3.3 Rules For Differentiation

The Product Rule

6. Product Rule

If both $f(x)$ and $g(x)$ are differentiable functions, then if $y = f(x) \bullet g(x)$, then

$$y' = f(x)g'(x) + g(x)f'(x)$$

Ex. 1 Differentiate the following:

$$f(x) = (2x^3 + 5)(3x^2 - x)$$

a b

$$* \; f'(x) = (2x^3+5)(6x-1) + (3x^2-x)(6x^2)$$

$$f'(x) = 12x^4 - 2x^3 + 30x - 5 + 18x^4 - 6x^3$$

$$= 30x^4 - 8x^3 + 30x - 5$$

$y = (x^2+3)^4(2x-7)^5$

Ex. 2 Differentiate the following:

$$y = \sqrt{x}(2-3x)$$

$y = x^{1/2}(2-3x)$

$y' = x^{1/2}(-3) + (2-3x)\cdot\frac{1}{2}x^{-1/2}$

$= -\frac{6}{2}x^{1/2} + x^{-1/2} - \frac{3}{2}x^{1/2}$

$= x^{-1/2} - \frac{9}{2}x^{1/2}$

$= \frac{1}{x^{1/2}} - \frac{9}{2}x^{1/2}$

Ex.3 Find the equation of the tangent line to the curve $f(x)=(3x^2+2)(2x^3-1)$ at the point (1,5)

$f'(x)=(3x^2+2)(6x^2)+(2x^3-1)(6x)$ ✱

$f'(1)=(3(1)^2+2)(6(1)^2)+(2(1)^3-1)(6(1))$

$=(5)(6)+(6)$

$=36$

$y-y_1=m(x-x_1)$

$y-5=36(x-1)$

$y-5=36x-36$

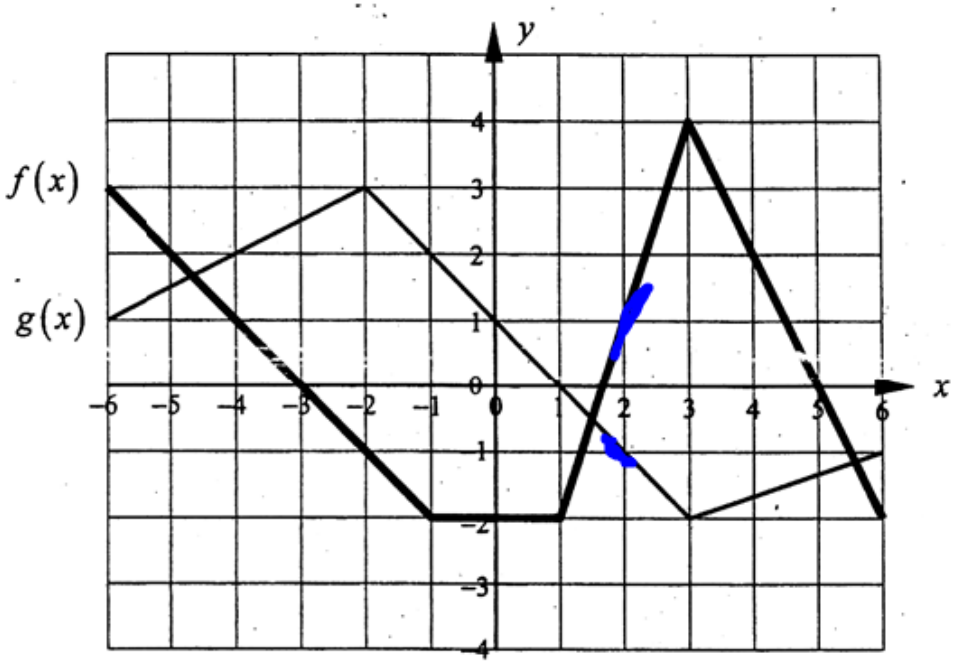
$0=36x-y-31$

The Product Rule Song

http://archives.math.utk.edu/visual.calculus/2/product_rule.1/index.html

Shown at right is a graph of the functions $f(x)$ and $g(x)$. Assume that $F(x)=f(x)\cdot g(x)$. By studying the graph and using the product rule, determine the value of each of the following.

21. $F'(2)$
22. $F'(-4)$
23. $F'(0)$
24. $F'(3)$
25. If $f(x)=(x^2+3x)^5$, use the product rule to show that $f'(x)=5(x^2+3x)^4(2x+3)$.



(21) $F'(2)$

$F(x) = f(x)\,g(x)$

$F'(x) = f(x)\cdot g'(x) + g(x)\,f'(x)$

$$F'(2) = f(2)g'(2) + g(2)f'(2)$$

$$= (1)(-1) + (-1)(3)$$

$$= -1 + -3 = -4$$

8. Let f and g be differentiable functions with the following properties:

(i) $g(x) > 0$ for all x

(ii) $f(0) = 1$

If $h(x) = f(x)g(x)$ and $h'(x) = f(x)g'(x)$, then $f(x) =$

(A) $f'(x)$ (B) $g(x)$ (C) e^x (D) 0 (E) 1

$h(x) = f(x)g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

$g(x)f'(x) = 0$

$f'(x) = 0$

Assignment
Handout #'s
1 odd
2odd
3odd
5,6

Calc 30 Text
Page 191 #'s 11,12,16,17,19