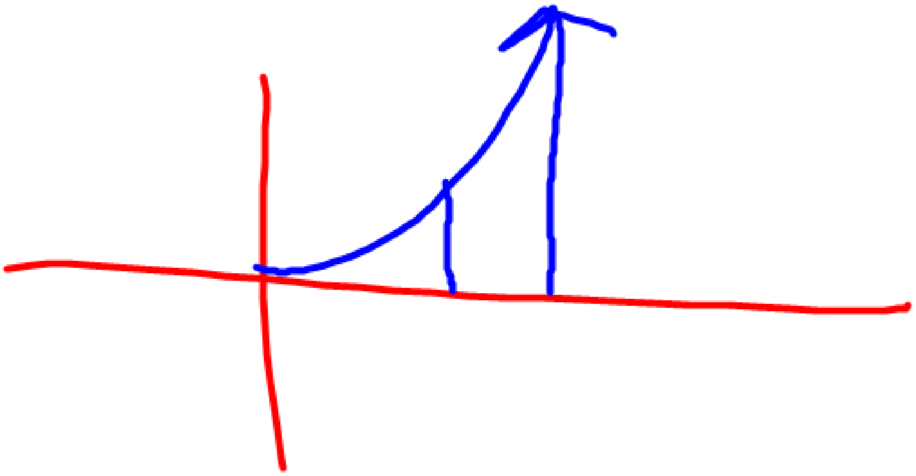
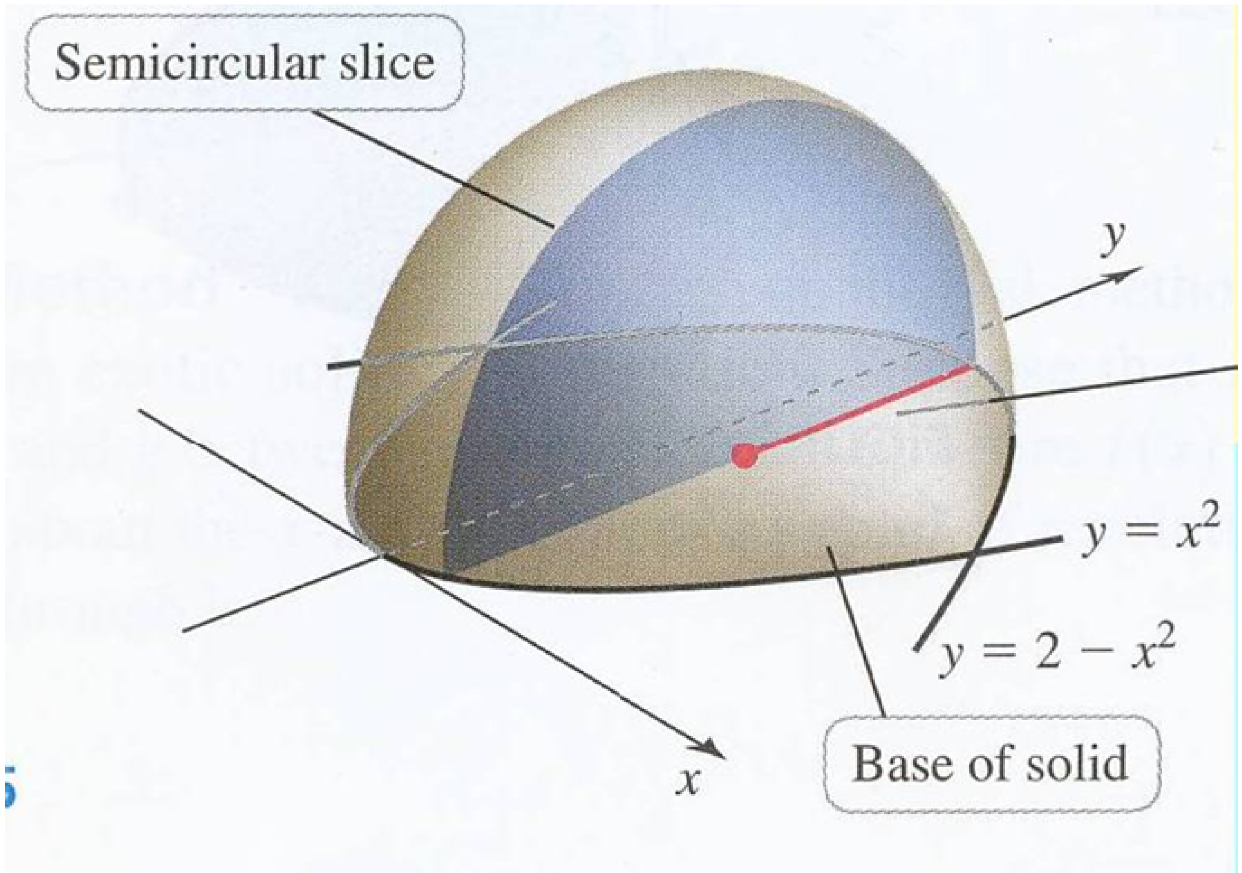
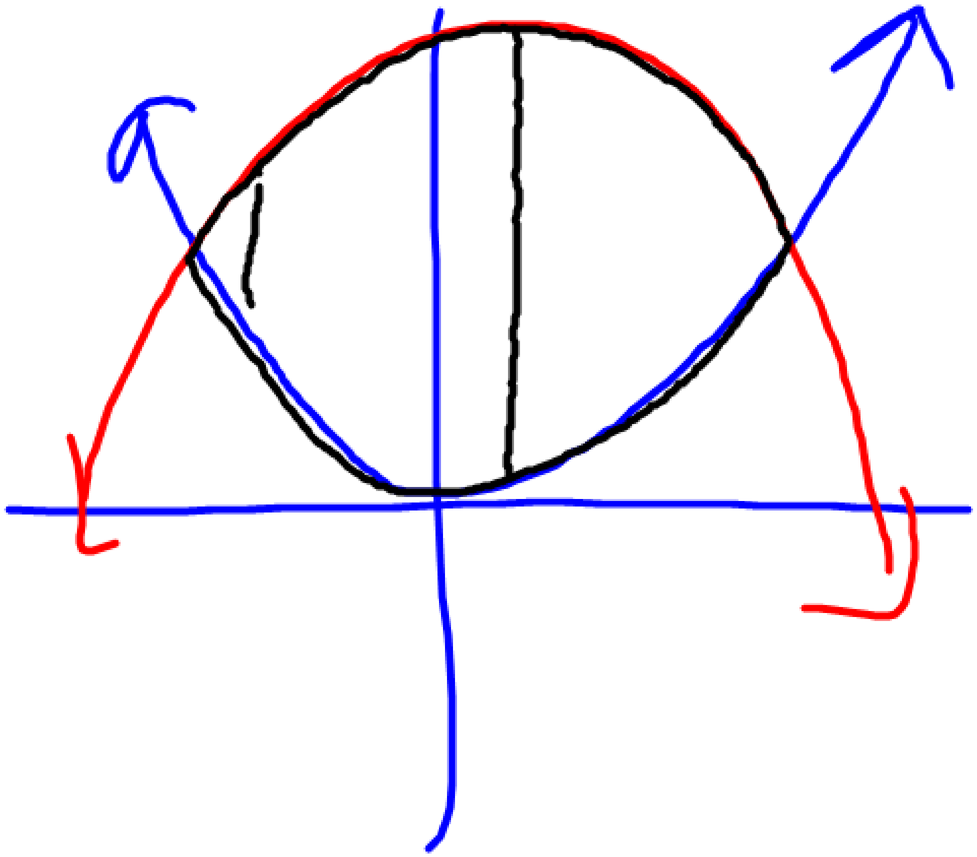
7.4 Solids of Known Cross Sections

With the disk method we found the volume of a solid having a circular cross section whose area is $A = \pi r^2$. This method could be applied to solids of any shape as long as we know a formula of any arbitrary cross section.



Let the base of a solid in quadrant I be enclosed by $y = 2-x^2$ and $y = x^2$. If all cross sections perpendicular to the x-axis are semi circles, find the volume of the solid.





For cross sections $\perp$ to the x *axis* we use:

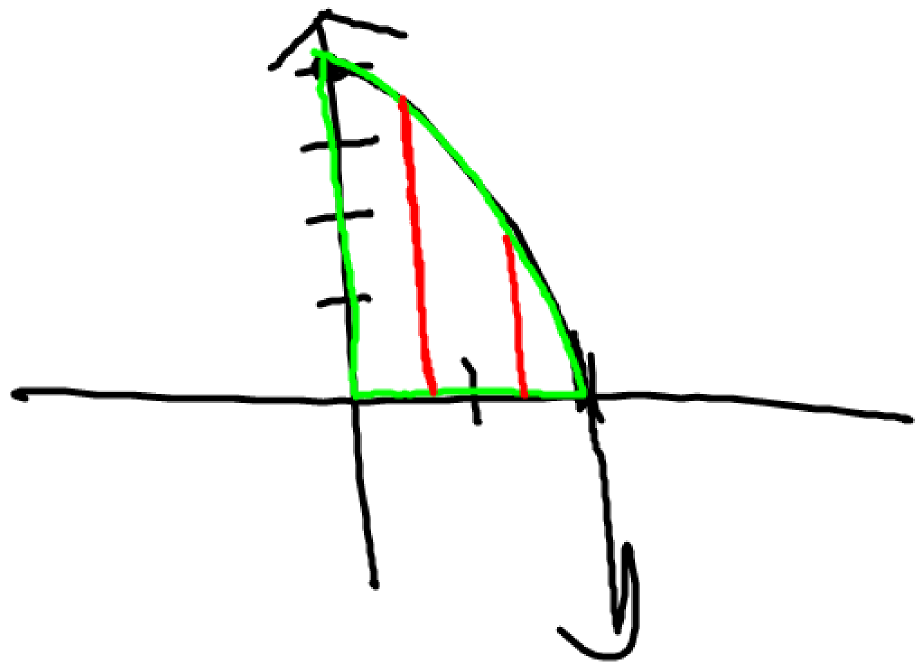
$$V = \int_a^b A(x)dx.$$

For cross sections $\perp$ to the y *axis* we use:

$$V = \int_c^d A(y)dy.$$

Example 1: Let the base of a solid in quadrant I enclosed by the *x-axis* and the *y axis* and the graph $y = 4\text{-}x^2$. If all of the cross sections $\perp$ to the x *axis* are squares, find the volume of the solid.

(See demo next slide)!



$$V = \int_0^2 (\text{side})^2 \, dx$$

$$= \int_0^2 (4-x^2)^2 \, dx = 17.067$$

http://mathdemos.gcsu.edu/mathdemos/sectionmethod/equsqrcross75slab.gif

Example 1: Let the base of a solid in quadrant I enclosed by the *x-axis* and the *y axis* and the graph $y = 4\text{-}x^2$. If all of the cross sections $\perp$ to the *x axis* are squares, find the volume of the solid.

Example 2 : Let the base of a solid be the first quadrant region enclosed by the *x axis*, the *y axis*, and the graph $y = 1 - \frac{x^2}{4}$. If all cross sections $\perp$ to the *y axis* are squares, find the volume of the solid.

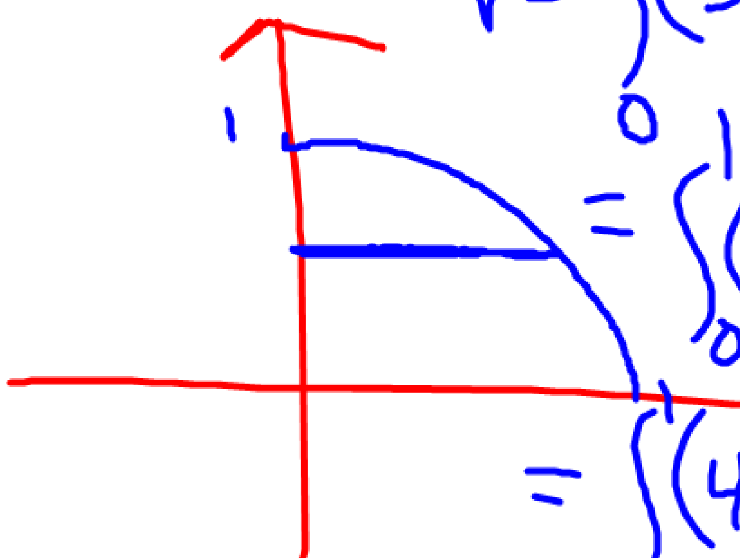
$$V = \int_0 (\text{side})^2\, dy$$

$$4y = 4 - x^2$$

$$x^2 = 4 - 4y$$

$$x = \pm\sqrt{4-4y}$$

$$x = \sqrt{4-4y}$$



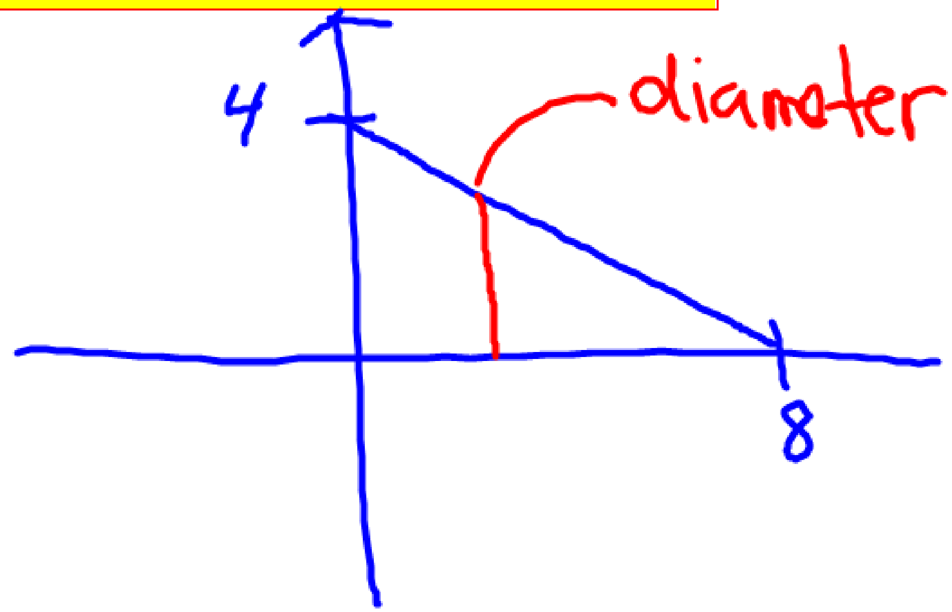
$$= \int_0^1 \left(\sqrt{4-4y}\right)^2 dy$$

$$= \int_0^1 (4-4y)\, dy = 2$$

Example 3: The base of a solid is a region in the first quadrant bounded by the x *axis* and the line $x + 2y = 8$. If cross sections $\perp$ to the x *axis* are semi circles, what is the volume of the solid?

$$2y = -x + 8$$

$$y = -\frac{1}{2}x + 4$$



$$V = \int_a^b \frac{1}{2}\pi r^2 \, dx$$

$$V = \frac{1}{2}\pi \int_0^8 (\text{radius})^2 \, dx$$

$$V = \frac{1}{2}\pi \int_0^8 \left(\frac{-\frac{1}{2}x+4}{2}\right)^2 dx$$

$$= \frac{1}{2}\pi \int_0^8 \frac{\left(-\frac{1}{2}x+4\right)^2}{4}$$

$$= \frac{\pi}{8} \int_0^8 \left(-\frac{1}{2}x+4\right)^2 dx = 16.755$$

$$=$$

$$V=\int_a^b \pi R^2-\pi r^2$$

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Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

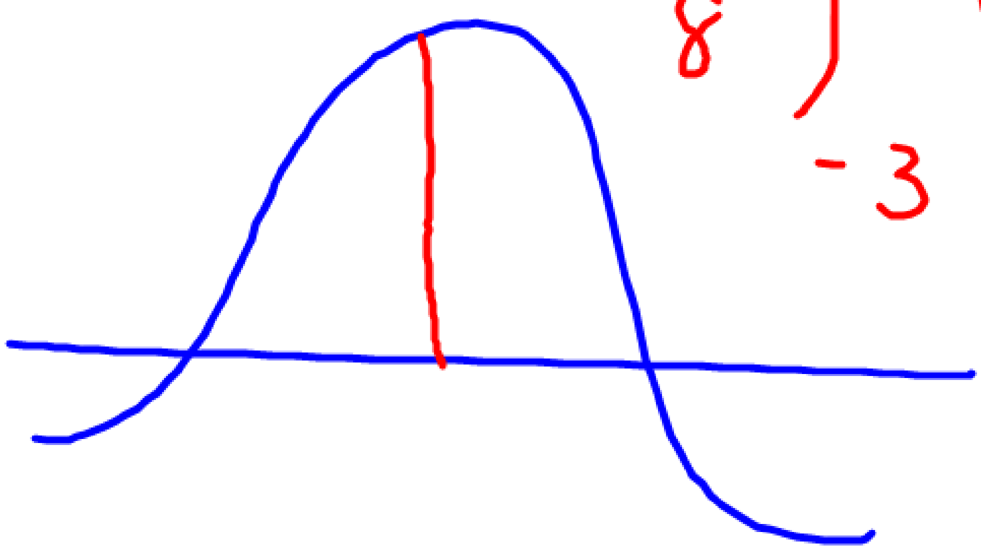
(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

a) $A=\int_{-3}^{3}\left(\frac{20}{1+x^2}\right)-2\,dx=37.962$

b) $V=\pi\int_{-3}^{3}\left(\frac{20}{1+x^2}\right)^2-(2)^2\,dx=1871.190$

c) $V = \frac{\pi}{2}\int_{-3}^{3} \left(\frac{\frac{20}{1+x^2} - 2}{2}\right)^2$

$= \frac{\pi}{8}\int_{-3}^{3} \left(\frac{20}{1+x^2} - 2\right)^2 dx$



$= 174.268$

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Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

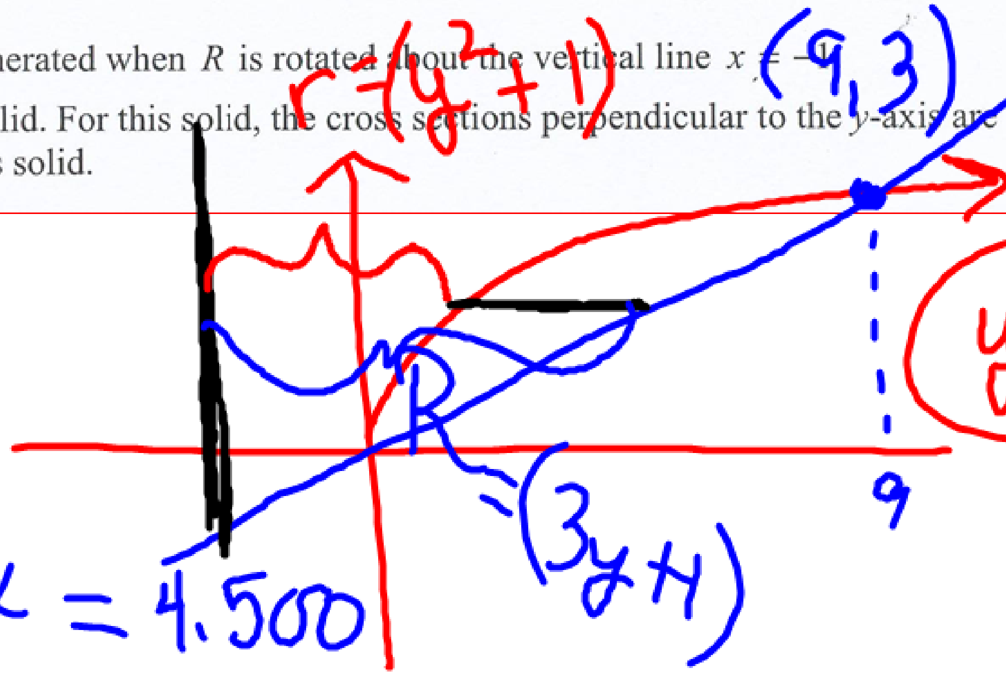
(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

$3y = x$

$y = \frac{x}{3}$

$y^2 = x$



a)

$$A = \int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.500$$

b) $V = \pi \int_0^3 (3y+1)^2 - (y^2+1)^2 \, dy$

$= 130.062$

c) $V = \int_0^3 (\text{side})^2 \, dy = \int_0^3 (3y - y^2)^2 \, dy$

$= 8.1$

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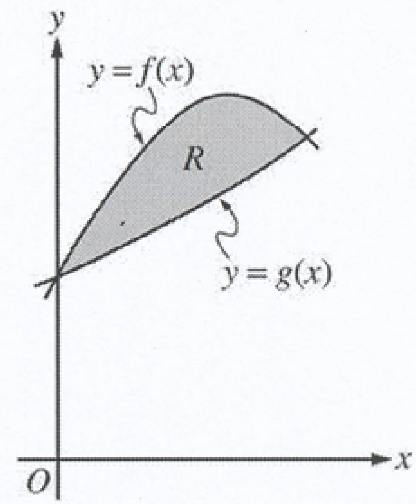
Question 1

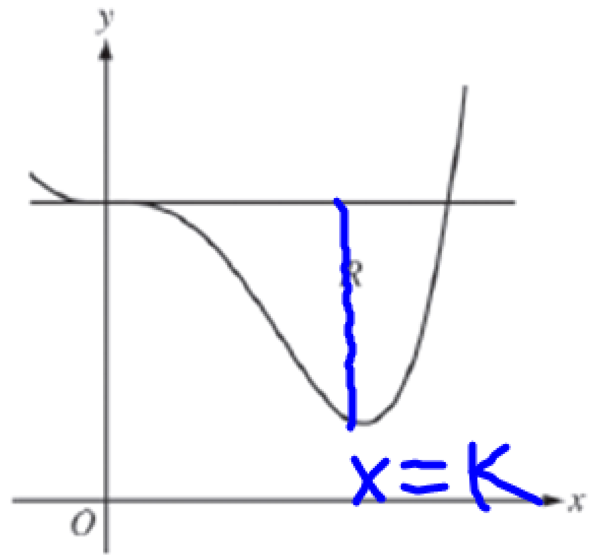
Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

 (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

 (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

 (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

a) $V = \pi \int_0^{2.3} (6)^2 - (f(x)+2)^2 \, dx$

$= 98.868$

b) $V = \int_a^b (\text{Area})\,dx$

$$V = \int_0^{2.3} \frac{1}{2}(b)(h)\,dx$$

$$= \frac{1}{2}\int_0^{2.3} (4 - f(x))^2\,dx = 3.574$$

c) $\int_0^K (4 - f(x))\,dx = \int_K^{2.3} (4 - f(x))\,dx$

Assignment Handout