3.8 Factoring Special Polynomials

Lesson Focus

Investigate some special factoring problems

Factor the following:

$x^2 - 16$

$4x^2 - 12x + 9$

Perfect Square Trinomial

- In factored form, a perfect square trinomial is:

$$a^2 + 2ab + b^2 = (a + b)(a + b)$$
$$a^2 + 2ab + b^2 = (a + b)^2$$

- And

$$a^2 - 2ab + b^2 = (a - b)(a - b)$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Example - PST

Factor each trinomial. Verify by multiplying the factors.

a) $4x^2 + 12x + 9$

$(2x)(3) \cdot 2$

PST

$(2x + 3)(2x + 3)$

$= (2x+3)^2$

b) $4 - 20x + 25x^2$

$(2)(5x) \cdot 2$

$= (2 - 5x)(2 - 5x)$

Example – PST – Your Turn

Factor each trinomial. Verify by multiplying the factors.

a) $36x^2 + 12x + 1$

$(6x)(1) \cdot 2$

$= (6x + 1)(6x + 1)$

b) $16 - 56x + 49x^2$

$(4)(7x) \cdot 2$

$= (4 - 7x)(4 - 7x)$

Difference of Squares

DOS

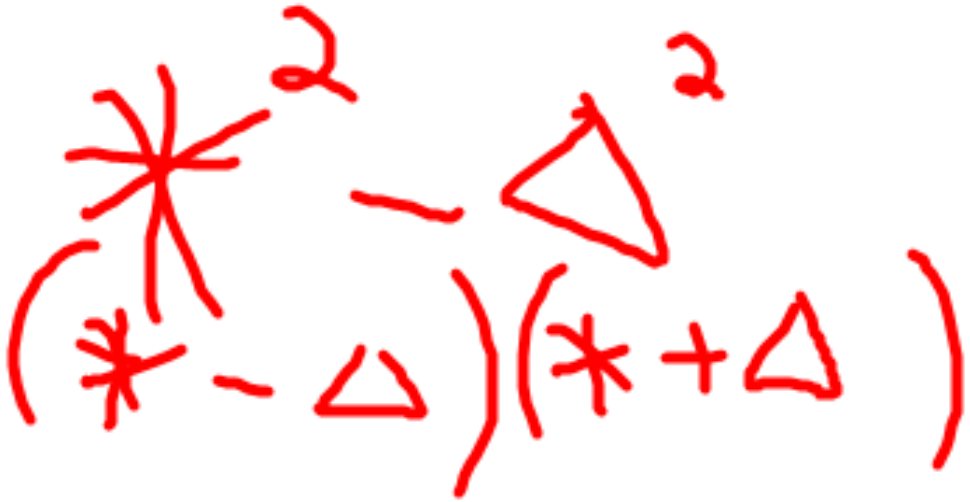
Subtracting 2 terms all perfect squares

- A difference of squares has the form:

$$a^2 - b^2$$

- In factored form:

$$a^2 - b^2 = (a + b)(a - b)$$



Example – DOS

Factor each binomial.

a) $25 - 36x^2$

$(5-6x)(5+6x)$

b) $5x^4 - 80y^4$

$5(x^4 - 16y^4)$

$5(x^2 - 4y^2)(x^2 + 4y^2)$

$5(x-2y)(x+2y)(x^2+4y^2)$

Example – DOS – Your Turn

Factor each binomial.

a) $81m^2 - 49$

$(9m-7)(9m+7)$

b) $162v^4 - 2w^4$

$2(81v^4 - w^4)$

$2(9v^2 - w^2)(9v^2 + w^2)$

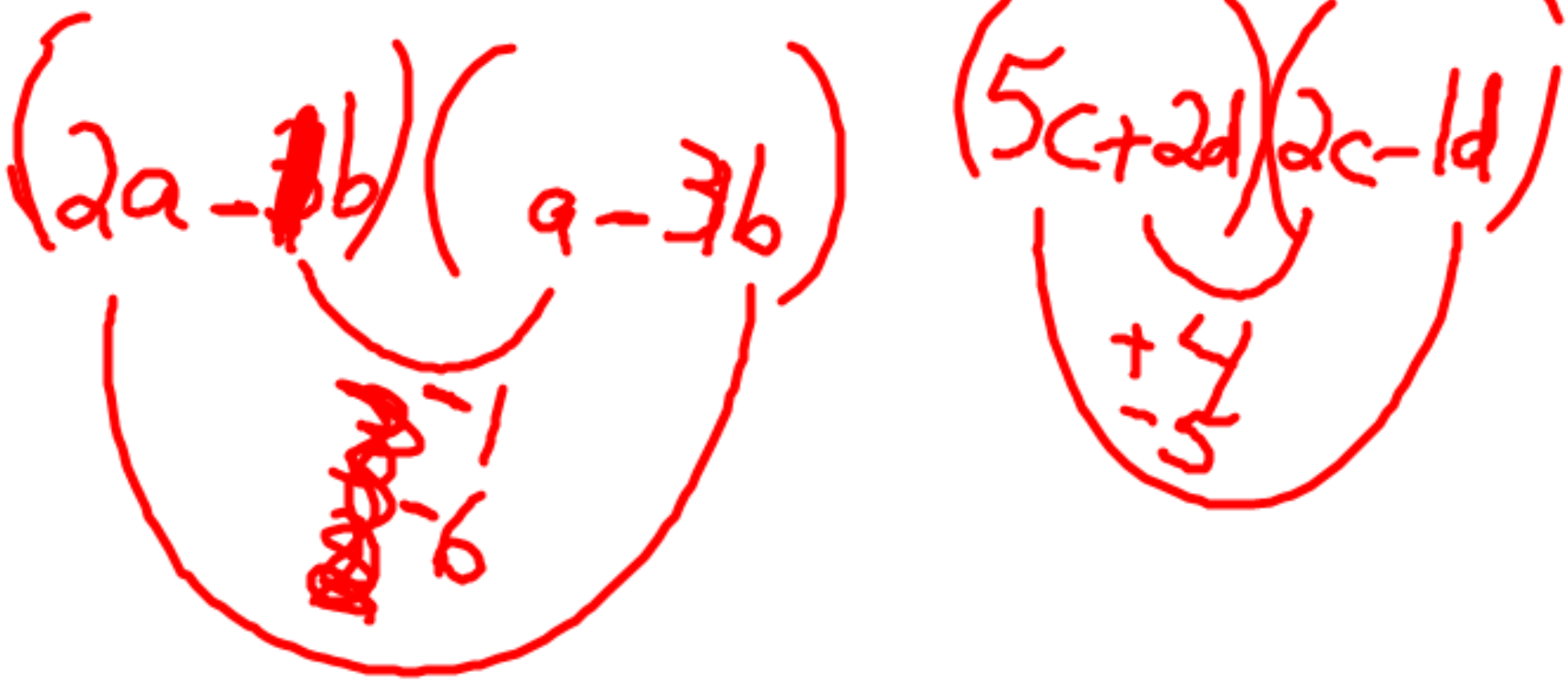
$2(3v - w)(3v + w)(9v^2 + w^2)$

Example – Two Variables

Factor each trinomial. Verify by multiplying the factors.

a) $2a^2 - 7ab + 3b^2$

b) $10c^2 - cd - 2d^2$



Example – Two Variables – Your Turn

Factor each trinomial. Verify by multiplying the factors.

a) $5c^2 - 13cd + 6d^2$

b) $3p^2 - 5pq - 2q^2$

Homework

P. 194-195

4, 5, 6, 8, 10, 11, 13

Handouts

Bonus.

$25x^2 - 100$

$= 25(x^2 - 4)$

$= 25(x + 2)(x - 2)$

Factoring the Difference of Squares

Factor each completely.

1) $9x^2 - 1$

2) $4n^2 - 49$

3) $36k^2 - 1$

4) $p^2 - 36$

5) $2x^2 - 18$

6) $196n^2 - 144$

7) $180m^2 - 5$

8) $294r^2 - 150$

9) $150k^2 - 216$

10) $20a^2 - 45$

11) $3n^2 - 75$

12) $24x^3 - 54x$

13) $a^2 - 25b^2$

14) $4x^2 + 49y^2$

15) $25x^2 + 16y^2$

16) $6a^2 + 96b^2$

17) $x^2 - 9y^2$

18) $49x^2 - 25y^2$

19) $9x^2 - 16y^2$

20) $54v^2 - 6u^2$

21) $36a^4 - 25b^4$

22) $2x^4r - 72y^4r$

23) $125m^4 - 20n^4$

24) $216x^4ay - 6y^5a$

25) $4x^4 - 144y^4$

26) $4x^4m - 36y^4m$

27) $7x^4 - 28y^4$

28) $7x^4 - 343y^4$

29) $16m^5 - n^6$

Answers to Factoring the Difference of Squares

1) $(3x+1)(3x-1)$
2) $(2n+7)(2n-7)$
3) $(6k+1)(6k-1)$
4) $(p+6)(p-6)$
5) $2(x+3)(x-3)$
6) $4(7n+6)(7n-6)$
7) $5(6m+1)(6m-1)$
8) $6(7r+5)(7r-5)$
9) $6(5k+6)(5k-6)$
10) $5(2a+3)(2a-3)$
11) $3(n+5)(n-5)$
12) $6x(2x+3)(2x-3)$
13) $(a+5b)(a-5b)$
14) Not factorable
15) Not factorable
16) $6(a^2+16b^2)$
17) $(x+3y)(x-3y)$
18) $(7x+5y)(7x-5y)$
19) $(3x+4y)(3x-4y)$
20) $6(3v+u)(3v-u)$
21) $(6a^2+5b^2)(6a^2-5b^2)$
22) $2r(x^2+6y^2)(x^2-6y^2)$
23) $5(5m^2+2n^2)(5m^2-2n^2)$
24) $6ay(6x^2+y^2)(6x^2-y^2)$
25) $4(x^2+6y^2)(x^2-6y^2)$
26) $4m(x^2+3y^2)(x^2-3y^2)$
27) $7(x^2+2y^2)(x^2-2y^2)$
28) $7(x^2+7y^2)(x^2-7y^2)$
29) $(4m^3+n^3)(4m^3-n^3)$

FACTORING PERFECT SQUARE TRINOMIALS

Courtesy of Harold Hiken

Factor each of the following perfect square trinomials. In the last two problems, look for a greatest common factor to remove first.

1) $a^2 + 4a + 4$

2) $p^2 + 2p + 1$

3) $x^2 - 10x + 25$

4) $y^2 - 8y + 16$

5) $r^2 + 24r + 144$

6) $k^2 + 121 + 22k$

7) $y^2 - 20y + 100$

8) $49 + 14a + a^2$

9) $9x^2 + 24x + 16$

10) $16t^2 - 40t + 25$

11) $25k^2 - 20k + 4$

12) $4c^2 + 12c + 9$

13) $100 - 20m + m^2$

14) $64y^2 - 48ya + 9a^2$

15) $100a^2 - 140ab + 49b^2$

16) $49x^2 + 28xy + 4y^2$

17) $x^3 + 6x^2y^2 + 9xy^3$

18) $4k^3w + 20k^2w^2 + 25kw^3$

1) $(a + 2)(a + 2)$
2) $(p + 1)(p + 1)$
3) $(x - 5)(x - 5)$
4) $(y - 4)(y - 4)$
5) $(r + 12)(r + 12)$
6) $(k + 11)(k + 11)$
7) $(y - 10)(y - 10)$
8) $(7 + a)(7 + a)$
9) $(3x + 4)(3x + 4)$
10) $(4t - 5)(4t - 5)$
11) $(5k - 2)(5k - 2)$
12) $(2c + 3)(2c + 3)$
13) $(10 - m)(10 - m)$
14) $(8y - 3a)(8y - 3a)$
15) $(10a - 7b)(10a - 7b)$
16) $(7x + 2y)(7x + 2y)$
17) $x(x + 3)(x + 3)$
18) $kw(2k + 5w)(2k + 5w)$