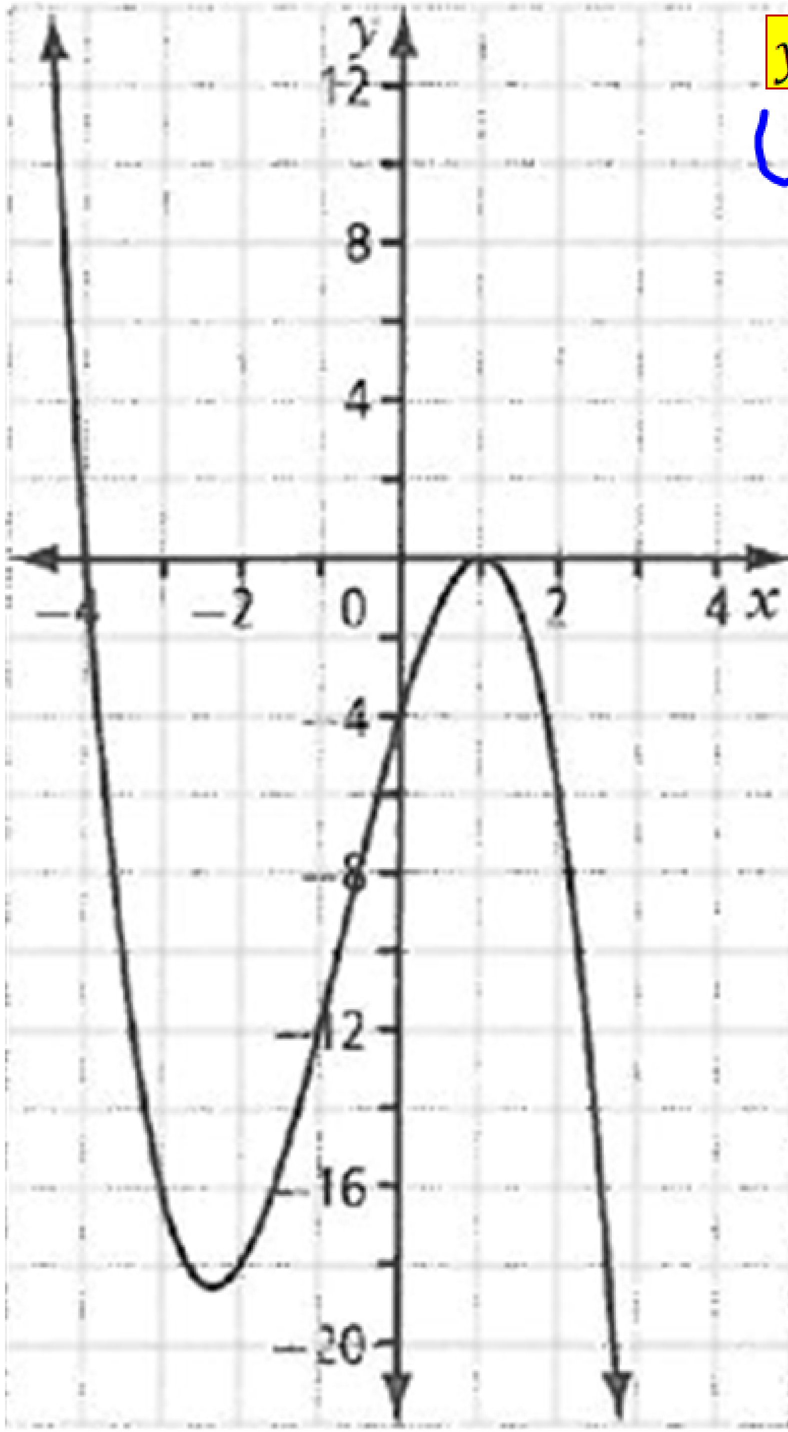
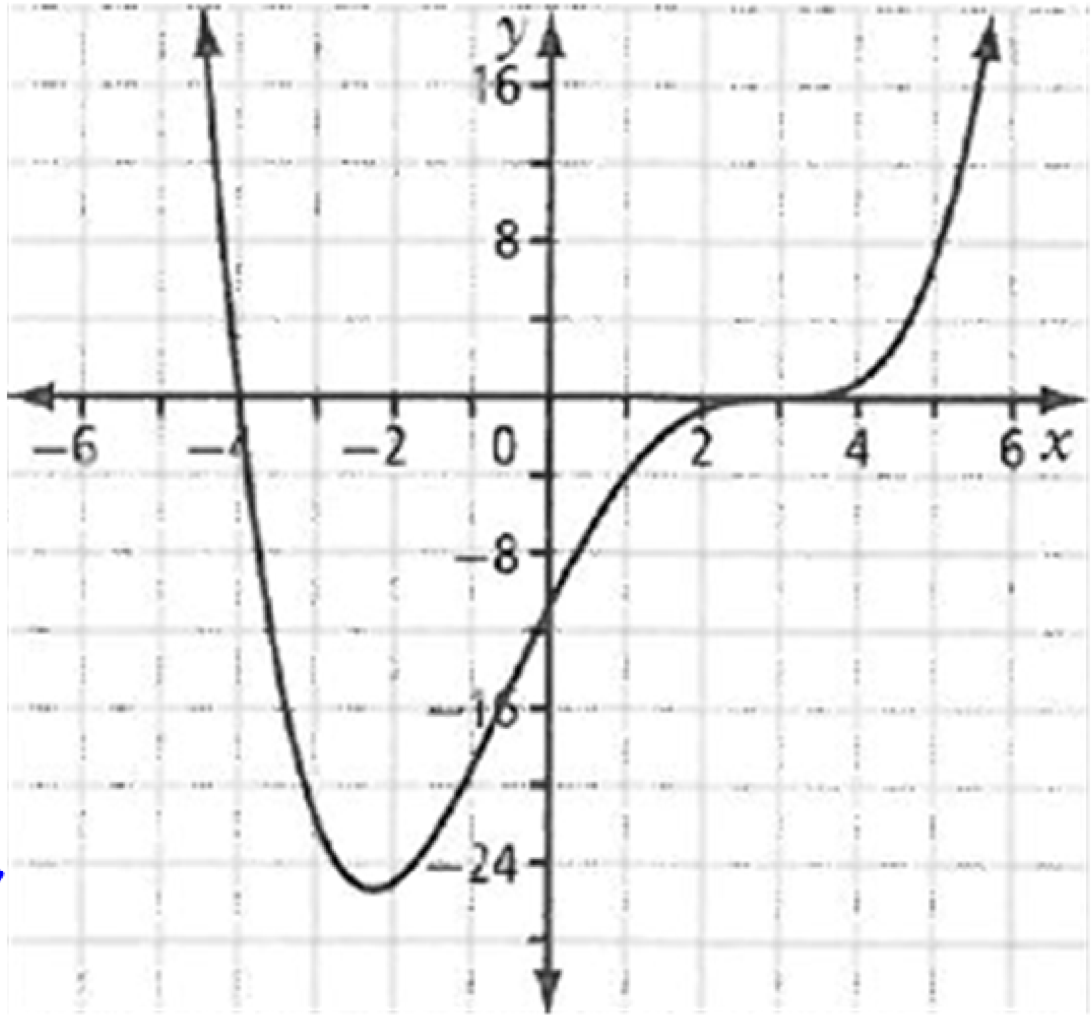
3.4 Graphs of Polynomial Functions



$$y = -(x+4)(x-1)^2$$

$y=(1/9)(x+4)(x-3)^3$



Zeros

x = −4
mult 1

x = 3
mult 3

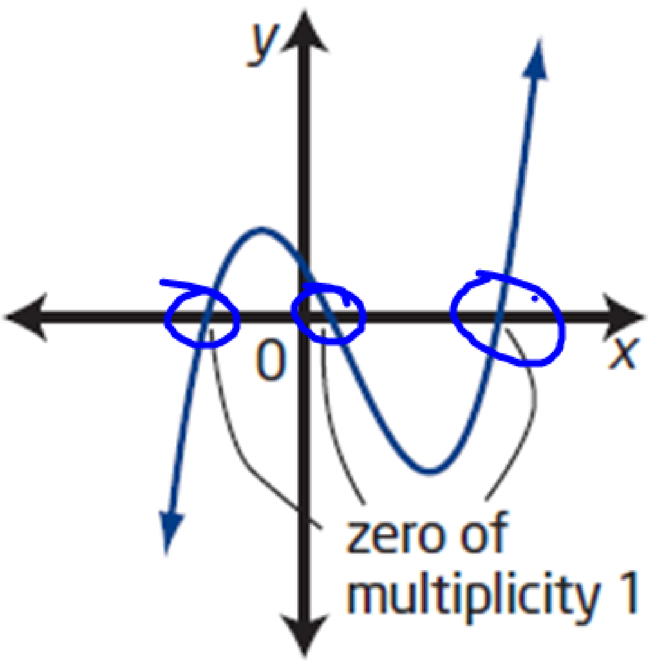
Degree 4

LC +

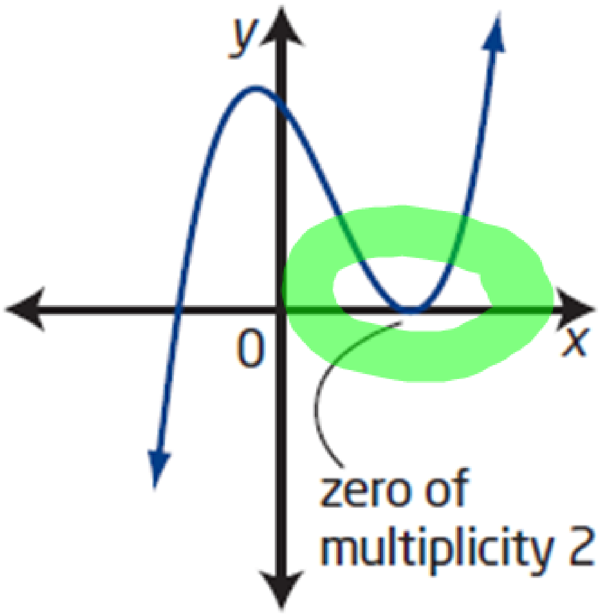
S2 E1

When dealing with graphs of polynomials , we not only are dealing with the zeros of the function, but also with the **multiplicity** of that zero. **Multiplicity** is how the graph reacts when it gets to the x-axis. **Multiplicity** is determined by the exponent on the factor of each zero.

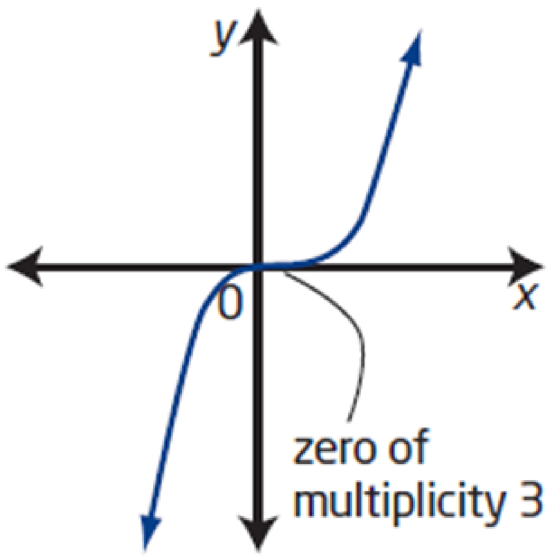
Multiplicity of 1: Say we have a factor of our polynomial being (x-2). When the graph reaches the zero of x=2 the graph acts like a line going through the point x=2

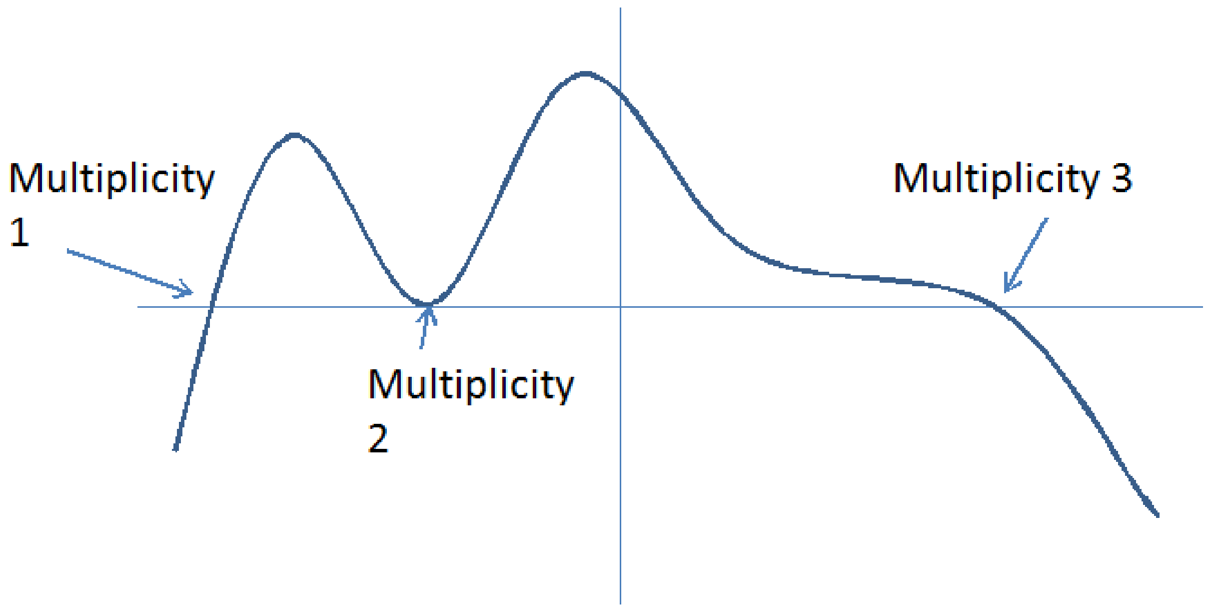


Multiplicity of 2: Say we have a factor of our polynomial being $(x-2)^2$. When the graph reaches the zero of x=2 the graph acts like a parabola at the point x=2. Any even degree multiplicity will act the same, for example degree 4,6,8.



Multiplicity of 3: Say we have a factor of our polynomial being x^3. When the graph reaches the zero of x=0 the graph acts like a "bump and squiggle". Any odd degree multiplicity greater than 1 will act the same, for example degree ~~4,6,8.~~ 3, 5, 7





By looking at the graph of a polynomial, we can determine the multiplicity of each zero.

Example 1 For the following graphs identify the:

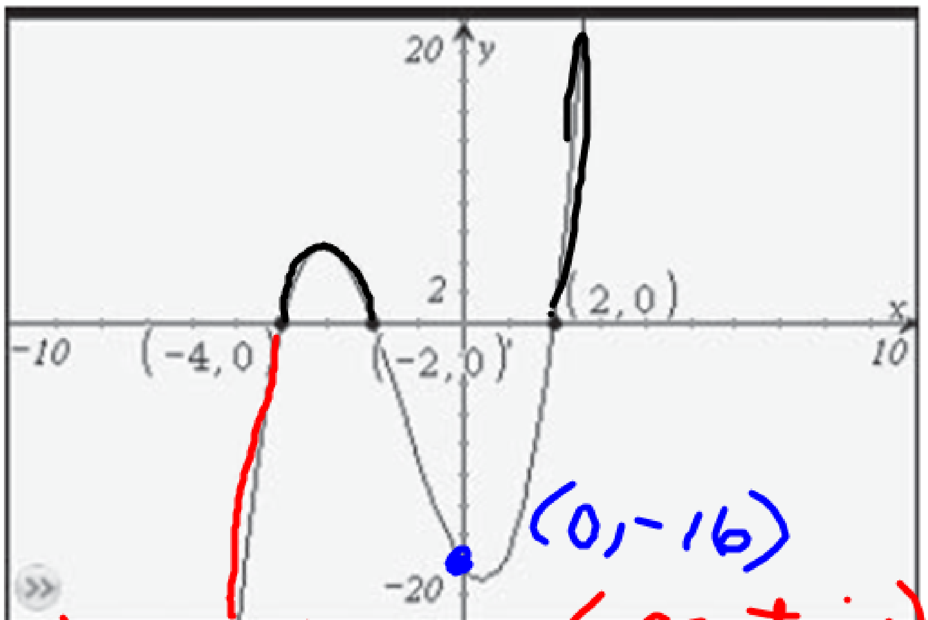
a) least possible degree ODD 3

b) sign of the leading coefficient +

c) the x intercepts and the factors with the least degree

d) the interval where the function is positive and the intervals where it is negative

* e) Find the equation of the polynomial including the leading coefficient



c) $x = -4$
$(x+4)$ factor
mult 1
$x = -2$
$(x+2)$ factor
mult 1
$x = 2$
$(x-2)$
mult 1

d) $f(x) < 0$ (negative)
$x < -4 \cup -2 < x < 2$
$f(x) > 0$ positive
$-4 < x < -2 \cup x > 2$

e) $f(x) = a(x+4)^{1}(x+2)^{1}(x-2)^{1}$

$$-16 = a(0+4)(0+2)(0-2)$$

$$\frac{-16}{-16} = \frac{-16a}{-16}$$

$$1 = a$$

$$\boxed{f(x) = (x+4)(x+2)(x-2)}$$

Example 2 For the following graphs identify the:

a) least possible degree 4

b) sign of the leading coefficient −

c) the x intercepts and the factors with the least degree

d) the interval where the function is positive and the intervals where it is negative

e) Find the equation of the polynomial including the leading coefficient

c) $x = -5$

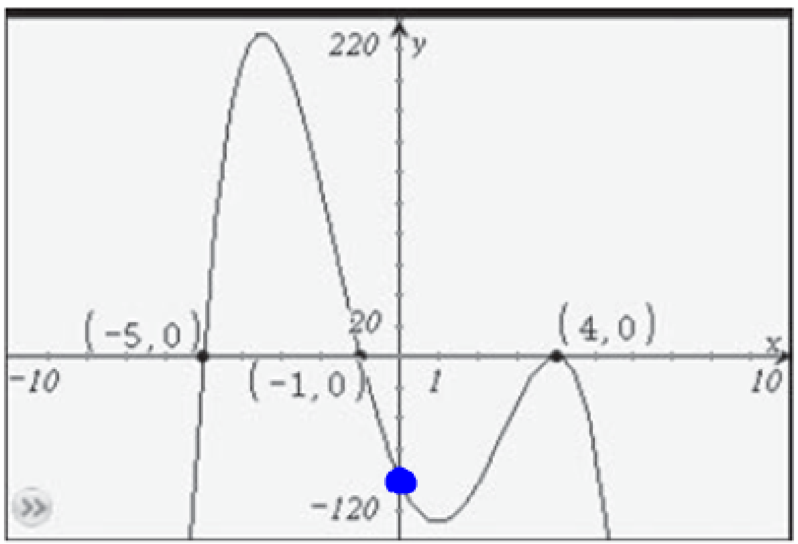
$(x+5)$ mult 1

$x = -1$

$(x+1)$ mult 1

$x = 4$

$(x-4)$ mult 2



(0, −80)

d) f(x) positive

$-5 < x < -1$

f(x) negative

$x < -5 \cup -1 < x < 4 \cup x > 4$

$$y = a(x+5)^1(x+1)^1(x-4)^2$$

$$-80 = a(0+5)(0+1)(0-4)^2$$

$$-80 = 80a$$

$$-1 = a$$

$$\underline{y = -(x+5)(x+1)(x-4)^2}$$

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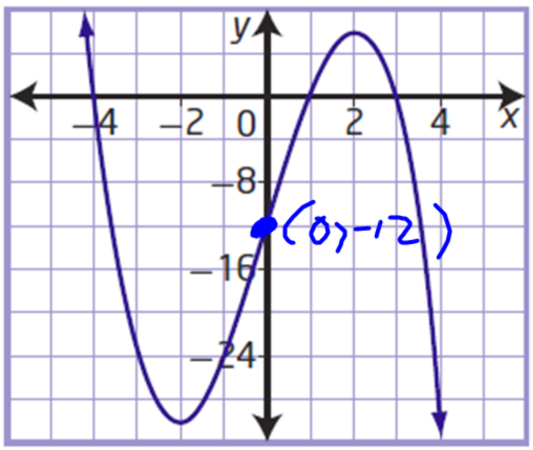
#'S 1-5

b)



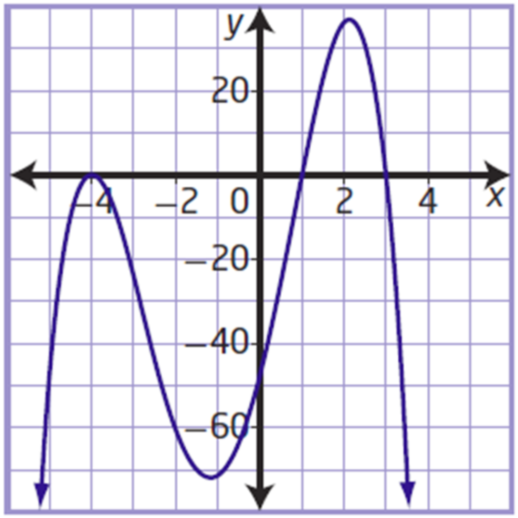
$$y = \overset{-}{\cancel{a}}^{\,1}(x+4)^1(x-1)^1(x-3)^1$$

$$-12 = a(4)(-1)(-3)$$

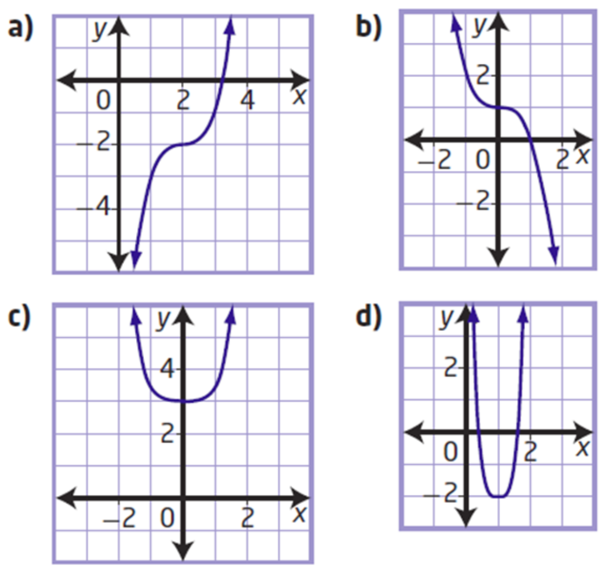
$$-12 = 12a$$

$$-1 = a$$

c)



5. Without using technology, match each graph with the corresponding function. Justify your choice.



A $y = (2(x - 1))^4 - 2$ **B** $y = (x - 2)^3 - 2$

C $y = 0.5x^4 + 3$ **D** $y = (-x)^3 + 1$

Example 3 Sketch the graphs of the polynomial function $y = -(x+2)^3(x-4)^1$

a) The degree 4

b) The leading coefficient $-$

c) End Behaviour S3 E4

d) Zeros

e) Y-intercept

f) Intervals the function is positive or negative

factored form.

d) $x=-2$ mult 3 $\quad x=4$ mult 1

e) let $x=0$

$y=-(0+2)^3(0-4)$

$y=(-8)(-4)=32$

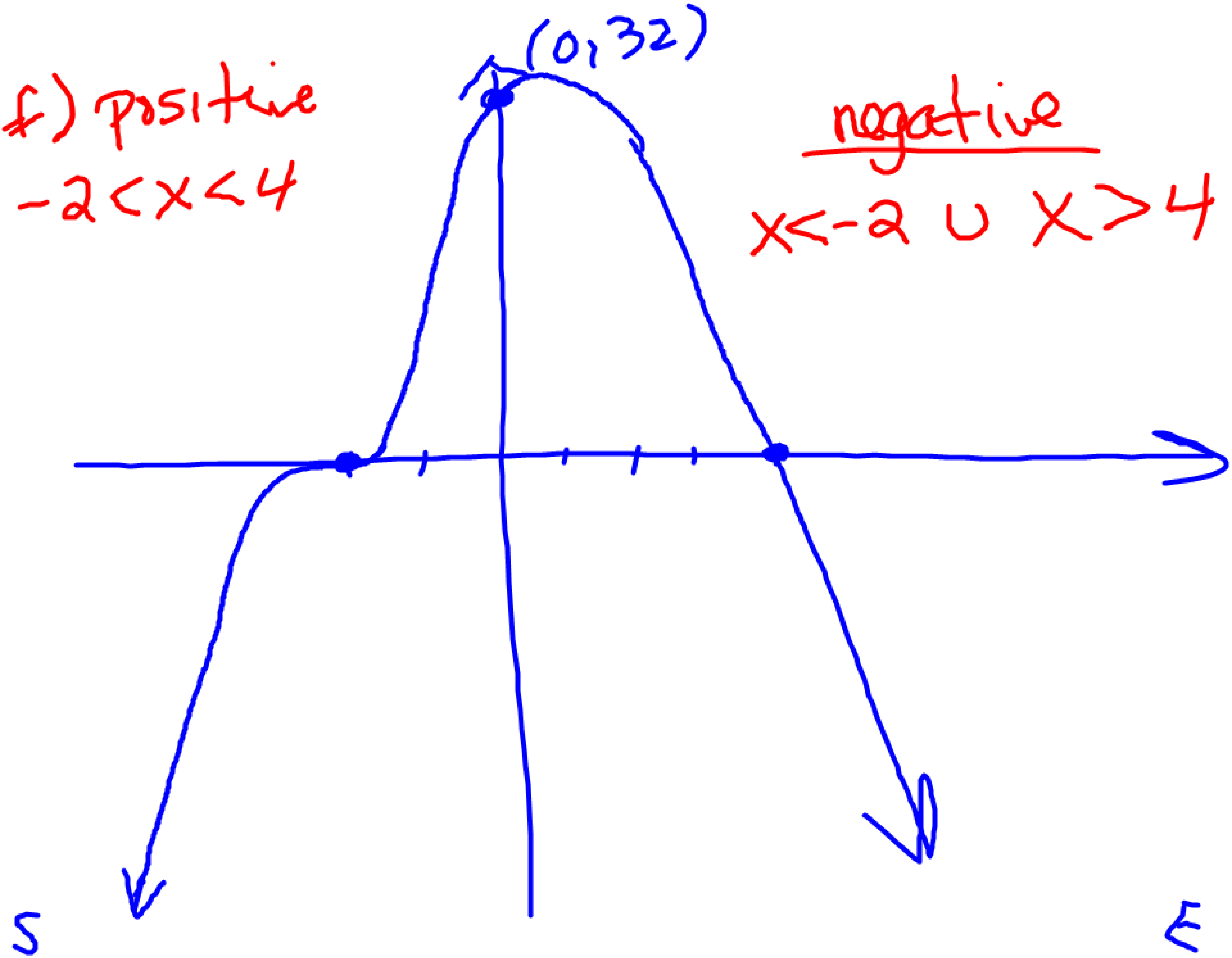
$(0,32)$

f) positive

$-2<x<4$

<u>negative</u>

$x<-2 \cup x>4$



$y = (x+4)(x-2)^2(x+1)^3(x-3)$

Degree

LC

S

E

Zeroes/mult

y int

$f(x)$

Example 4 Sketch the graphs of the polynomial function $y = -2x^3 + 6x - 4$

a) The degree 3

b) The leading coefficient —

c) End Behaviour S2   E4

d) Zeros

e) Y-intercept $(0,-4)$

f) Intervals the function is positive or negative

d) $f(1) = -2(1)^3 + 6(1) - 4$

$= 0$

$(x-1)$ is a factor

$$\begin{array}{r|rrrr} 1 & -2 & 0 & 6 & -4 \\ & \downarrow & -2 & -2 & 4 \\ \hline & -2 & -2 & 4 & \boxed{0} \end{array}$$

$-2x^2 - 2x + 4$

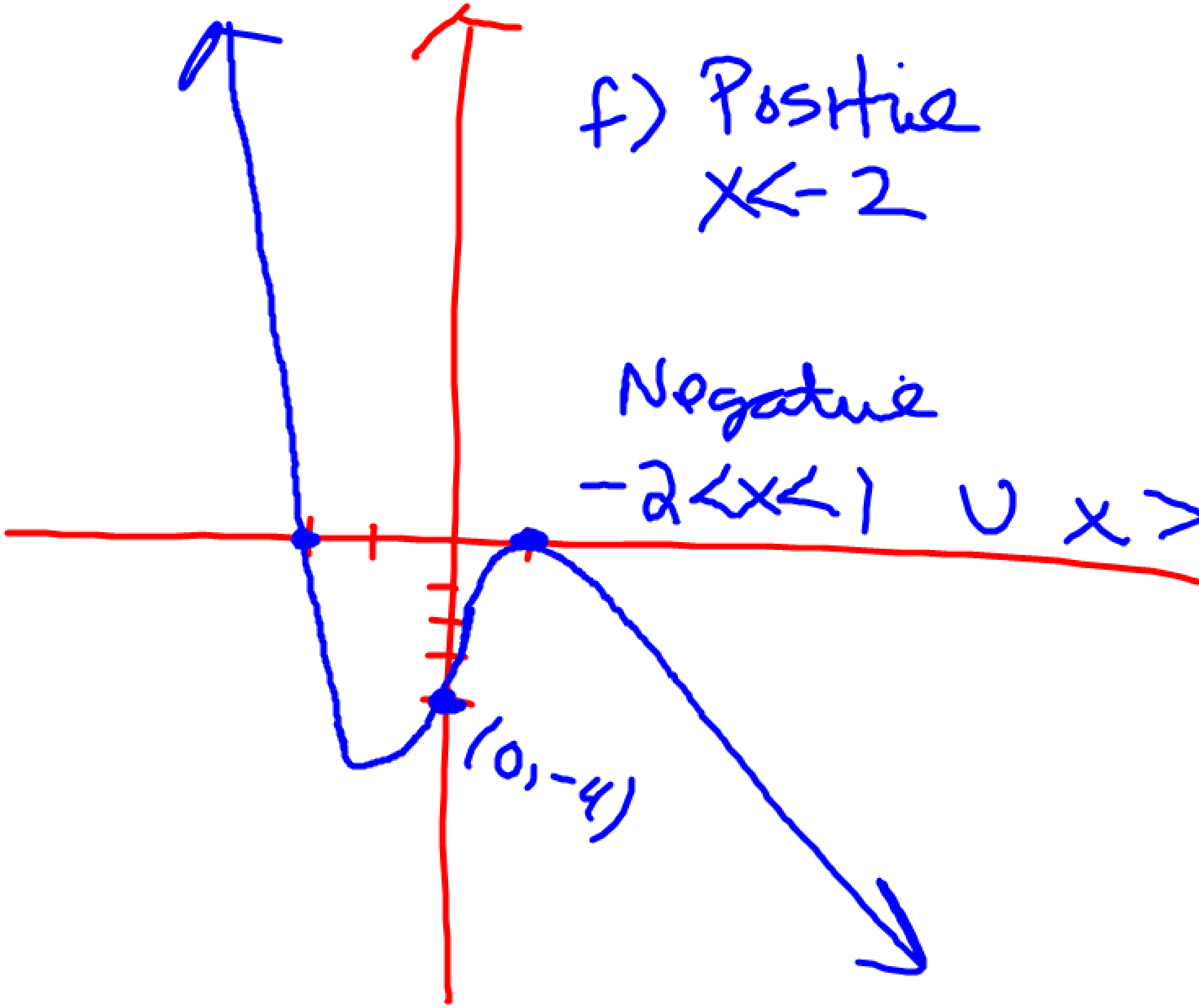
$$f(x) = (x-1)(\underbrace{-2x^2-2x+4})$$

$$f(x) = -2(x-1)(x^2+x-2)$$

$$= -2(x-1)(x+2)(x-1)$$

$$\boxed{f(x) = -2(x+2)(x-1)^2}$$

S



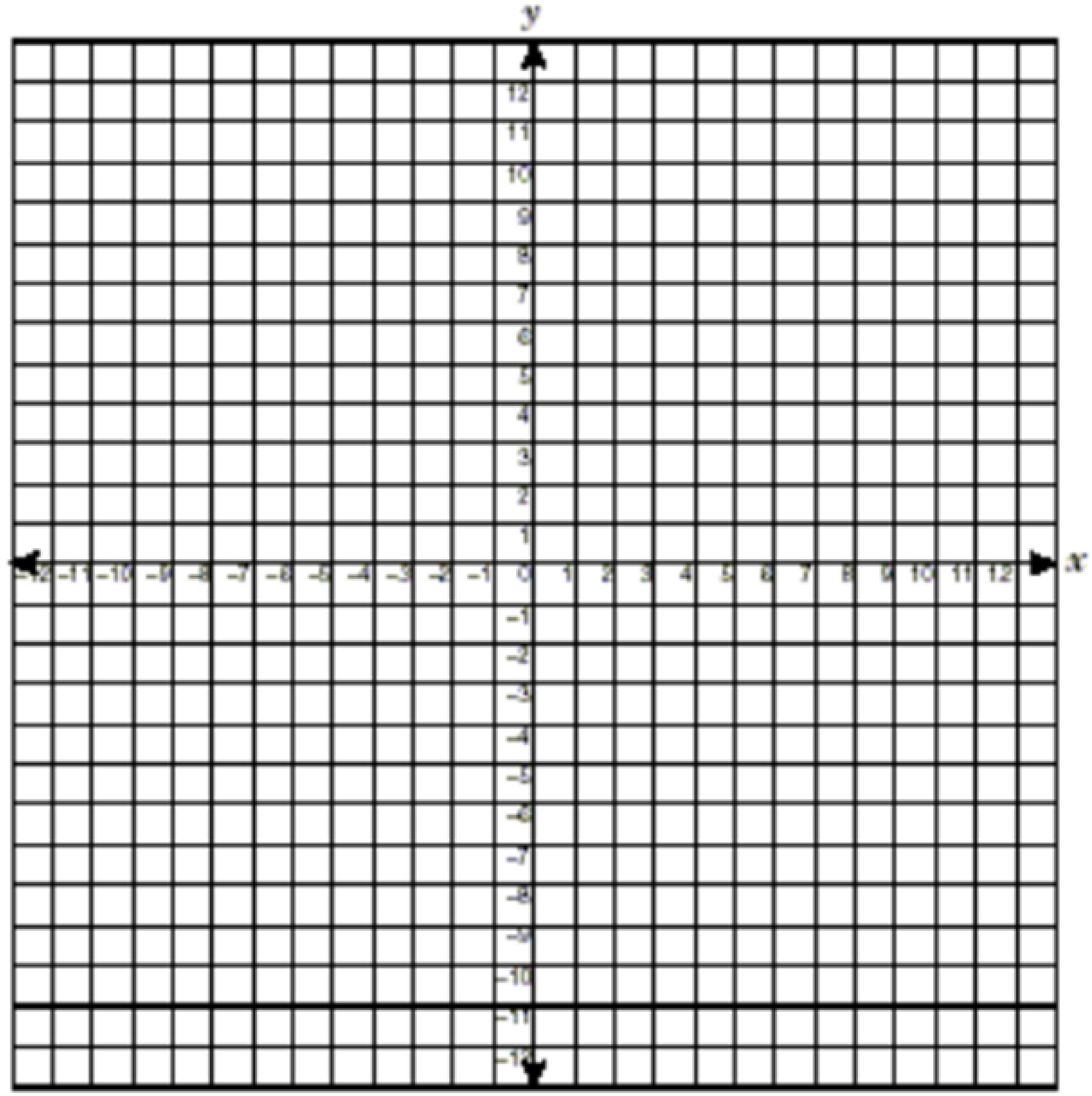
f) Positive

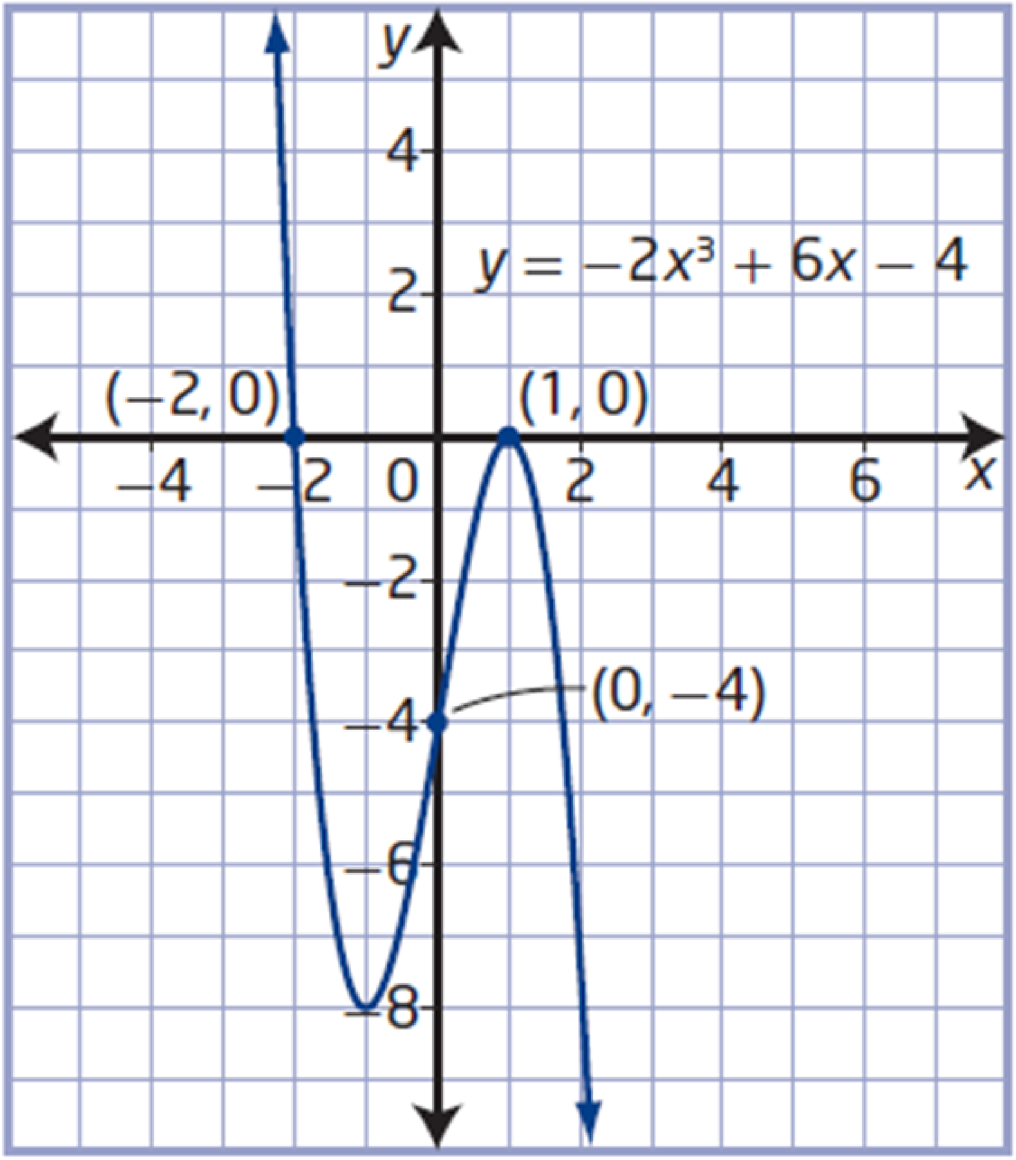
$x < -2$

Negative

$-2 < x < 1 \cup x > 1$

E


y
x



y
x
4
2
−2
−4
−6
−8
−4
−2
0
2
4
6
$y = -2x^3 + 6x - 4$
(−2, 0)
(1, 0)
(0, −4)

Your Turn: Sketch the graphs of the polynomial function $g(x) = (x-2)^3(x+1)$

a) The degree

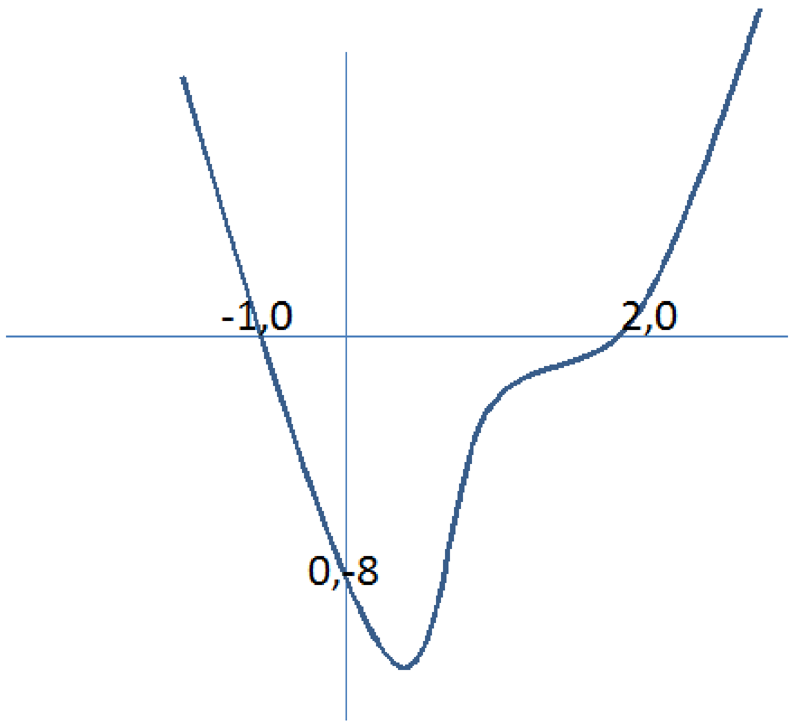
b) The leading coefficient

c) End Behaviour

d) Zeros

e) Y-intercept

f) Intervals the function is positive or negative


-1,0
2,0
0,-8

Recall our transformations used earlier this year!

Parameter	Transformation
k	• Vertical translation up or down • $(x, y) \rightarrow (x, y + k)$
h	• Horizontal translation left or right • $(x, y) \rightarrow (x + h, y)$
a	• Vertical stretch about the x-axis by a factor of $\|a\|$ • For $a < 0$, the graph is also reflected in the x-axis. • $(x, y) \rightarrow (x, ay)$
b	• Horizontal stretch about the y-axis by a factor of $\frac{1}{\|b\|}$ • For $b < 0$, the graph is also reflected in the y-axis. • $(x, y) \rightarrow \left(\frac{x}{b}, y\right)$

Example 5

Apply Transformations to Sketch a Graph

Graph $y=x^3$ then graph the transformations for

$Y=-2(4(x-1))^3+3$

$y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

$a=-2$

$b=4$

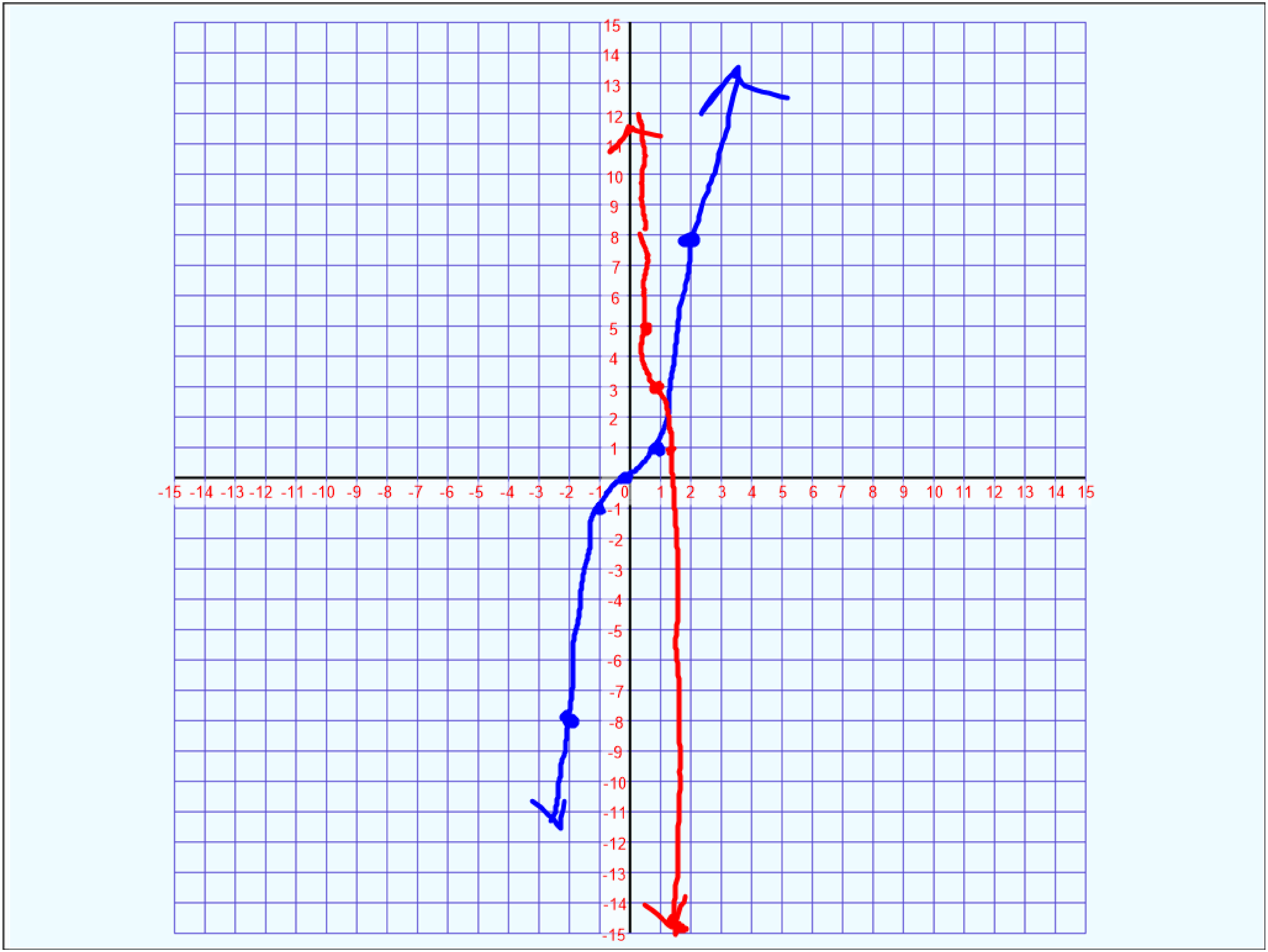
$h=1$

$k=3$

$\left(\frac{x}{b}+h, ay+k\right)$

$\left(\frac{x}{4}+1, -2y+3\right)$

x	y
$\frac{1}{2}$	19
$\frac{3}{4}$	5
1	3
$\frac{5}{4}$	1
$\frac{3}{2}$	-15


15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
-15 -14 -13 -12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
-1
-2
-3
-4
-5
-6
-7
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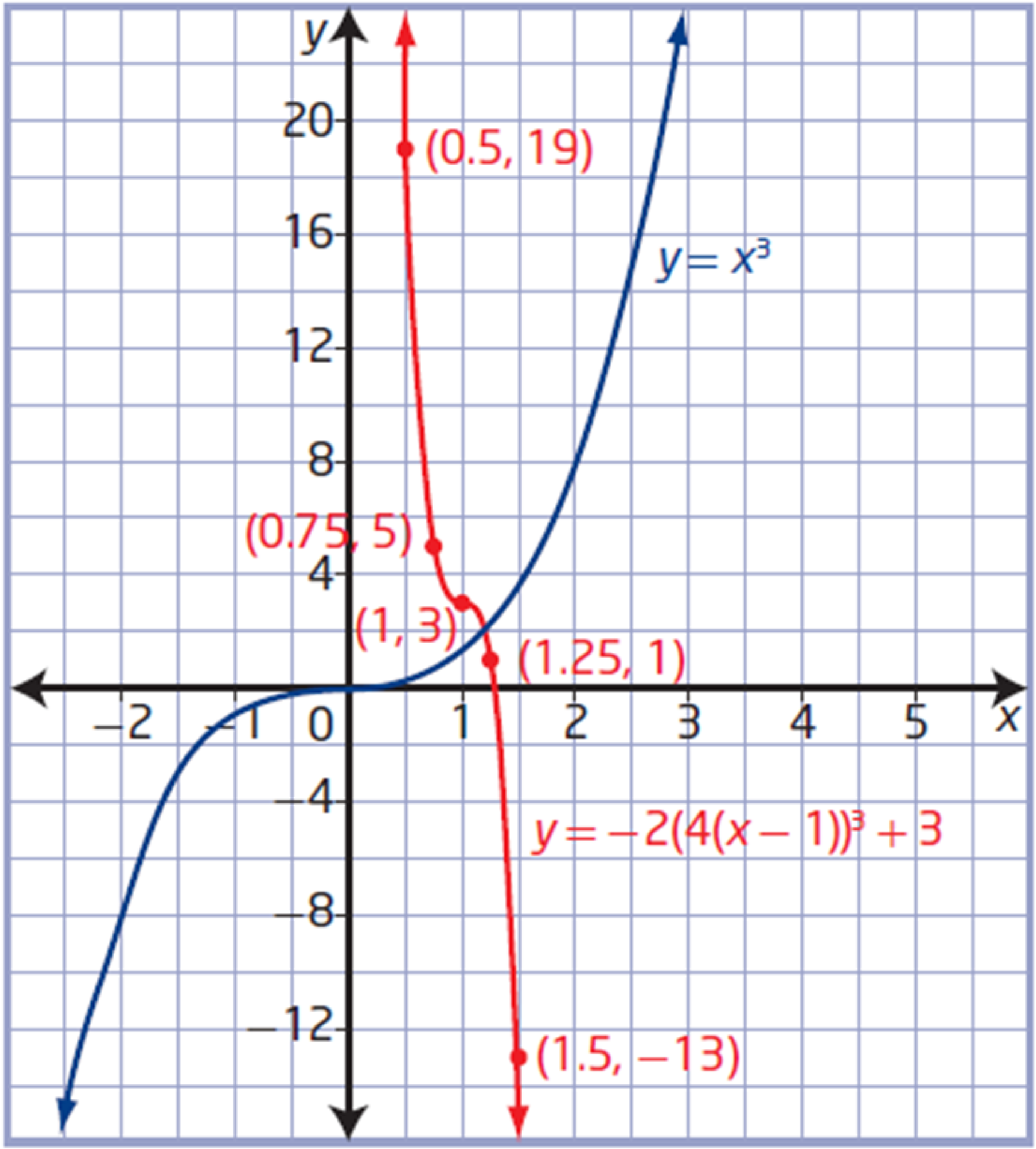
-12
-13
-14
-15



y
20
16
12
8
4
0
−4
−8
−12
−2
−1
1
2
3
4
5
x
(0.5, 19)
(0.75, 5)
(1, 3)
(1.25, 1)
(1.5, −13)
y = x³
y = −2(4(x − 1))³ + 3

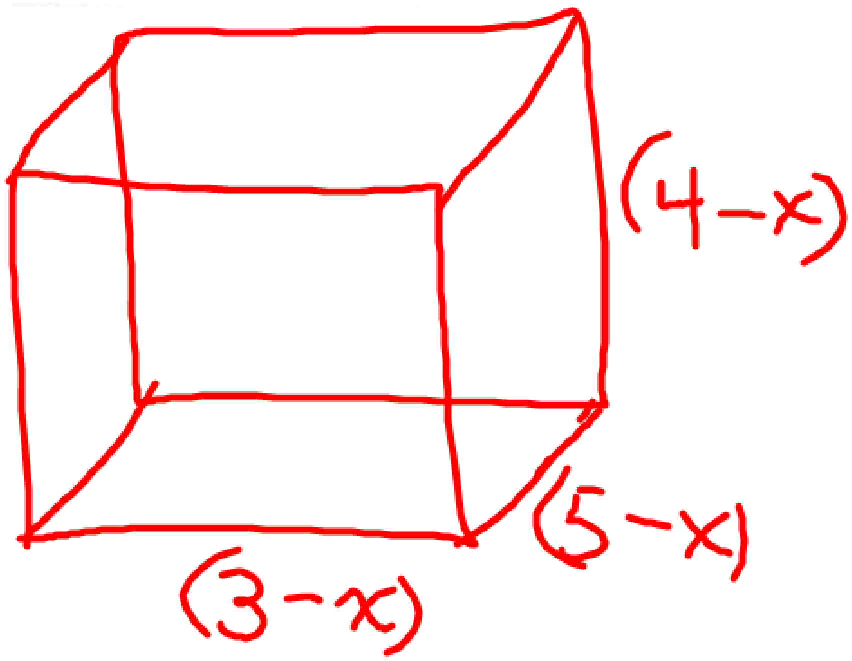
Example 4

Model and Solve Problems Involving Polynomial Functions

Bill is preparing to make an ice sculpture. He has a block of ice that is 3 ft wide, 4 ft high, and 5 ft long. Bill wants to reduce the size of the block of ice by removing the same amount from each of the three dimensions. He wants to reduce the volume of the ice block to 24 ft^3.

a) Write a polynomial function to model this situation.

b) How much should he remove from each dimension?



let x = amount removed

$$(3-x)(5-x)(4-x)=24$$

$$(3-x)(5-x)(4-x)=24$$

$$(15-3x-5x+x^2)(4-x)=24$$

$$(x^2-8x+15)(4-x)=24$$

$$\cancel{4x^2}-\cancel{32}x+60-\cancel{x^3}+\cancel{8x^2}-15x=24$$

$$0=x^3-12x^2+47x-36$$

$$(1)^3-12(1)^2+47(1)-36=0$$

$(x-1)$ is a factor.

$$
\begin{array}{r|rrrr}
1 & 1 & -12 & 47 & -36 \\
 & \downarrow & 1 & -11 & 36 \\
\hline
 & 1x^2 & -11x & +36 & \boxed{0}
\end{array}
$$

$$(x-1)(x^2-11x+36)=P(x)$$

$x=1$ only zero.

$\therefore$ New dimensions are 2 ft x 3 ft x 4 ft.

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6,7a,c 8,a,c 9b,d,e,10,13,14

Review P.153
1,2, 4-14,
Practice Test P.155