\right]$

$y' = \dfrac{1}{(x-3)(x+2)}[2x-1] = \dfrac{2x-1}{(x-3)(x+2)}$

9. $\cancel{f(x)}$ $y = \ln\left(\frac{x+4}{x-8}\right)$

$$y' = \frac{1}{\left(\frac{x+4}{x-8}\right)}\left[\frac{(x-8)(1)-(x+4)}{(x-8)^2}\right]$$

$$y' = \left(\frac{\cancel{x-8}}{x+4}\right)\left(\frac{-12}{(x-8)^{\cancel{2}}}\right) = \frac{-12}{(x+4)(x-8)}$$

10. $f(x) = \ln\left(\frac{x^2-2}{x+2}\right)$

$$f'(x) = \frac{1}{\left(\frac{x^2-2}{x+2}\right)}\left[\frac{(x+2)(2x)-(x^2-2)(1)}{(x+2)^2}\right]$$

$$= \left(\frac{\cancel{x+2}}{x^2-2}\right)\left[\frac{2x^2+4x-x^2+2}{(x+2)^{\cancel{2}}}\right]$$

$$= \frac{x^2+4x+2}{(x^2-2)(x+2)}$$

11. $f(x) = x\ln x$

$$f'(x) = x\left(\frac{1}{x}\right) + \ln x(1)$$

$$= 1 + \ln x$$

(12) $f(x) = x^3 \ln(1-x^3)$

$$f'(x) = x^3\left(\frac{1}{1-x^3} \cdot -3x^2\right) + \ln(1-x^3)(-3x^2)$$

$$= -\frac{3x^5}{1-x^3} - 3x^2 \ln(1-x^3)$$

$$= -3x^2\left(\frac{x^3}{1-x^3} + \ln(1-x^3)\right)$$

(13) $f(x) = (x+3)^{-1} \ln(x+3)$

$$f(x) = \frac{\ln(x+3)}{(x+3)}$$

$$f'(x) = \frac{(x+3)\left(\frac{1}{x+3}\right) - \ln(x+3)}{(x+3)^2}$$

$$= \frac{1 - \ln(x+3)}{(x+3)^2}$$

(14) $f(x) = [\ln(x^3-1)]^3$

$$f'(x) = 3[\ln(x^3-1)]^2 \cdot \frac{1}{x^3-1} \cdot 3x^2$$

$$= \frac{9x^2[\ln(x^3-1)]^2}{x^3-1}$$

(15) $f(x) = \ln(e^{x^3})$

$$f'(x) = \frac{1}{e^{x^3}} \cdot e^{x^3} \cdot 3x^2$$

$$= 3x^2$$

(16) $f(x) = \ln 5 \qquad f'(x) = 0$

(17) $f(x) = \ln(\ln x)$

$$= \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$= \frac{1}{x \ln x}$$

(18) $f(x) = \ln[\ln(x^3 + 9x)]$

$$f'(x) = \frac{1}{\ln(x^3+9x)} \cdot \frac{1}{(x^3+9x)} \cdot 3x^2+9$$

$$= \frac{(3x^2+9)}{\ln(x^3+9x)(x^3+9x)}$$

(19) $f(x) = x^3 \ln x - \frac{1}{2}x^2$

$$f'(x) = x^3\left(\frac{1}{x}\right) + \ln x\,(3x^2) - x$$

$$= x^2 + \ln x\,(3x^2) - x$$

(20) $x^2 + \ln(x + y^2) = 4$

$$2x + \frac{1}{x+y^2}\left(1 + 2y\frac{dy}{dx}\right) = 0$$

$$2x(x+y^2) + 1 + 2y\frac{dy}{dx} = 0$$

$$2x^2 + 2xy^2 + 1 + 2y\,dy/dx = 0$$

$$2y\frac{dy}{dx} = -2x^2 \mp 2xy^2 - 1$$

$$\frac{dy}{dx} = \frac{-2x^2 - 2xy^2 - 1}{2y}$$