Unit 3 Limits and Continuity

3.2 Limits - A Visual Approach

3.2 Visual Limits

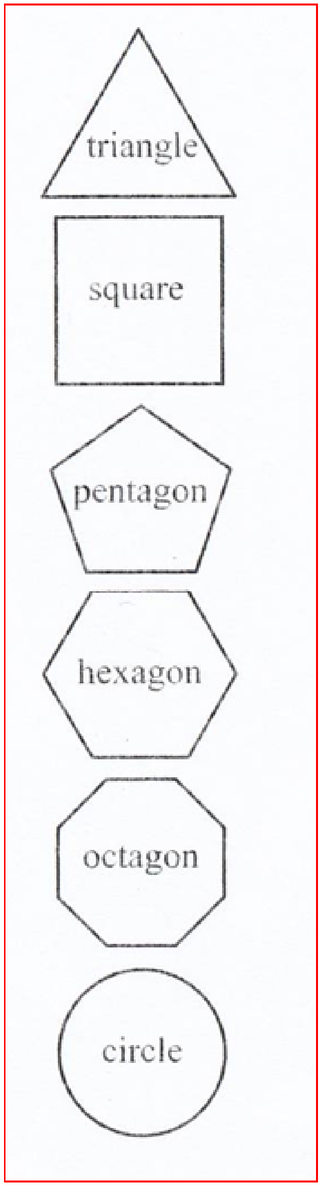
Learning Targets:

1. SWBAT define a limit.

2. SWBAT visually determine the limit of a function.

Introduction

The first five figures at right show a series of regular polygons (n-gons) having 3, 4, 5, 6, and 8 sides respectively. Notice that as the number of sides increases, the regular polygon's shape becomes more like that of a circle. If n represents the number of sides in the regular polygon, we could say that as n approaches infinity, the regular polygon's shape approaches that of a circle. We will use the arrow symbol, $\rightarrow$, to represent the word "approaches". Thus the earlier sentence can be abbreviated by writing "as $n \rightarrow \infty$, the regular n-gon $\rightarrow$ a circle." The circle is said to be the limiting shape of the regular n-gons, as $n \rightarrow \infty$.

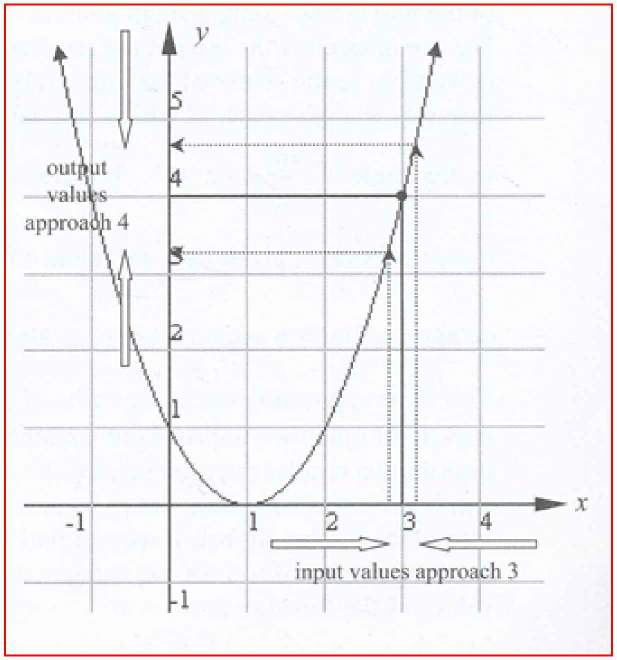


http://www.youtube.com/watch?v=W0VWO4asgmk&feature=youtube_gdata

Informal Definition of a Limit

Shown at right is the graph of the function $f(x)=(x-1)^2$. By examining the graph, what is $\lim_{x\to3}(x-1)^2$? That is, as input values, x, approach 3 *from either side*, what do the output values, $(x-1)^2$, approach? By examining the graph, we see that as $x\to3$, $(x-1)^2\to4$. Thus we write $\lim_{x\to3}(x-1)^2=4$.

4 is said to be the limit of the expression $(x-1)^2$ as x approaches 3.



Intuitive Limit Definition

If, as x approaches b from both the right and the left, $f(x)$ approaches the single real number L, then L is called the limit of the function $f(x)$ as x approaches b, and we write

$\lim_{x \to b} f(x) = L$.

Ex.1 By examining the graph determine the following limits:

(a) $\lim_{x\to 0} f(x)$ 1

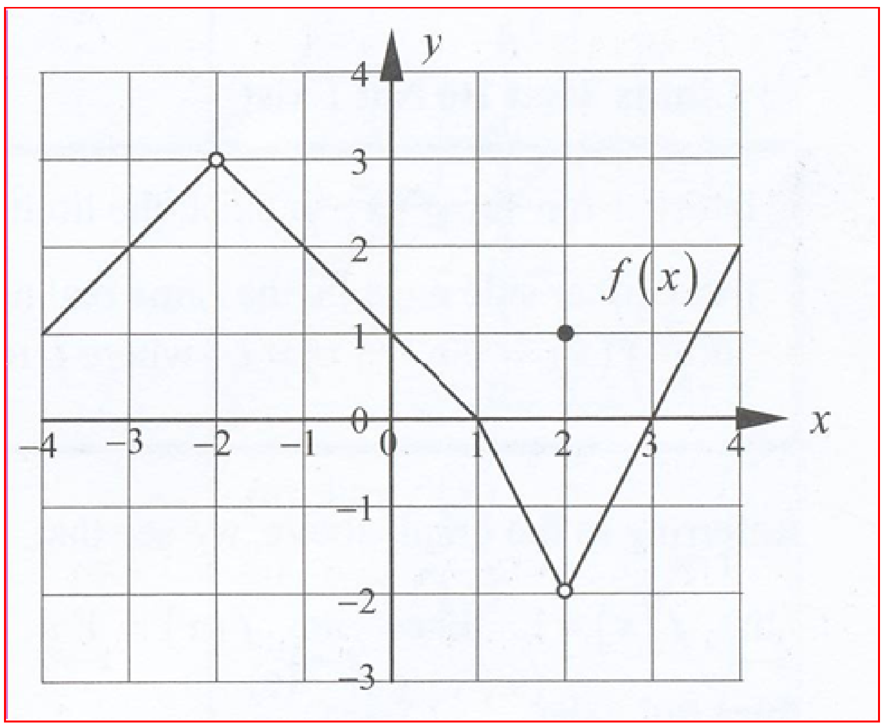
(b) $\lim_{x\to 3} f(x)$ 0

(c) $\lim_{x\to -1} f(x)$ 2

(d) $\lim_{x\to -2} f(x)$ 3

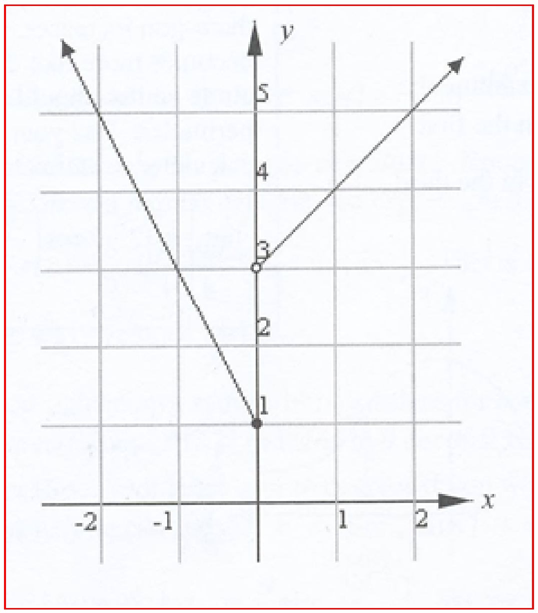
(e) $\lim_{x\to 2} f(x)$ -2

(f) $\lim_{x\to -3} f(x)$ 2



Limits That Don't Exist

In order for $\lim_{x\to b} f(x)$ to exist, the limit as you approach b from either side must be the same real number. That is $\lim_{x\to b^+} f(x) = \lim_{x\to b^-} f(x) = L$, where L is a real number.

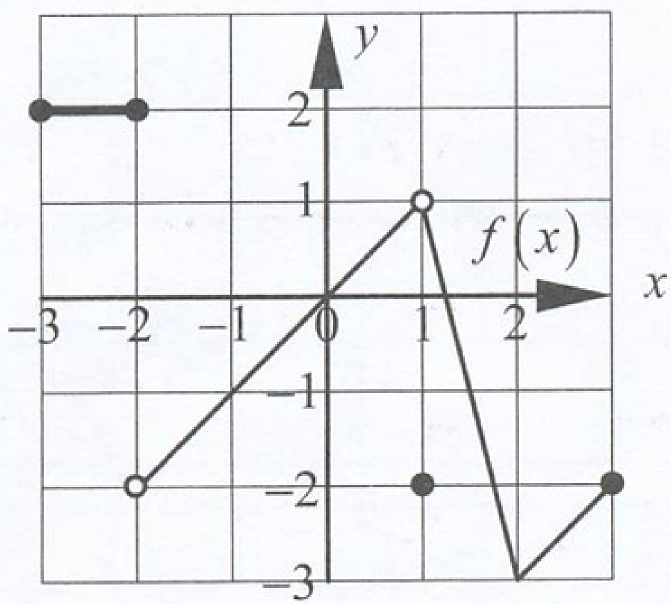


$$\lim_{x\to 0^+} f(x) = 3$$

$$\lim_{x\to 0^-} f(x) = 1$$

$$\lim_{x\to 0} f(x) = \text{DNE}$$

By referring to the graph of $f(x)$ below, determine each of the following.



(a) $\lim_{x\to 2} f(x)$ −3

(b) $\lim_{x\to 0} f(x)$ 0

(c) $\lim_{x\to -2.5} f(x)$ 2

(d) $\lim_{x\to 1} f(x)$ 1

(e) $f(1)$ −2

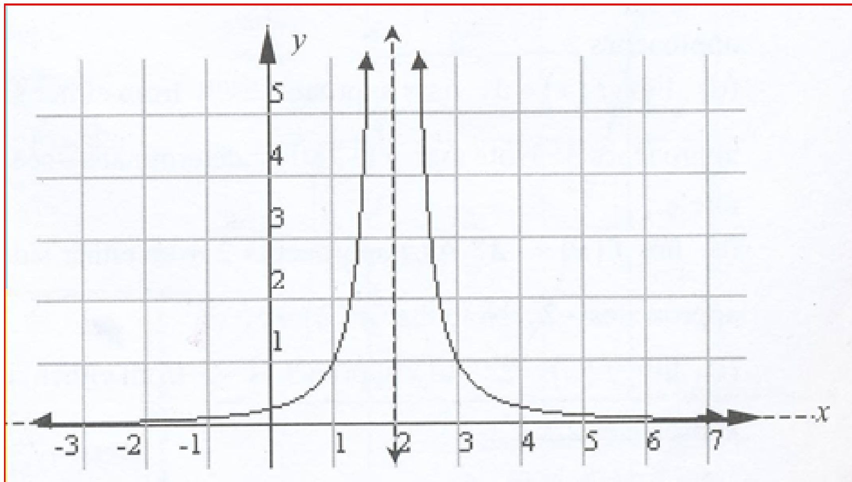
(f) $f(-2)$ 2

(g) $\lim_{x\to -2^+} f(x)$ −2

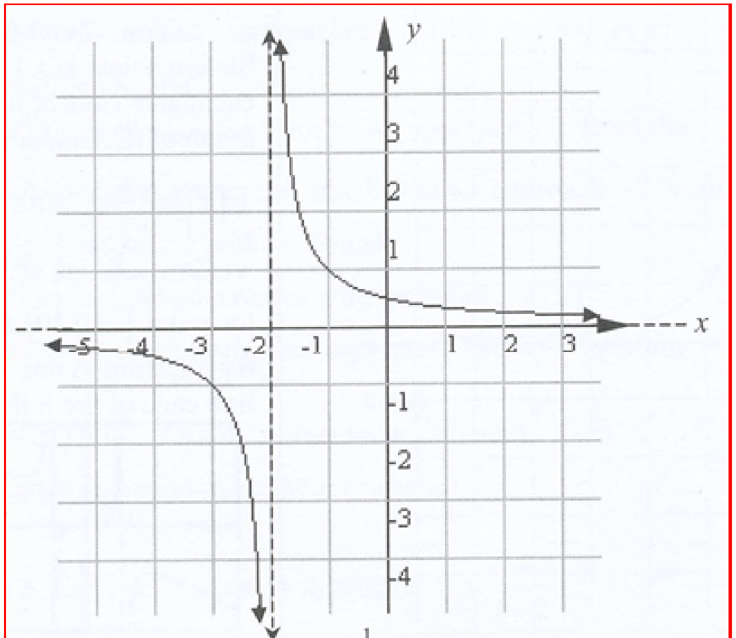
(h) $\lim_{x\to -2^-} f(x)$ 2

(i) $\lim_{x\to -2} f(x)$ DNE

Infinite Limits

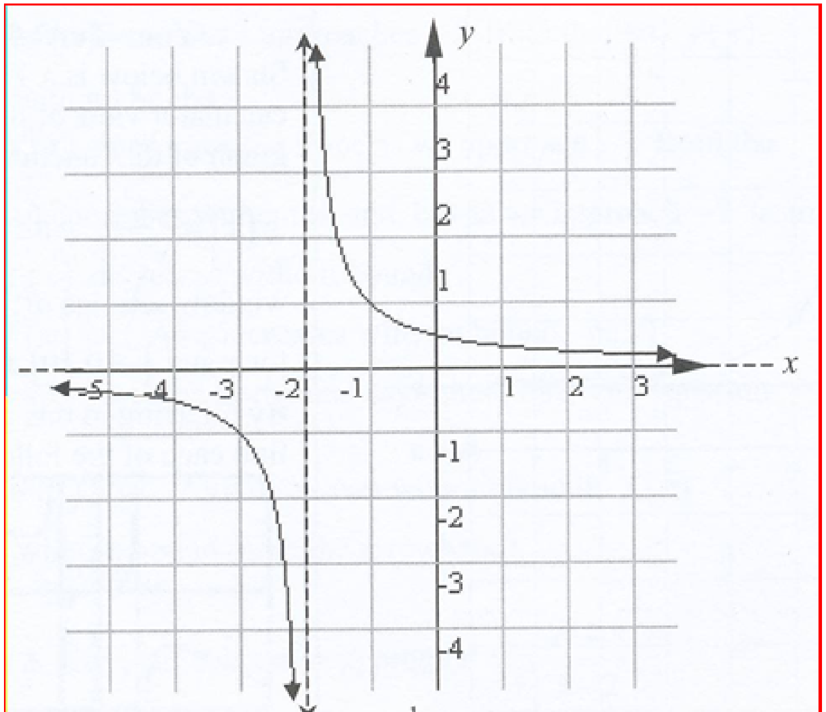


$f(x) = \frac{1}{(x-2)^2}$



$g(x) = \frac{1}{x+2}$

Limits at Infinity

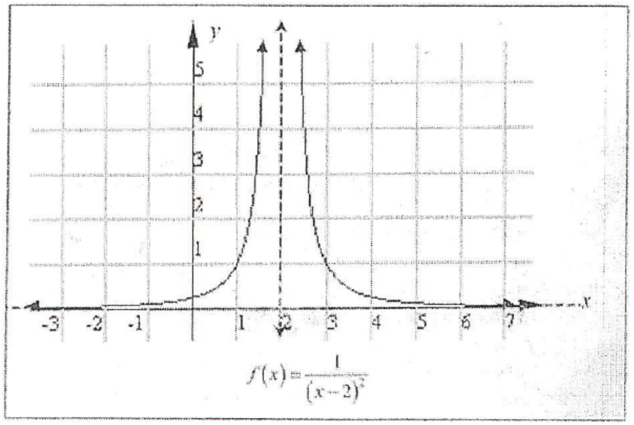


$g(x) = \frac{1}{x+2}$

$$\lim_{x\to\infty} \frac{1}{x+2} = 0$$

$$\lim_{x\to-\infty} \frac{1}{x+2} = 0$$

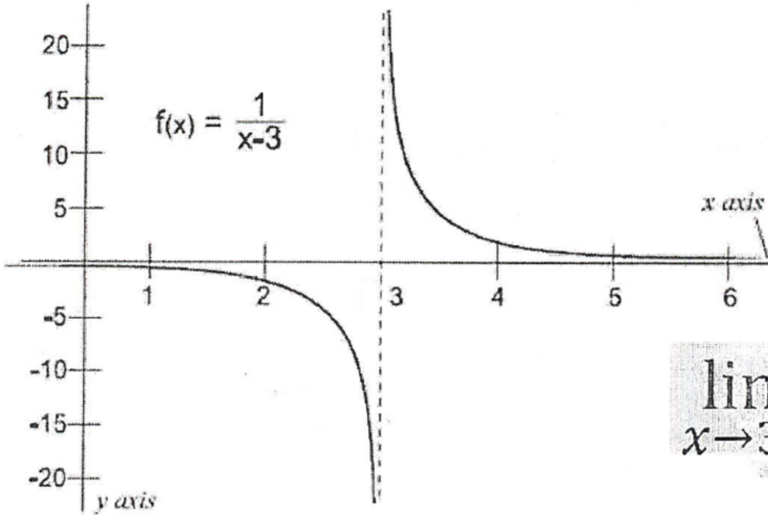
For rationals $\lim_{x\to\pm\infty} f(x) = HA$



$\lim_{x\to 2^+} f(x)$ ∞

$\lim_{x\to 2^-} f(x)$ ∞

$\lim_{x\to 2} f(x)$ ∞



$$\lim_{x\to 3^+} f(x)$$ ∞

$$\lim_{x\to 3^-} f(x)$$ $-\infty$

$$\lim_{x\to 3} f(x)$$ DNE

Example 2 By examining the graph of $f(x)$ below, determine each of the following.

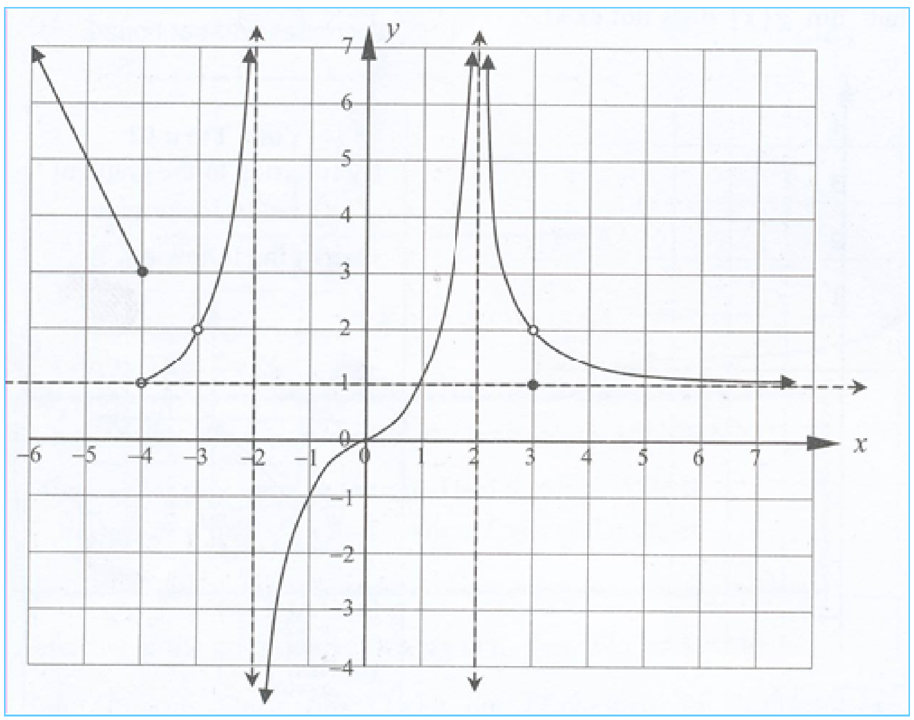
(a) $f(-6)$ 7 (b) $f(0)$ 0 (c) $f(3)$ 1 (d) $f(-4)$ 3 (e) $f(-3)$ DNE

(f) $\lim_{x \to 1} f(x)$ 1 (g) $\lim_{x \to -5} f(x)$ 5 (h) $\lim_{x \to 0} f(x)$ 0 (i) $\lim_{x \to 3} f(x)$ 2

(j) $\lim_{x \to -4^+} f(x)$ 1 (k) $\lim_{x \to -4^-} f(x)$ 3 (l) $\lim_{x \to -4} f(x)$ DNE (m) $\lim_{x \to 2^+} f(x)$ ∞

(n) $\lim_{x \to 2^-} f(x)$ ∞ (o) $\lim_{x \to 2} f(x)$ ∞ (p) $\lim_{x \to -2^+} f(x)$ $-\infty$ (q) $\lim_{x \to -2^-} f(x)$ ∞

(r) $\lim_{x \to -2} f(x)$ DNE (s) $\lim_{x \to \infty} f(x)$ 1 (t) $\lim_{x \to -\infty} f(x)$ ∞



Assignment
Page 122 Oral Exercises
#'s 3-8, 12,13-20
Page 123
#'s 8-13

8. For the function $f(x)$ whose graph is shown, give the value of each quantity, if it exists. If it does not exist, explain why.

(a) $f(-3)$

(b) $f(0)$

(c) $f(2)$

(d) $f(-1)$

(e) $\lim_{x\to -2} f(x)$

(f) $\lim_{x\to 1} f(x)$

(g) $\lim_{x\to 3^-} f(x)$

(h) $\lim_{x\to -3^+} f(x)$

(i) $\lim_{x\to 2^+} f(x)$

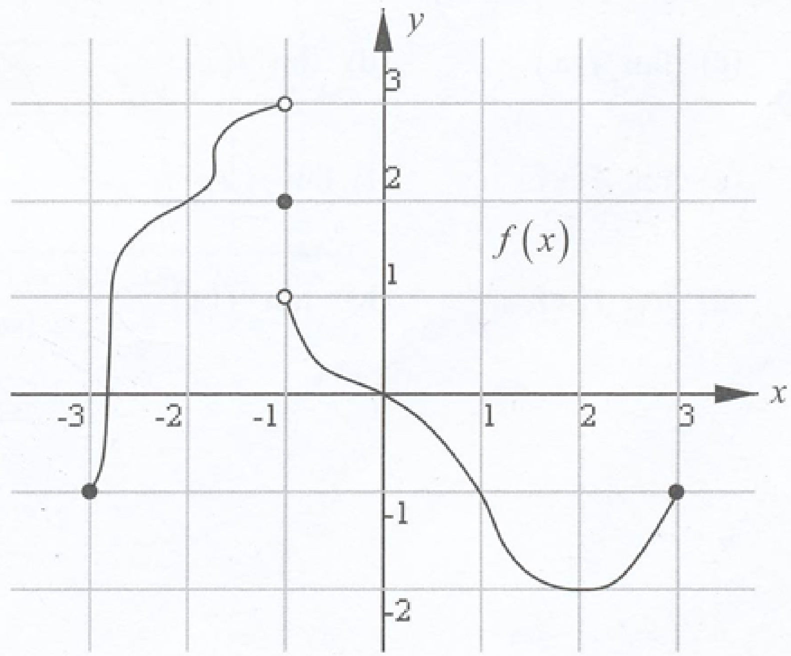
(j) $\lim_{x\to 2^-} f(x)$

(k) $\lim_{x\to 2} f(x)$

(l) $\lim_{x\to -1^+} f(x)$

(m) $\lim_{x\to -1^-} f(x)$

(n) $\lim_{x\to -1} f(x)$



9. For the function $g(x)$ whose graph is shown, give the value of each quantity, if it exists. If it does not exist, explain why.

(a) $g(-1)$

(b) $g(1)$

(c) $\lim_{x\to 0} g(x)$

(d) $\lim_{x\to 1^+} g(x)$

(e) $\lim_{x\to 1^-} g(x)$

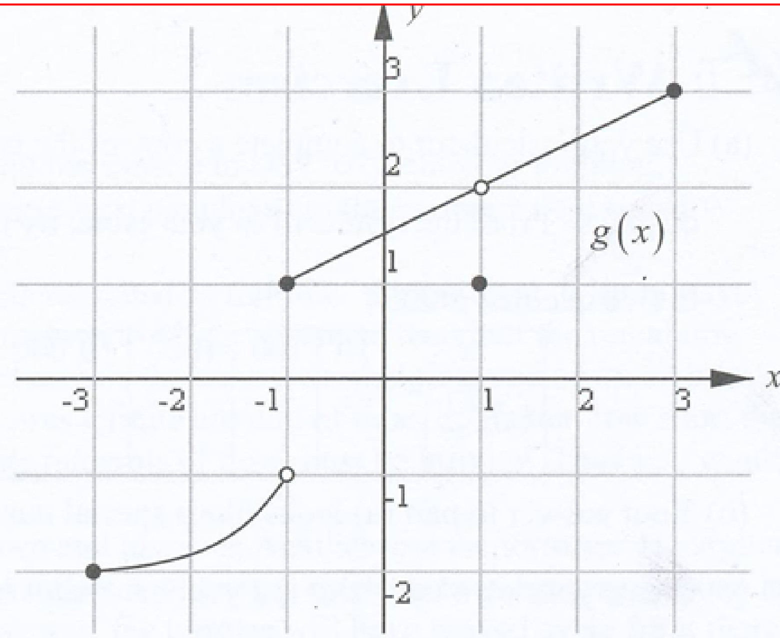
(f) $\lim_{x\to 1} g(x)$

(g) $\lim_{x\to -3^+} g(x)$

(h) $\lim_{x\to -1^+} g(x)$

(i) $\lim_{x\to -1^-} g(x)$

(j) $\lim_{x\to -1} g(x)$

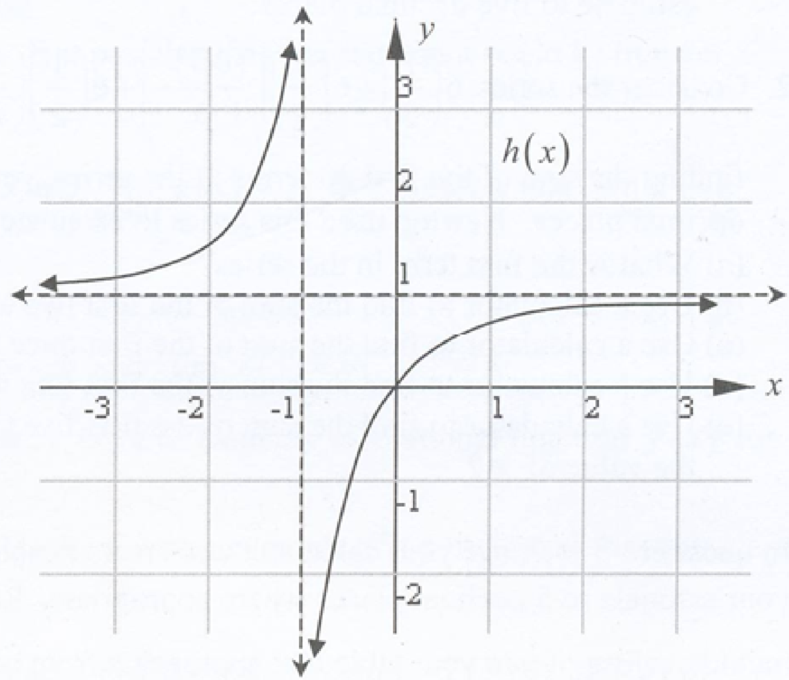


10. For the function $h(x)$ whose graph is shown, find each of the following.

(a) $\lim_{x\to 0} h(x)$ (b) $\lim_{x\to -1^-} h(x)$

(c) $\lim_{x\to -1^+} h(x)$ (d) $\lim_{x\to -1} h(x)$

(e) $\lim_{x\to \infty} h(x)$ (f) $\lim_{x\to -\infty} h(x)$



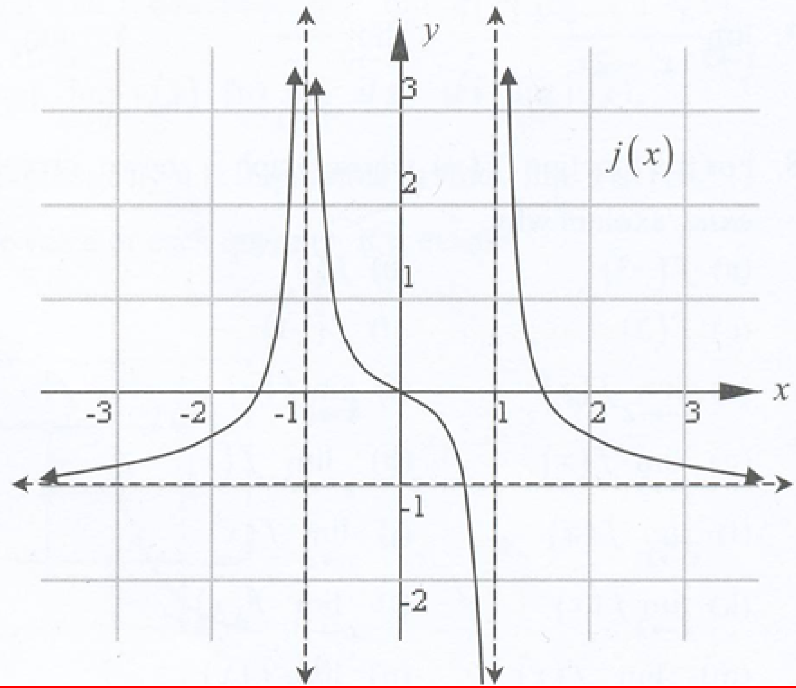
11. For the function $j(x)$ whose graph is shown, find each of the following.

(a) $\lim_{x\to -1^+} j(x)$ (b) $\lim_{x\to -1^-} j(x)$

(c) $\lim_{x\to -1} j(x)$ (d) $\lim_{x\to 1^+} j(x)$

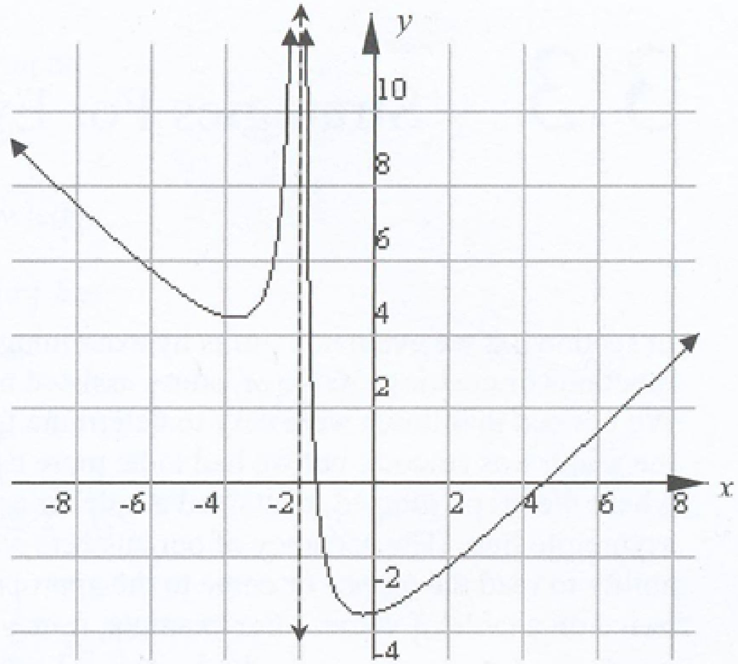
(e) $\lim_{x\to 1^-} j(x)$ (f) $\lim_{x\to 1} j(x)$

(g) $\lim_{x\to \infty} j(x)$ (h) $\lim_{x\to -\infty} j(x)$



12. Shown at right is the graph of the function $f(x)=\dfrac{x^2-1}{|x+2|}-3$. Examine the graph and determine each of the following.

(a) $\lim_{x\to -2} f(x)$ (b) $\lim_{x\to -\infty} f(x)$ (c) $\lim_{x\to\infty} f(x)$



$f(x)=\dfrac{x^2-1}{|x+2|}-3$

Application

13. Catering a banquet for one thousand people requires a great deal of organization and coordination. More than one person would be required just to make the coffee and place it into urns so that those serving can get these urns to the tables. Suppose that the graph below shows the total number of litres of coffee, $c(t)$, that are on the banquet tables or in the kitchen at any time t.

    (a) Based on the shape of the graph, describe what is happening.

    (b) Find $\lim_{t \to 30^+} c(t)$ and $\lim_{t \to 30^-} c(t)$ and explain the significance of these one-sided limits.

    (c) When was the last batch of fresh coffee ready?