3.3 Rules For Differentiation

The Quotient Rule

7. Quotient Rule

If both $f(x)$ and $g(x)$ are differentiable functions, then if $y = \dfrac{f(x)}{g(x)}$, then

$$y' = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Low de Hi minus Hi de Low

http://www.youtube.com/watch?v=DdV2UZV7AoA&feature=related

Ex.1 Differentiate:

$$F(x)=\frac{x^2+2x-3}{x^3+1}$$

$$F'(x)=\frac{(x^3+1)(2x+2)-(x^2+2x-3)(3x^2)}{(x^3+1)^2}$$

$$F'(x)=\frac{2x^4+2x^3+2x+2-3x^4-6x^3+9x^2}{(x^3+1)^2}$$

$$=\frac{-x^4-4x^3+9x^2+2x+2}{(x^3+1)^2}$$

Ex. 2 Differentiate

$$y = \frac{\sqrt{x}}{1+2x} = \frac{x^{1/2}}{1+2x}$$

$$y' = \frac{(1+2x)\cdot\left(\frac{1}{2x^{1/2}}\right) - x^{1/2}(2)}{(1+2x)^2}$$

$$= \frac{\frac{1+2x}{2x^{1/2}} - 2x^{1/2}(2x^{1/2})}{2x^{1/2}(1+2x)^2}$$

$$= \frac{1 + 2x - 4x}{2x^{1/2}(1+2x)^2}$$

$$= \frac{1 - 2x}{2x^{1/2}(1+2x)^2}$$

Suppose $f(x) = \dfrac{x^2 - x + 1}{3x + 2}$;

what is the equation of the line tangent to f at the point $(0, 1/2)$?

$$f'(x) = \frac{(3x+2)(2x-1) - (x^2 - x + 1)(3)}{(3x+2)^2}$$

$$f'(0) = \frac{(3(0)+2)(3(0)-1) - (0^2 - 0 + 1)(3)}{(3(0)+2)^2}$$

$$= \frac{-2-3}{4} = -\frac{5}{4}$$

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = -\frac{5}{4}(x - 0)$$

$$\boxed{y = -\frac{5}{4}x + \frac{1}{2}}$$

10. What is the instantaneous rate of change at $x = 2$ of the function f given by $f(x) = \frac{x^2 - 2}{x - 1}$?

(A) -2 (B) $\frac{1}{6}$ (C) $\frac{1}{2}$ (D) 2 (E) 6

$$f'(x) = \frac{(x-1)(2x) - (x^2-2)(1)}{(x-1)^2}$$

$$f'(2) = \frac{(2-1)(2(2)) - (2^2-2)(1)}{(2-1)^2}$$

$$= 4 - 2 = 2$$

3. If $f(x) = \dfrac{5 - x}{x^3 + 2}$, then $f'(x) =$

(A) $\dfrac{-4x^3 + 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(B) $\dfrac{-2x^3 + 15x^2 + 2}{\left(x^3 + 2\right)^2}$

(C) $\dfrac{2x^3 - 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(D) $\dfrac{4x^3 - 15x^2 + 2}{\left(x^3 + 2\right)^2}$

$$f'(x) = \frac{(x^3+2)(-1) - (5-x)(3x^2)}{(x^3+2)^2}$$

$$= \frac{-x^3 - 2 - 15x^2 + 3x^3}{(x^3+2)^2}$$

$$= \frac{2x^3 - 15x^2 - 2}{(x^3+2)^2}$$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

   (a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

   (b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

$$h'(x) = \frac{f(x)\,g'(x) - g(x)\,f'(x)}{[f(x)]^2}$$

$$h'(1) = \frac{f(1)\,g'(1) - g(1)\,f'(1)}{(f(1))^2}$$

$$h'(1) = \frac{(-6)(8) - (2)(3)}{(-6)^2}$$

$$= \frac{-48-6}{36} = -\frac{54}{36}$$

$$= -\frac{6}{4} = -\frac{3}{2}$$

Assignment
Handout #'s 2 a,b,c,e (Just derivative), 3 - 7

Calc 30 Text
Page 195 #'s 9-11, 18, 19

AP Text
Page 120 #'s 27, 28, 30