3.3 The Factor Theorem

Just like the remainder theorem if the divisor of a polynomial is x-a, and you evaluate F(a) and get a remainder of 0, then x-a is a factor.

The Factor Theorem

If $p(a)=0$ then $x - a$ is a factor of $p(x)$.

Example 1

Determine if $x - 2$ a factor of $2x^3 - 10 - 3x$? $= P(x)$

$$P(2) = 2(2)^3 - 10 - 3(2)$$

$$= 16 - 10 - 6 = 0$$

$\therefore$ $x - 2$ is a factor

Students Turn

Is $x - 1$ a factor of $x^3 + 2x^2 - 5x - 6$? $= P(x)$

$P(1) = (1)^3 + 2(1)^2 - 5(1) - 6$

$= -8$

$\therefore$ x − 1 is not a factor

Your Turn

Determine which of the following binomials are factors of the polynomial $P(x) = x^3 + 2x^2 - 5x - 6$.

$x - 1, x + 1, x - 2, x + 2, x - 3, x + 3, x - 6, x + 6$

Possible Factors of a Polynomial

When factoring a polynomial, $P(x)$, it is helpful to know which integer values of a to try when determining if $P(a) = 0$.

Consider the polynomial $P(x) = x^3 - 7x^2 + 14x - 8$. If $x = a$ satisfies $P(a) = 0$, then $a^3 - 7a^2 + 14a - 8 = 0$, or $a^3 - 7a^2 + 14a = 8$. Factoring out the common factor on the left side of the equation gives the product $a(a^2 - 7a + 14) = 8$. Then, the possible integer values for the factors in the product on the left side are the factors of 8. They are ±1, ±2, ±4, and ±8.

The relationship between the factors of a polynomial and the constant term of the polynomial is stated in the **integral zero theorem**.

The integral zero theorem states that if $x - a$ is a factor of a polynomial function $P(x)$ with integral coefficients, then a is a factor of the constant term of $P(x)$.

Example 2: Factor fully using the factor theorem:

$p(x) = 2x^3 - 5x^2 - 4x + 3$

$\pm 1 \quad \pm 3$

$p(1) = 2\cancel{(1)^3} - 5(1)^2 - 4(1) + 3$

$p(-1) = 2(-1)^3 - 5(-1)^2 - 4(-1) + 3$

$= -2 - 5 + 4 - 3 = 0$

$\therefore \; x - (-1) = \underline{x+1}$ is a factor

$$
\begin{array}{r|rrrr}
-1 & 2 & -5 & -4 & 3 \\
 & \downarrow & -2 & 7 & -3 \\
\hline
 & 2 & -7 & 3 & \boxed{0}
\end{array}
$$

$(x+1)(2x^2 - 7x + 3)$

$(x+1)(2x-1)(x-3)$

$\pm 1 \quad \pm 2 \quad \pm 4$

Example 3: Factoring a Quartic Function

Factor fully $x^4 - 3x^3 - x^2 + 7x - 4 = p(x)$

$$p(1) = (1)^4 - 3(1)^3 - (1)^2 + 7(1) - 4$$

$$= 0$$

$\therefore$ $x - 1$ is a factor

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -1 & 7 & -4 \\ & \downarrow & 1 & -2 & -3 & 4 \\ \hline & 1 & -2 & -3 & 4 & \boxed{0} \end{array}$$

$$(x-1)(x^3-2x^2-3x+4) = g(x)$$

$$g(1) = (1)^3 - 2(1)^2 - 3(1) + 4 = 0$$

$\therefore$ $\underline{x-1}$ is a factor of $g(x)$

$$\begin{array}{c|cccc} 1 & 1 & -2 & -3 & 4 \\ & \downarrow & 1 & -1 & 4 \\ \hline & 1 & -1 & -4 & \boxed{0} \end{array}$$

$$(x-1)(x-1)(x^2-x-4)$$

$$(x-1)^2(x^2-x-4)$$

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#'s 1,2a,b,e 3,a,b,e 4,5a,c,d6,c,e7,9,10,13, C1