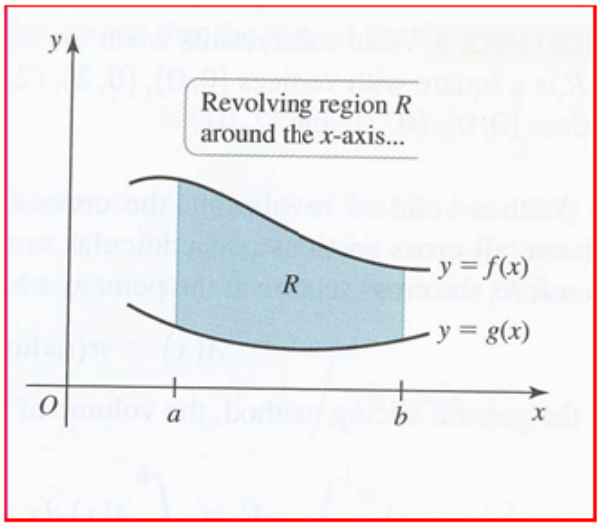
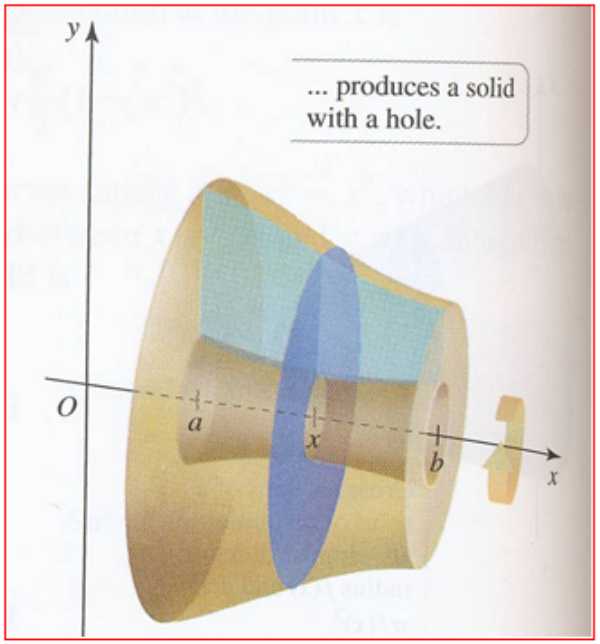
3.2 The Washer Method


y
Revolving region R
around the x-axis...
R
y = f(x)
y = g(x)
O
a
b
x


y
... produces a solid
with a hole.
O
a
x
b
x

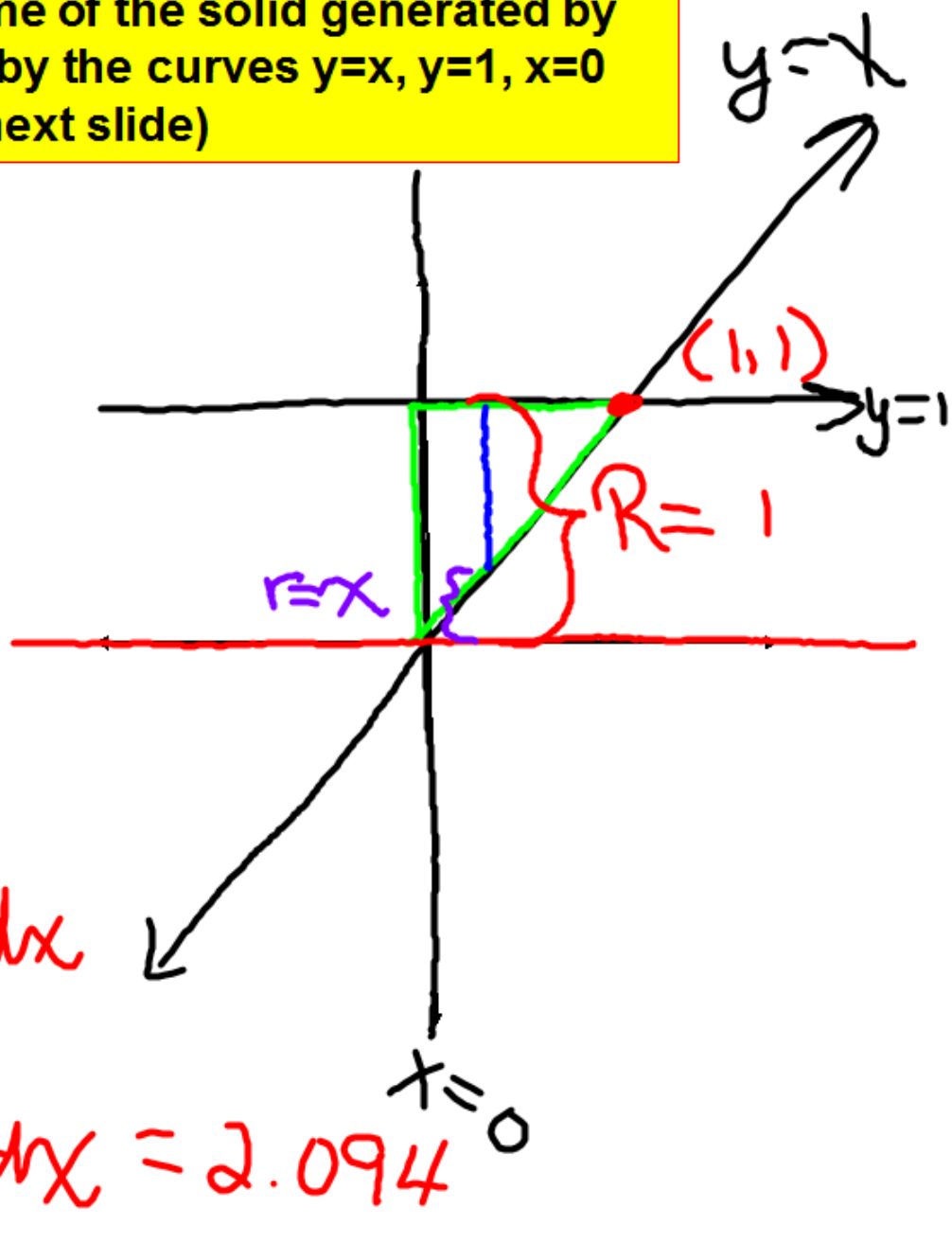
Say we asked to find the volume of the solid generated by revolving the region bounded by the curves y=x, y=1, x=0 about the x-axis. (See visual next slide)

$$V = \int_a^b \pi R^2 - \pi r^2 \, dx$$

$$= \pi \int_0^1 R^2 - r^2 \, dx$$

$$= \pi \int_0^1 (1)^2 - (x)^2 \, dx$$

$$= \pi \int_0^1 (1 - x^2) \, dx = 2.094$$



AP Central Examples

http://mathdemos.gcsu.edu/mathdemos/washermethod/

The process used to find the volume of revolution of a solid when the cross sectional areas do not touch the axis of rotation is called the Washer Method.

Washer Method

$$V = \int_a^b \pi\left(R(x)\right)^2 - \pi\left(r(x)\right)^2 dx$$

outer inner

where $R(x)$ is the outer radius and $r(x)$ is the inner radius.

$$V = \pi\int_a^b \left(R(x)\right)^2 - \left(r(x)\right)^2 dx$$

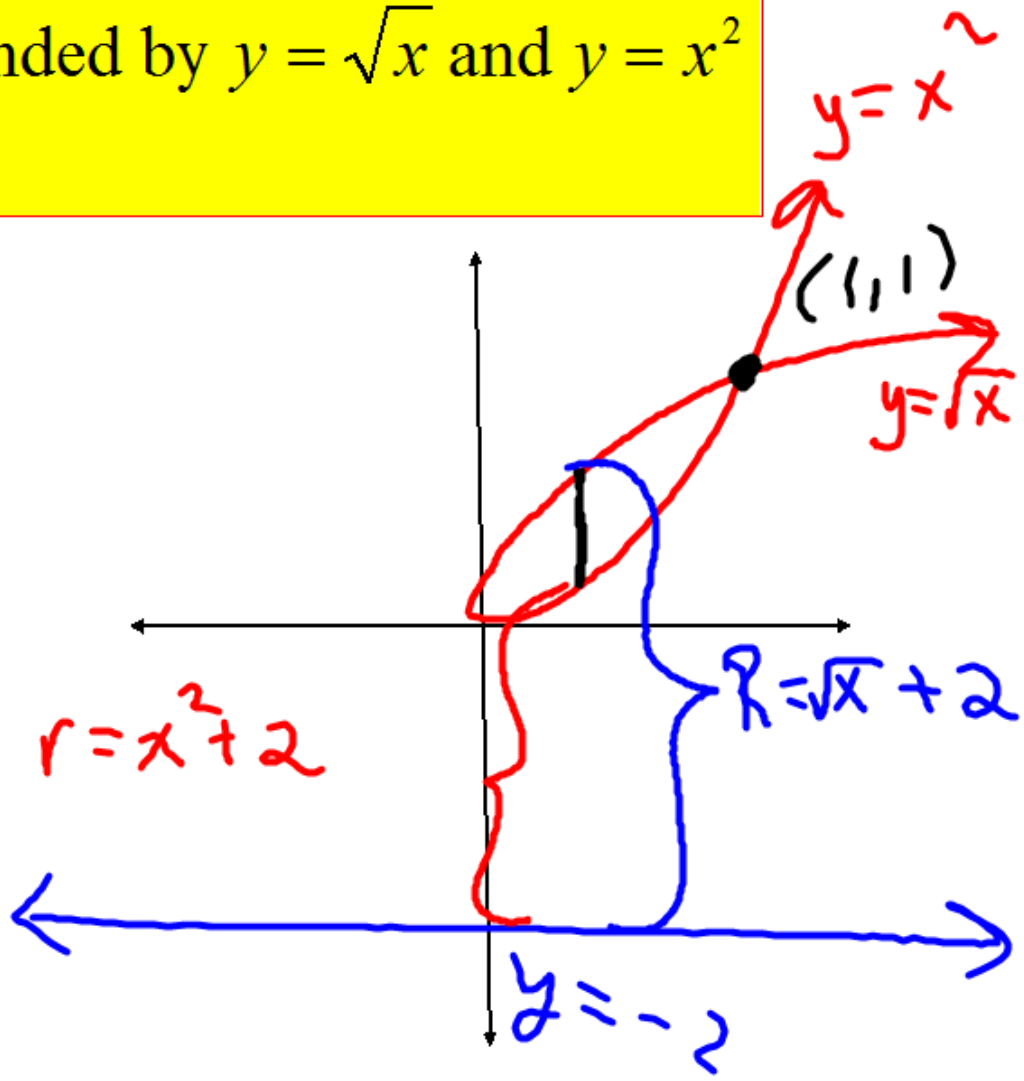
Example 1 : Find the volume of the solid formed by revolving the region bounded by $y = \sqrt{x}$ and $y = x^2$ about the line $y = -2$.

$$V = \pi \int_a^b R^2 - r^2 \, dx$$

$$V = \pi \int_0^1 (\sqrt{x}+2)^2 - (x^2+2)^2 \, dx$$

$$= 1.633\pi$$

$$\approx 5.131$$



$A = .178210$

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$B = 1$

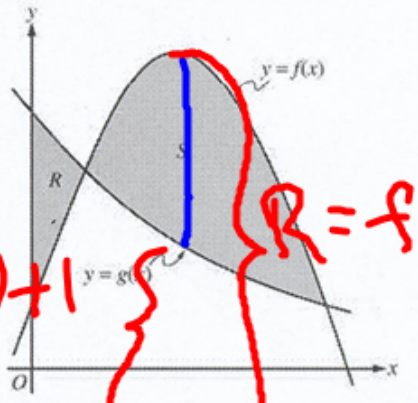
Question 1

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.



$R = f(x) + 1$

$r = g(x) + 1$

$y = -1$

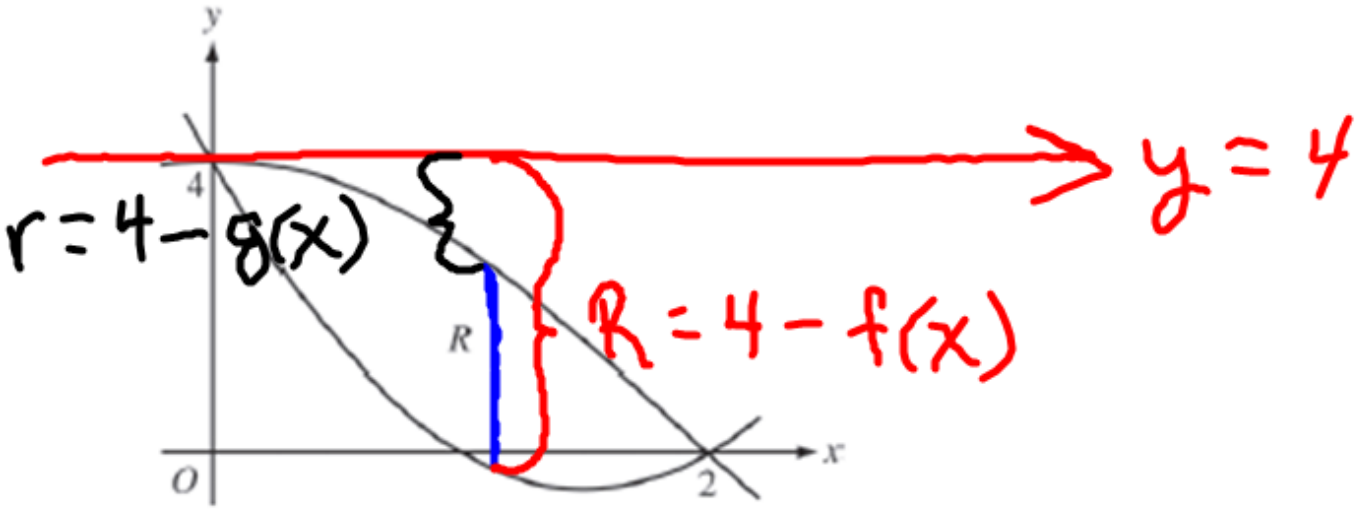
a) $\text{Area } R = \int_0^A 4^{-x} - \left(\frac{1}{4} + \sin(\pi x)\right) dx = .065$

b) $\text{Area } S = \int_A^1 \frac{1}{4} + \sin(\pi x) - 4^{-x} \, dx = .410$

$$V = \pi \int_{A}^{1} (f(x)+1)^2 - (g(x)+1)^2 \, dx$$

$$= 4.559$$

2013 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



5. Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

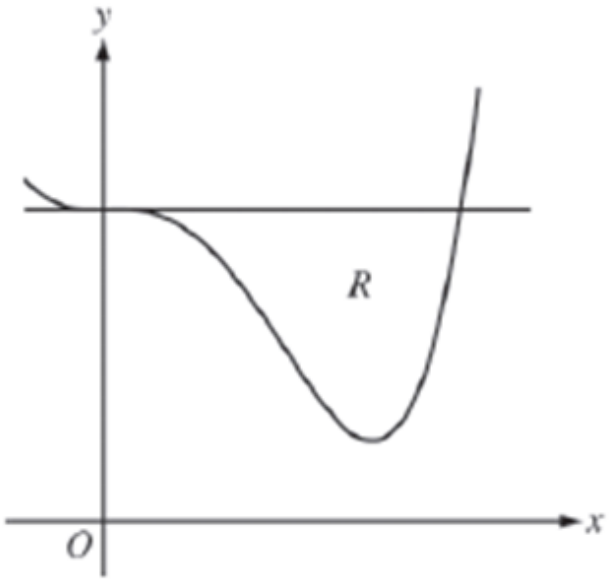
   (a) Find the area of R.

   (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

b) $V = \pi \int_0^2 (4 - f(x))^2 - (4 - g(x))^2 \, dx$

2014 AP® CALCULUS AB FREE-RESPONSE QUESTIONS



2. Let R be the region enclosed by the graph of $f(x) = x^4 - 2.3x^3 + 4$ and the horizontal line $y = 4$, as shown in the figure above.

   (a) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (b) Region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in R. Find the volume of the solid.

   (c) The vertical line $x = k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value k.

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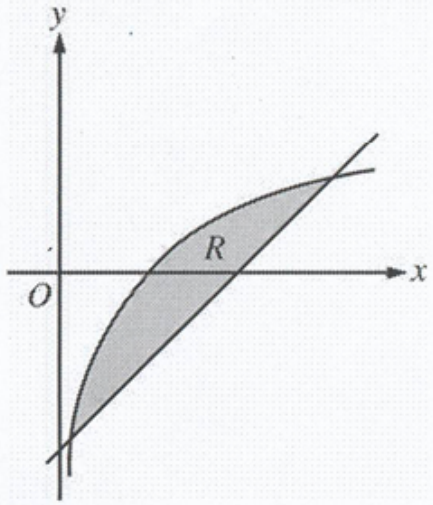
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



A = .2387...

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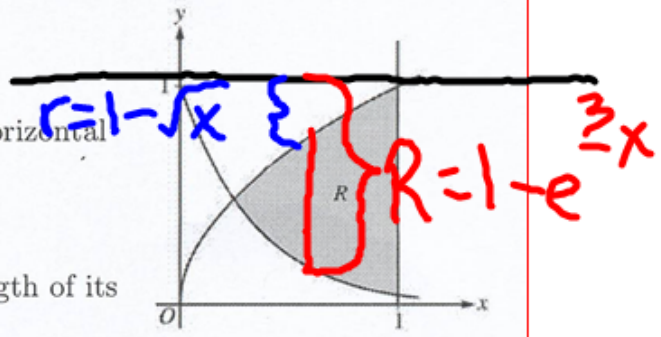
Question 1

Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

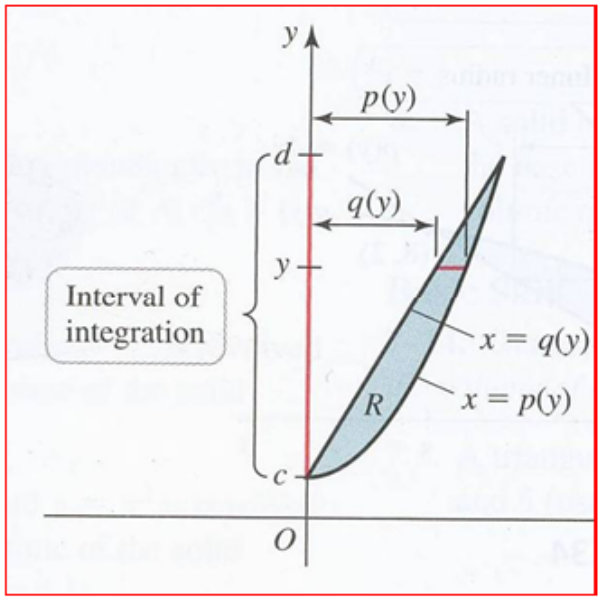
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

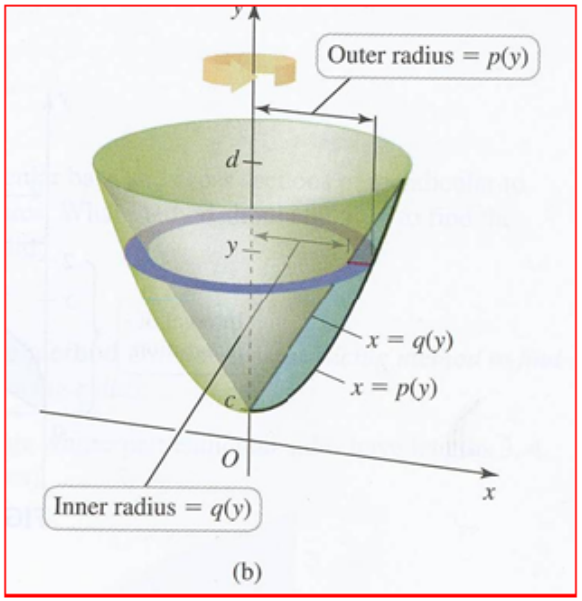


a) $A = \int_A^1 \sqrt{x} - e^{-3x}\,dx = .442$

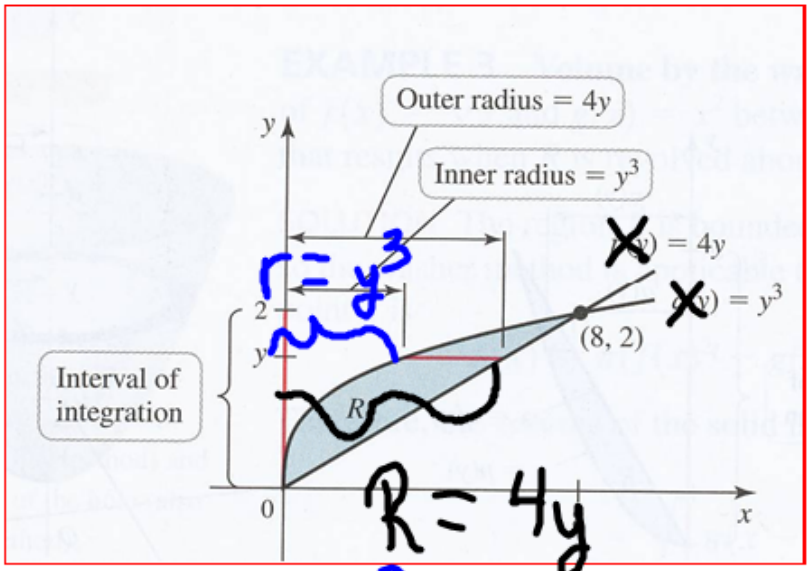
b) $V = \pi \int_A^1 (1-e^{-3x})^2 - (1-\sqrt{x})^2\,dx = 1.424$

Revolving About the Y-Axis





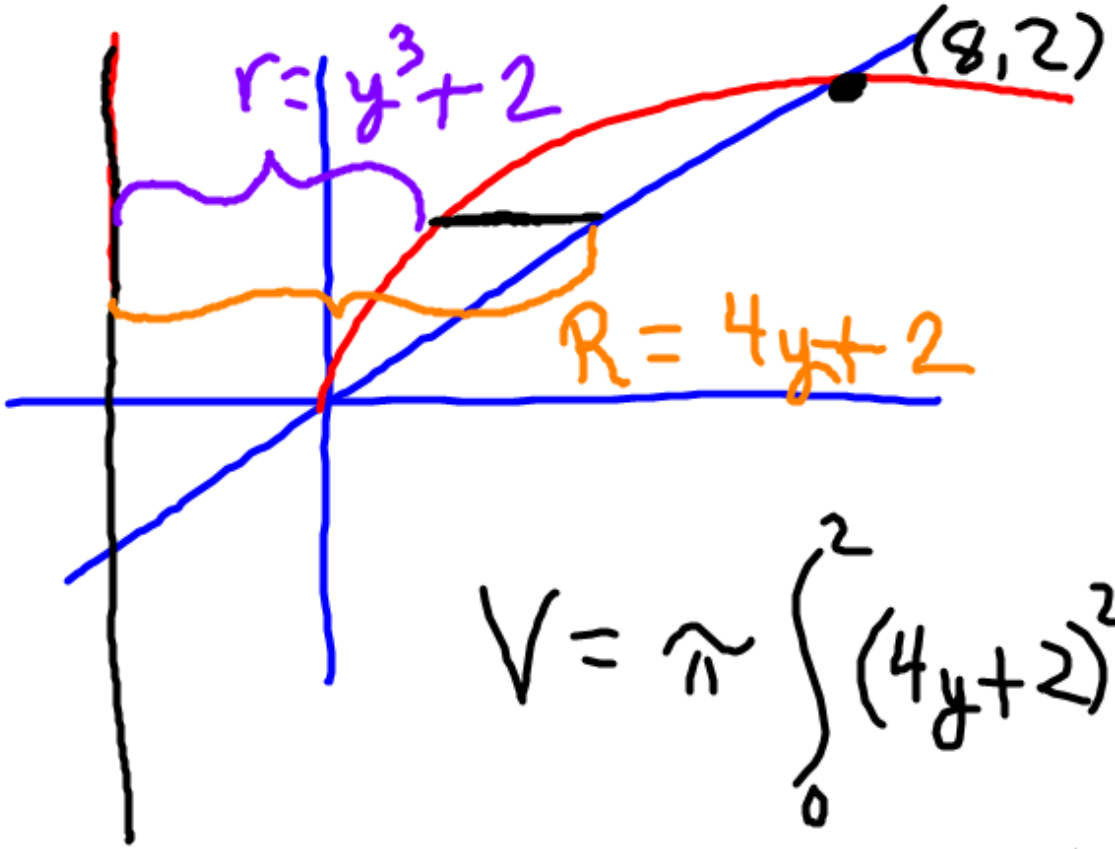
EXAMPLE 4 **Which solid has greater volume?** Let R be the region in the first quadrant bounded by the graphs of $x = y^3$ and $x = 4y$. Which is greater, the volume of the solid generated when R is revolved about the x-axis or the y-axis?



$$V = \pi \int_0^2 (4y)^2 - (y^3)^2 \, dy$$

$$= \pi \int_0^2 (16y^2 - y^6) \, dy = 76.595$$

Revolve around a vertical line x = -2 and x=10

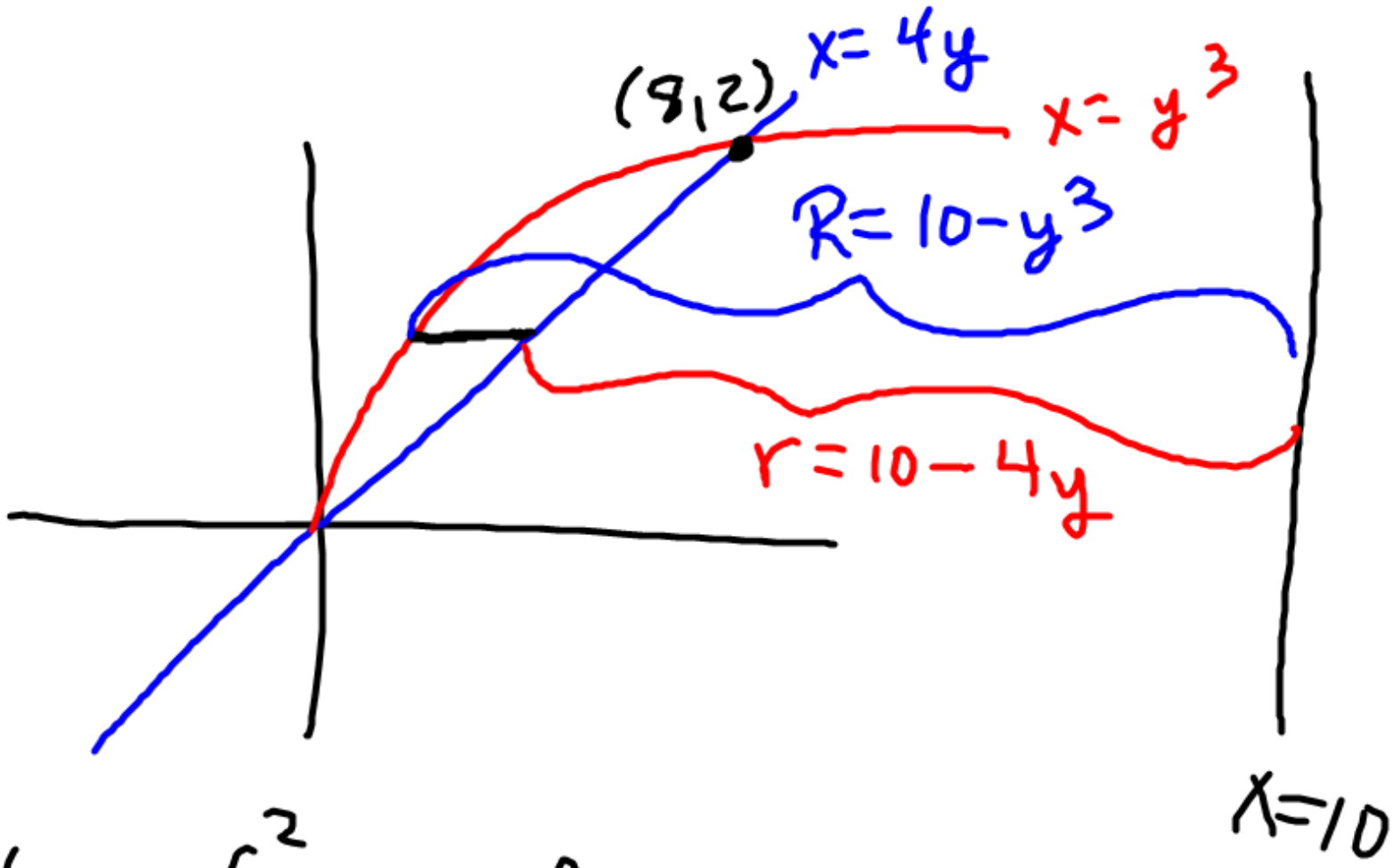


$x = 4y$

$x = y^3$

$$V = \pi \int_0^2 (4y+2)^2 - (y^3+2)^2 \, dy$$

$$= 126.861$$



$$V = \pi \int_0^2 (10 - y^3)^2 - (10 - 4y)^2 \, dy$$

Assignment

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AP #'s 21,22,23,24,25,35

Handout last 2 questions

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2005 SCORING GUIDELINES (Form B)

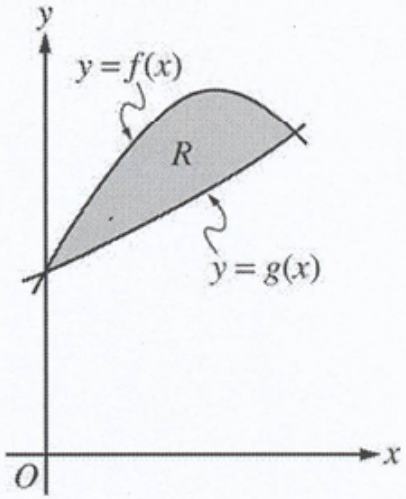
Question 1

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.



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2007 SCORING GUIDELINES (Form B)

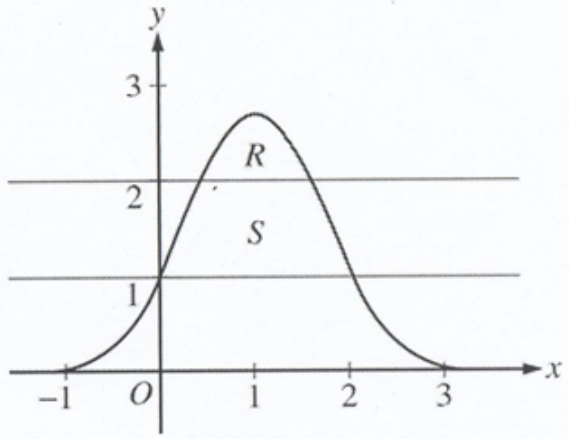
Question 1

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.

(a) Find the area of R.

(b) Find the area of S.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



AP® CALCULUS AB
2008 SCORING GUIDELINES (Form B)

Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.