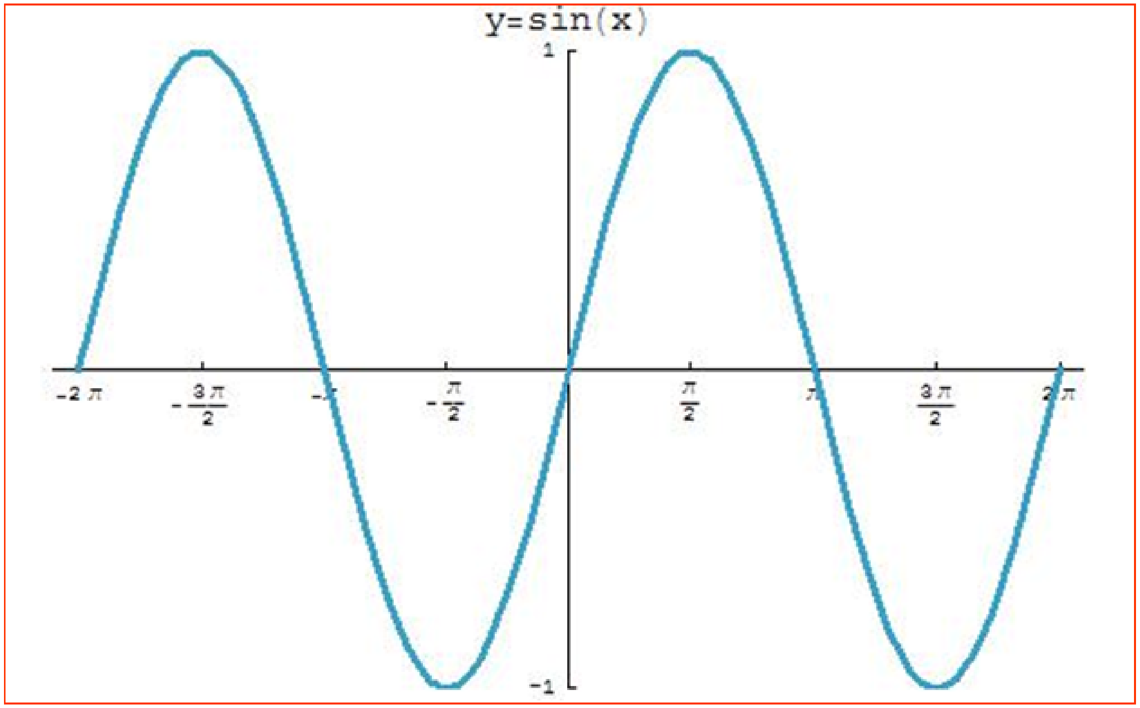
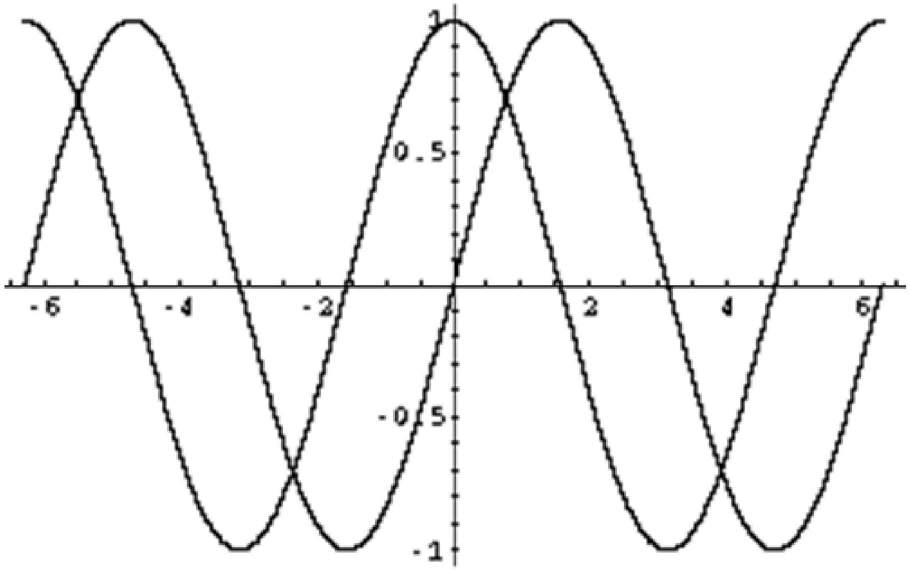
3.5 Derivatives of Trigonometric Functions

Lets revisit the sine curve





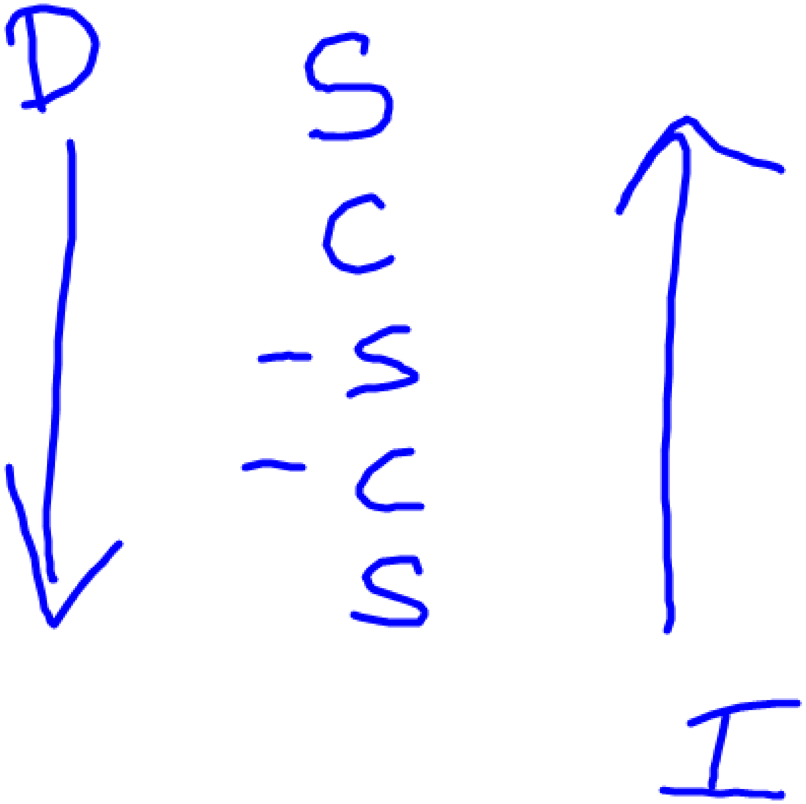
$$\frac{d}{dx}\sin x = \cos x$$

If $y = \sin u$

$$\frac{dy}{dx} = \cos u \cdot \frac{du}{dx}$$

If $y = \cos u$

$$\frac{dy}{dx} = -\sin u \frac{du}{dx}$$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Similarly it can be shown that:

$$\frac{d}{dx}\cos x = -\sin x$$

Ex. 1 Differentiate the following:

a) $y = \sin 3x$

$y' = (\cos 3x)3$

$= 3\cos(3x)$

b) $y = \cos(x^2 + 2)$

$y' = -\sin(x^2+2)(2x)$

$= -2x\sin(x^2+2)$

$c)\ f(x) = \sin(x^3)$

$f'(x) = \cos(x^3) \cdot 3x^2$

$= 3x^2 \cos(x^3)$

$d)\ f(x) = \sin^3 2x$

$f(x) = (\sin 2x)^3$

$f'(x) = 3(\sin 2x)^2 (\cos 2x)\, 2$

$= 6 \sin^2 2x \cos 2x$

$e)\ y=\cos^3\left(x^2-5\right)$

$$y=\left(\cos\left(x^2-5\right)\right)^3$$

$$y'=3\left(\cos\left(x^2-5\right)\right)^2\cdot\left(-\sin\left(x^2-5\right)\right)\left(2x\right)$$

$$=-6x\cos^2\left(x^2-5\right)\sin\left(x^2-5\right)$$

$f)\ y = x^2 \cos(4x+2)$

$$y' = x^2\left(-4\sin(4x+2)\right) + \cos(4x+2)\cdot 2x$$

$$= -4x^2 \sin(4x+2) + 2x\cos(4x+2)$$

$$= -2x\left(2x\sin(4x+2) - \cos(4x+2)\right)$$

$g)\ y = \cos(\sin 2x)$

$y' = -\sin(\sin 2x) \cdot (\cos 2x)\, 2$

$= -2\sin(\sin 2x)\cos 2x$

$h)\ y = \sin^3\left(x^2+1\right)\cos^2(3x)$

$$y = \left(\sin(x^2+1)\right)^3\left(\cos(3x)\right)^2$$

$$y' = \left(\sin(x^2+1)\right)^3 \cdot 2\cos(3x)(-\sin 3x)(3)$$
$$+ \left(\cos(3x)\right)^2 \cdot 3\left(\sin(x^2+1)\right)^2 \cos(x^2+1)(2x)$$

$$y' = -6\sin^3(x^2+1)\cos 3x \sin 3x$$
$$+ 6x\sin^2(x^2+1)\cos^2(3x)\cos(x^2+1)$$

$$= -6\sin^2(x^2+1)\cos 3x\left(\sin(x^2+1)\sin 3x - x\cos 3x\cos(x^2+1)\right)$$

$y - 1 = 1(x - 0)$

$y = x + 1$

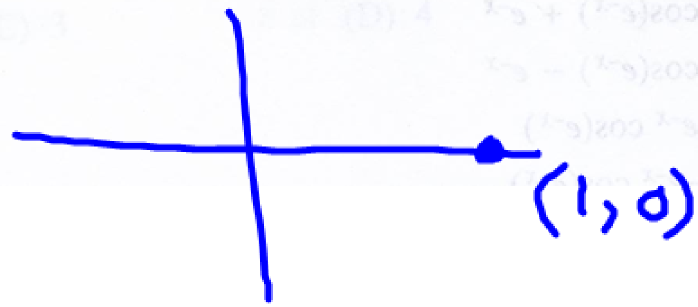
18. An equation of the line tangent to the graph of $y = x + \cos x$ at the point (0, 1) is

(A) $y = 2x + 1$ (B) $y = x + 1$ (C) $y = x$ (D) $y = x - 1$ (E) $y = 0$

$y' = 1 - \sin x$

$y'(0) = 1 - \sin 0$

$= 1$



14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x \cos 2x)$

(E) $2x(\sin 2x + x \cos 2x)$

$y - 0 = -2(x - \pi/4)$

10. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \frac{\pi}{4}$ is

(A) $y - 1 = -\left(x - \frac{\pi}{4}\right)$

(B) $y - 1 = -2\left(x - \frac{\pi}{4}\right)$

(C) $y = 2\left(x - \frac{\pi}{4}\right)$

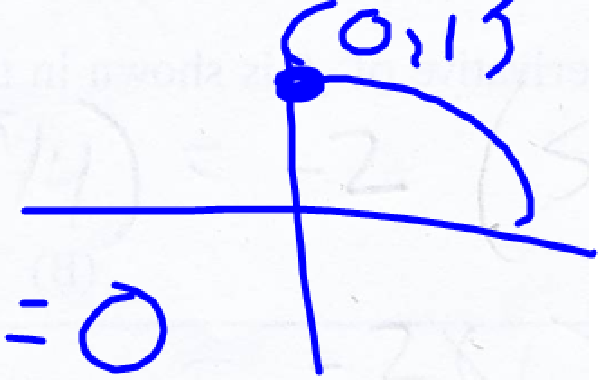
(D) $y = -\left(x - \frac{\pi}{4}\right)$

(E) $y = -2\left(x - \frac{\pi}{4}\right)$

$y(\pi/4) = \cos\left(2\frac{\pi}{4}\right)$

$= \cos(\pi/2)$

$= 0$



$(\pi/4, 0)$

$y' = -2\sin 2x$

$y'(\pi/4) = -2\sin 2(\pi/4) = -2$

http://archives.math.utk.edu/visual.calculus/2/product_rule.4/index.html

1997 MC Question

7. $\dfrac{d}{dx}\cos^2(x^3) =$

(A) $6x^2 \sin(x^3)\cos(x^3)$

(B) $6x^2 \cos(x^3)$

(C) $\sin^2(x^3)$

(D) $-6x^2 \sin(x^3)\cos(x^3)$

(E) $-2 \sin(x^3)\cos(x^3)$

If we know the derivative of sin x and cos x, we can develop the derivative for tan x.

$y = \tan x$

$y = \dfrac{\sin x}{\cos x}$

$y' = \dfrac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \sec^2 x$

If $y = \tan u$

$$\frac{dy}{dx} = \sec^2 u \cdot \frac{du}{dx}$$

Ex. Find the derivative

$$y = 3\tan x^2$$

$$y' = 3\left[\sec^2 x^2\right] \cdot 2x$$

$$= 6x\sec^2 x^2$$

What is the first derivative of a cow?

Prime Rib!

Ex. Find the derivative

$$y = \tan^4\left(3x^5\right)$$

28. If $f(x) = \tan(2x)$, then $f'\left(\frac{\pi}{6}\right) =$

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 4 (D) $4\sqrt{3}$ (E) 8

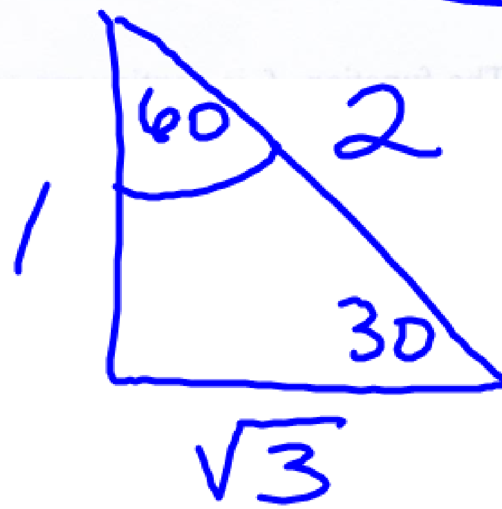
$$y = \tan 2x$$

$$y' = 2\sec^2 2x$$

$$y'(\pi/6) = 2\left(\sec 2(\pi/6)\right)^2$$

$$= 2\left(\sec \pi/3\right)^2$$

$$= 2(2)^2 = 8$$



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#'s 12, 16, 19, 31, 37

AP Text: Page 140
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Recall:

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

Let's develop the reciprocal trigonometric derivatives

$$y = \csc x$$

$$y = \sec x$$

$$y = \cot x$$

Ex.1 Find the derivative of the following:

$a)\ y = \csc\left(x^2 + 2\right)$

$$b)\ y = \sec^2(2x)$$

$$c)\ y = \cot\left(3x^2 - 4x\right)$$

Assignment Handout
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