3.2 Remainder Theorem

Divide $x^2 + 7x + 17$by $x + 3$ using the process of long division

$$\begin{array}{r} x + 4 \\ x+3 \overline{)\, x^2 + 7x + 17} \\ -\left(x^2 + 3x\right) \\ \hline 4x + 17 \\ -\left(4x + 12\right) \\ \hline \boxed{5} \end{array}$$

Remainder

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$$

$$\frac{x^2+7x+17}{x+3} = x+4 + \frac{5}{x+3}$$

The result of the division of a polynomial in x, $P(x)$, by a binomial of the form $x - a$, $a \in \mathrm{I}$, is $\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}$, where $Q(x)$ is the quotient and R is the remainder.

An easier way to divide is by using **synthetic division.**

Divide $x^2 + 7x + 17$ by $x + 3$ using the process of **synthetic division.**

① polynomial written in descending order

② use "0" place holders

$$
\begin{array}{r|rrr}
-3 & 1 & 7 & 17 \\
 & \downarrow & -3 & -12 \\
\hline
 & 1x & +4 & , \boxed{5}
\end{array}
$$

x x^2

remainder

$$x + 4 + \frac{5}{x+3}$$

Your Turn

a) Divide the polynomial $P(x) = x^4 - 2x^3 + x^2 - 3x + 4$ by $x - 1$.

Express the result in the form $\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}$.

b) Identify any restrictions on the variable.

c) Verify your answer.

1	1	-2	1	-3	4
	↓	1	-1	0	-3

$1x^3 - 1x^2 + 0x - 3$ $\boxed{1}$

a) $$\cancel{(x-1)}\,\frac{x^4-2x^3+x^2-3x+4}{\cancel{x-1}} = (x-1)(x^3-x^2-3) + \frac{1\,\cancel{(x-1)}}{\cancel{x-1}}$$

b) $x \neq 1$

c) $$x^4-2x^3+x^2-3x+4 = x^4-x^3-3x-x^3+x^2+3+1$$

$$= x^4-2x^3+x^2-3x+4$$

Example 1

Use synthetic division to divide for $2x^3 + x^2 + 4x - 10$ by $x + 2$

$$\begin{array}{r|rrrr} -2 & 2 & 1 & 4 & -10 \\ & \downarrow & -4 & 6 & -20 \\ \hline & 2 & -3 & 10 & \boxed{-30} \end{array}$$

$$2x^2 - 3x + 10 - \frac{30}{x+2}$$

$$x^3 + 6x^2 + 11x + 6$$

Example 2

Divide $x^3 + 11x + 6x^2 + 6$ by $x + 2$

$$\begin{array}{r|rrrr}
-2 & 1 & 6 & 11 & 6 \\
 & \downarrow & -2 & -8 & -6 \\
\hline
 & 1 & 4 & 3 & \boxed{0}
\end{array}$$

$$(x+2)(x^2+4x+3)$$

$$\boxed{(x+2)(x+3)(x+1)}$$

Your Turn

Use synthetic division to determine $\dfrac{x^3 + 7x^2 - 3x + 4}{x - 2}$.

$$
\begin{array}{r|rrrr}
2 & 1 & 7 & -3 & 4 \\
  & \downarrow & 2 & 18 & 30 \\
\hline
  & 1 & 9 & 15 & \boxed{34}
\end{array}
$$

$$x^2 + 9x + 15 + \frac{34}{x-2}$$

Now lets look back at our last example. We were asked to divide $x^3 + 7x^2 - 3x + 4$ by $x - 2$. What if we evaluated our polynomial at $x = 2$?

$$P(2) = (2)^3 + 7(2)^2 - 3(2) + 4$$
$$= 8 + 28 - 6 + 4$$
$$= 34$$

The result we just discovered is the **remainder theorem.**

The **remainder theorem** states that when a polynomial in x, $P(x)$, is divided by a binomial of the form $x - a$, the remainder is $P(a)$.

Example 3
For each dividend, determine the value of k, if the remainder is -2

$a)\ (2x^3 - 5x^2 - 4x + k) \div (x + 1)$

$$2(-1)^3 - 5(-1)^2 - 4(-1) + K = -2$$
$$-2 - 5 + 4 + K = -2$$
$$K = 1$$

Students try
b) $(3x^3 + kx^2 - 13x + 4) \div (x + 2)$

$$3(-2)^3 + K(-2)^2 - 13(-2) + 4 = -2$$
$$-24 + 4K + 26 + 4 = -2$$
$$4K + 6 = -2$$
$$4K = -8$$
$$K = -2$$

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#’s 2 use synthetic,4d,e,f ,5d,e,f 6a,c,e 7a,c
8,10,13a, 17b