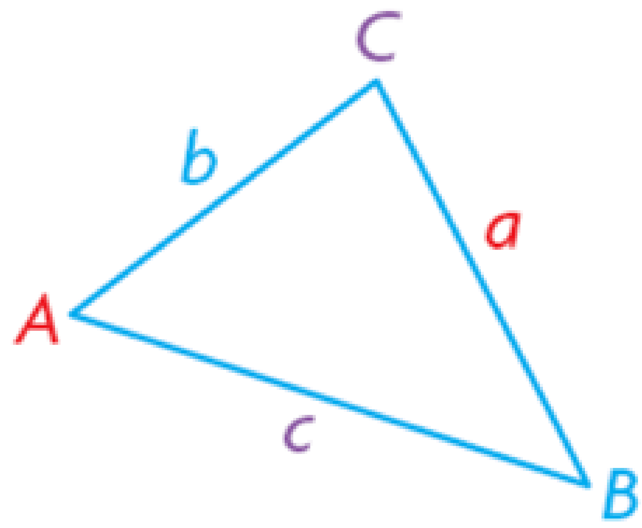
3.2 Proving and Applying the Sine Law

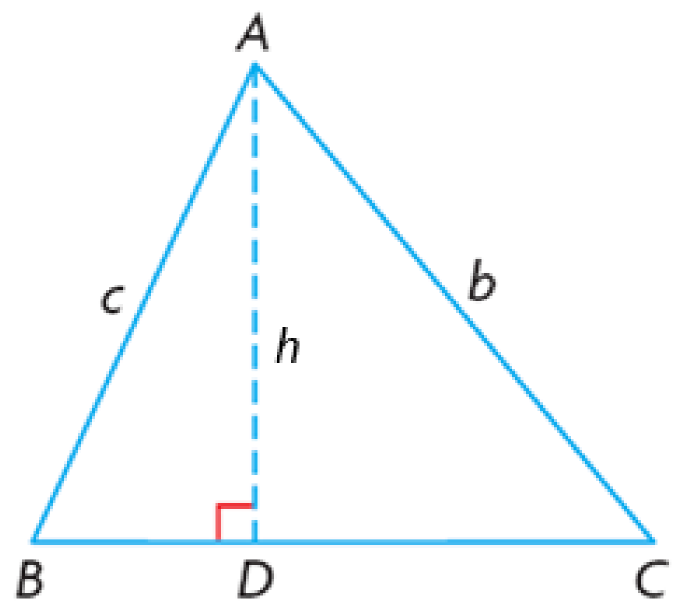
- What did we discover last class?
- In an acute triangle, *ΔABC,*
- $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



Proof: Sine Law

Use what we know to find *h*

$$\sin B = \frac{h}{c}$$



$$\sin c = \frac{h}{b}$$

Is *h* the same for *ΔADC* and *ΔABD*?

Continue proof

Solve for *h* for each equation

$$\sin B = \frac{h}{c} \qquad \sin c = \frac{h}{b}$$

$$h = (c)(\sin B) \qquad h = (b)(\sin C)$$

Make *h* equal to each other

$$(c)(\sin B) = (b)(\sin C)$$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

OR

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

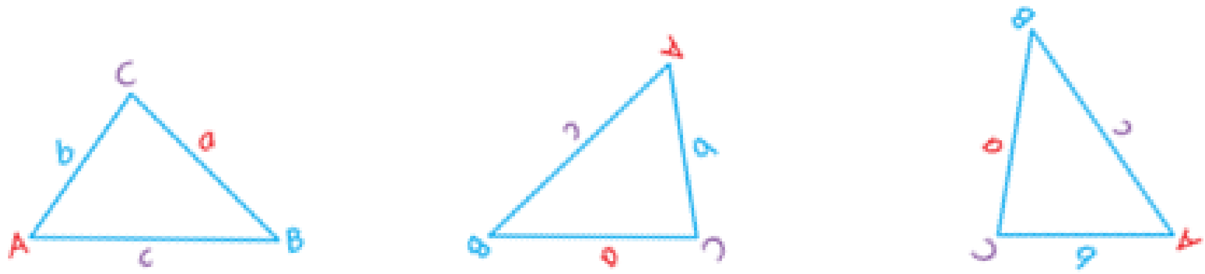
Go Deeper

Could you use the same proof to show that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{a}{\sin A} = \frac{c}{\sin C} \qquad \frac{\sin A}{a} = \frac{\sin C}{c}$$

YES, just rotate the triangle



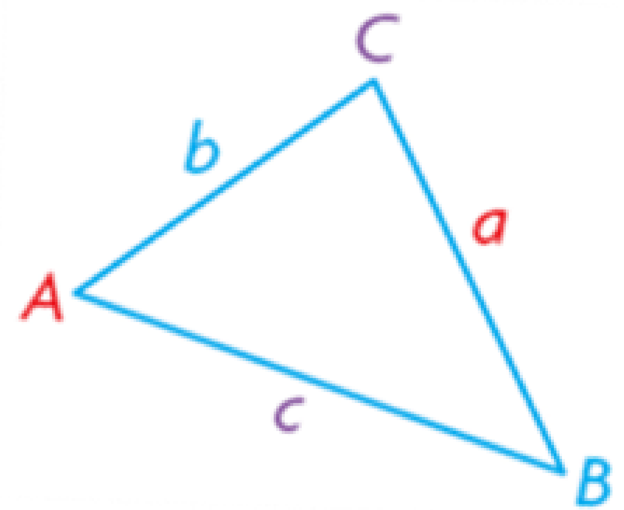
Sine Law

We have just proven:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

and

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



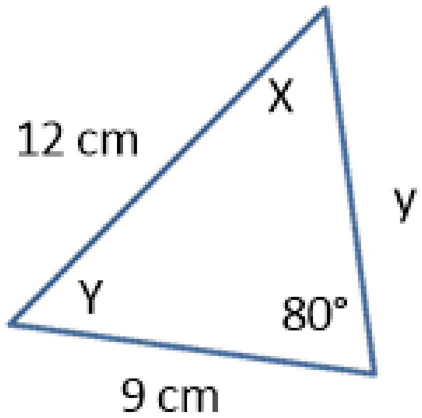
For acute triangles ΔABC

Example 1 (p. 120)

A triangle has angles measuring 80° and 55°. The side opposite the 80° angle is 12.0 m in length. Determine the length of the side opposite the 55° angle to the nearest tenth of a metre.

Example

Determine <*X*



If *y*=9.6 cm. Use the sine law to determine <*Y*. Verify your answer by determining the sum of the angles.

Homework

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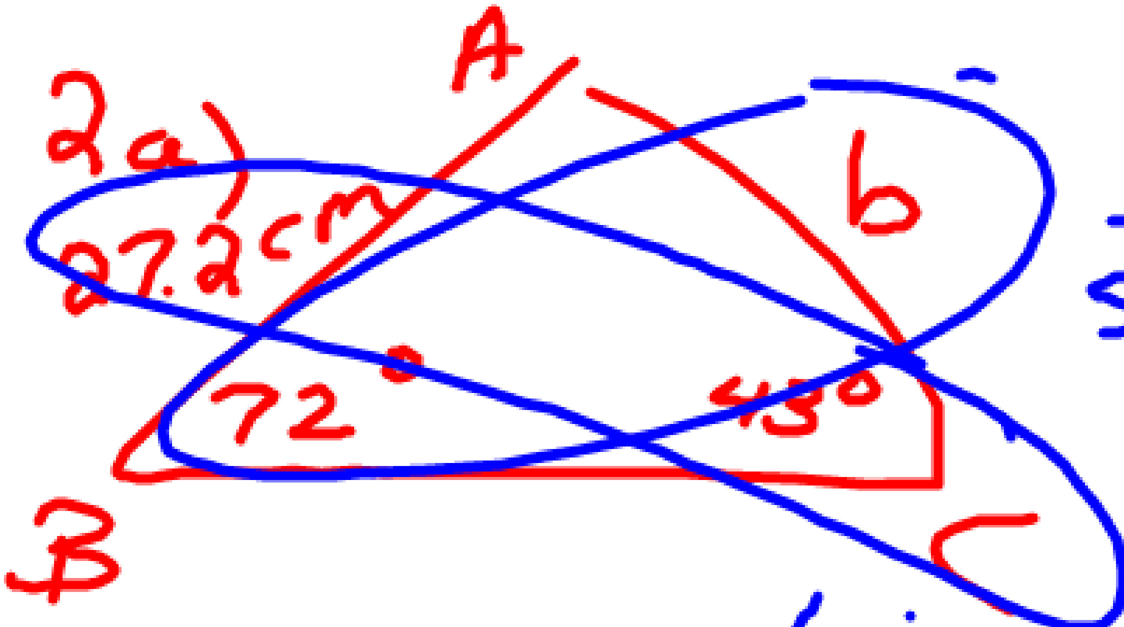
2, 3, 6, 7, 8, 11, 15

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$



$$\frac{27.2}{\sin 43^\circ} = \frac{b}{\sin 72^\circ}$$

$$37.9\text{cm} = \frac{(\sin 72)(27.2)}{\sin 43^\circ} = b$$

Key Idea

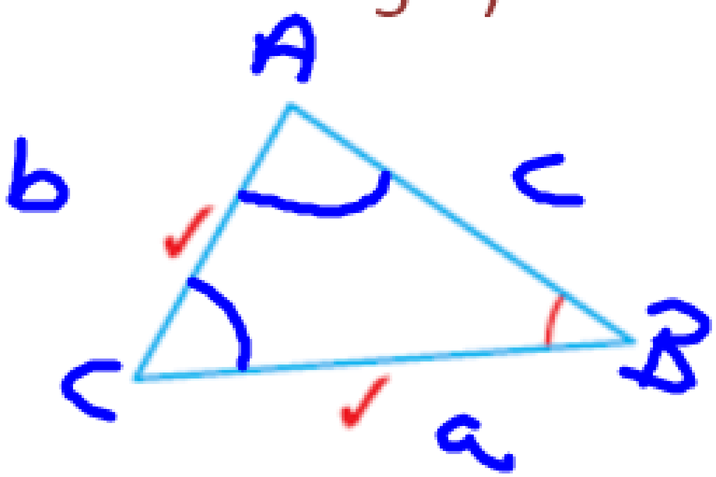
The sine law can be used to determine unknown side lengths or angle measures in acute triangles

What information do you need to use the sine law?

Need to Know

You can use the sine law when you know:

Two sides and the angle opposite to a known side (**SSA** – *side side angle*)

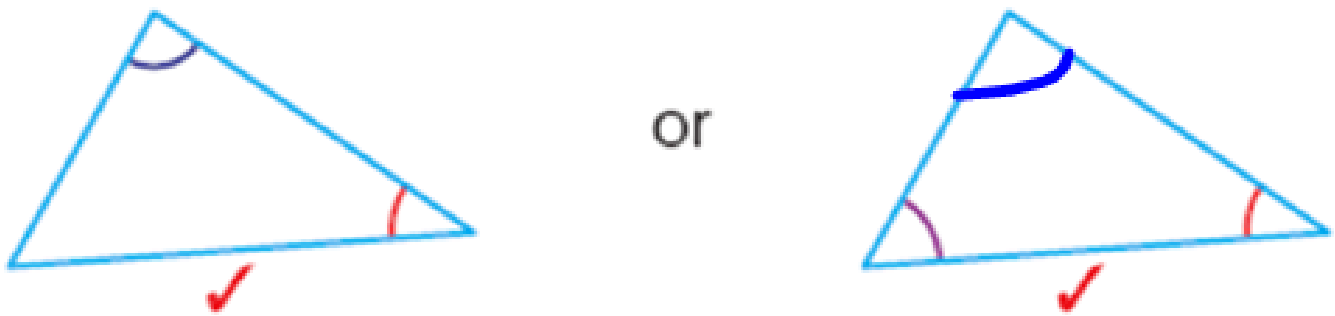


Which angle could you find?

Need to Know

You can use the sine law when you know:

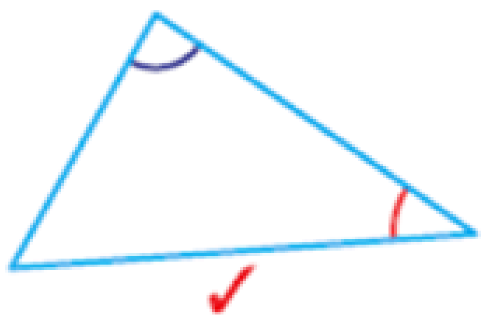
Two angles and any side (**SAA** – *side angle angle* or **ASA** – *angle side angle*)



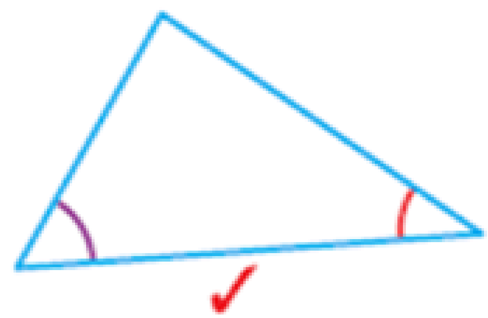
Why don't you need to know the angle opposite one of the sides??

Need to Know

If you know the measures of two angles in a triangle, you can determine the third angle because the angles must add to 180°



or



Need to Know

When finding side lengths, use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

When finding angles, use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

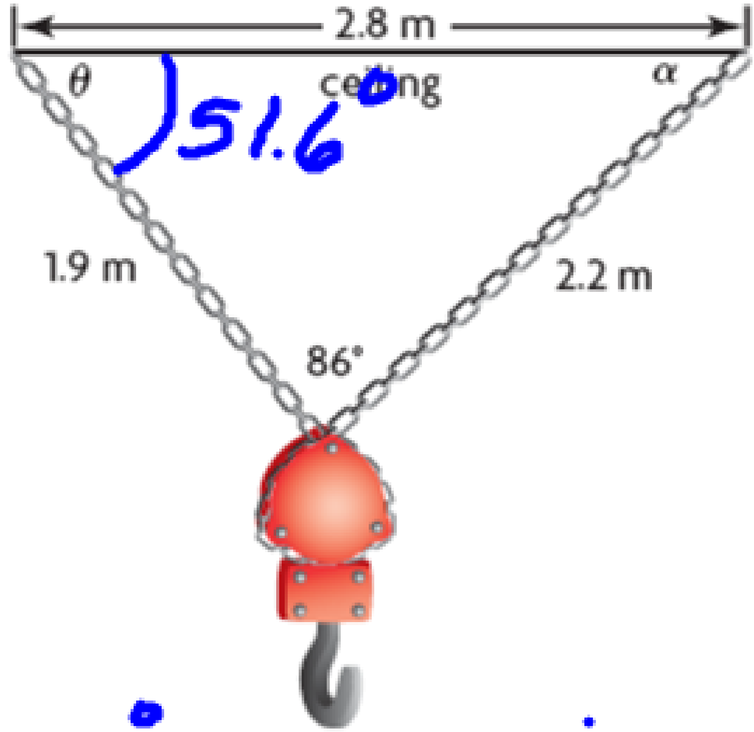
Applications

We can use the sine law to solve real life problems

Example 2 (p.121)

$\alpha = 180 - 86 - 51.6 = 42.4^\circ$

Toby uses chains attached to hooks on the ceiling and a winch to lift engines at his father's garage. The chains, the winch, and the ceiling are arranged as shown. Toby solved the triangle using the sine law to determine the angle that each chain makes with the ceiling to the nearest degree. He claims that $\theta = 40°$ and $\alpha = 54°$. Is he correct? Explain, and make any necessary corrections.



(2.2)

$$\frac{\sin 86^\circ}{2.8} = \frac{\sin \theta}{2.2}$$

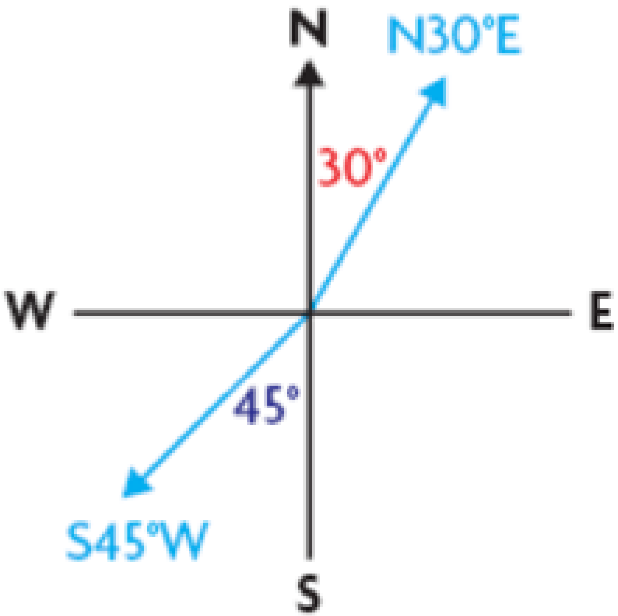
$$\sin\theta = \frac{(2.2)\sin 86^\circ}{(2.8)}$$

$$\sin\theta = 0.7838$$

$$\theta = \sin^{-1}(0.7838)$$

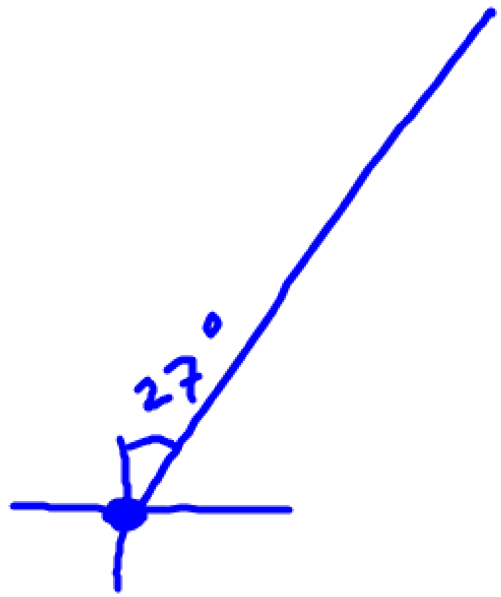
$$\theta = 51.6^\circ$$

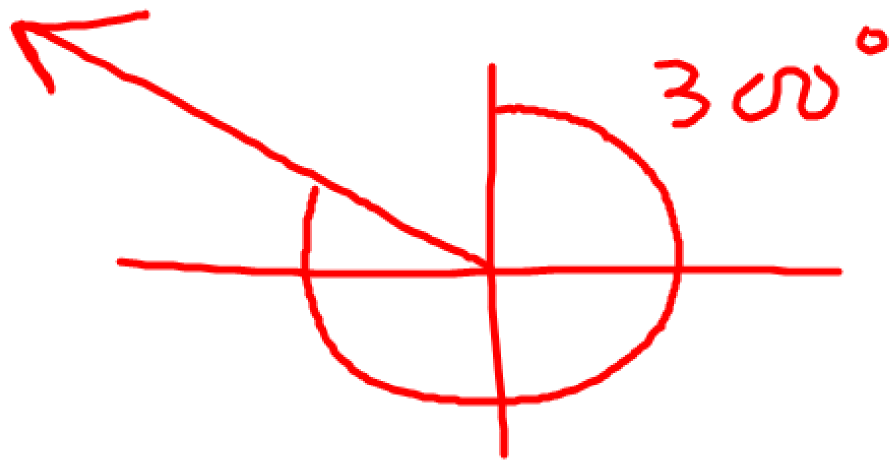
Never Eat Soggy Weiners



- Directions are often stated in terms of north and south on a compass.
- Ex: N30°E means start facing North, then turn 30° to the east.
- Ex: S45°W means start facing south, then turn 45° to the west.

N 27° E





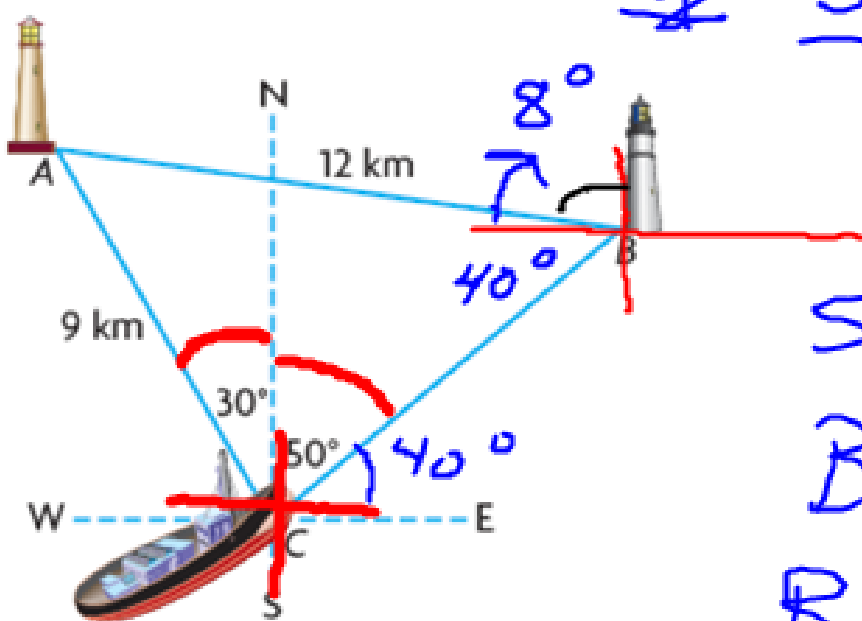
E 53° N

Example 3 (p. 122)

From B must travel W 8°N towards A.

N82°W

The captain of a small boat is delivering supplies to two lighthouses, as shown. His compass indicates that the lighthouse to his left is located at N30°W and the lighthouse to his right is located at N50°E. Determine the compass direction he must follow when he leaves lighthouse *B* for lighthouse *A*.



$$\frac{\sin B}{} = \frac{9 \sin 80^\circ}{12}$$

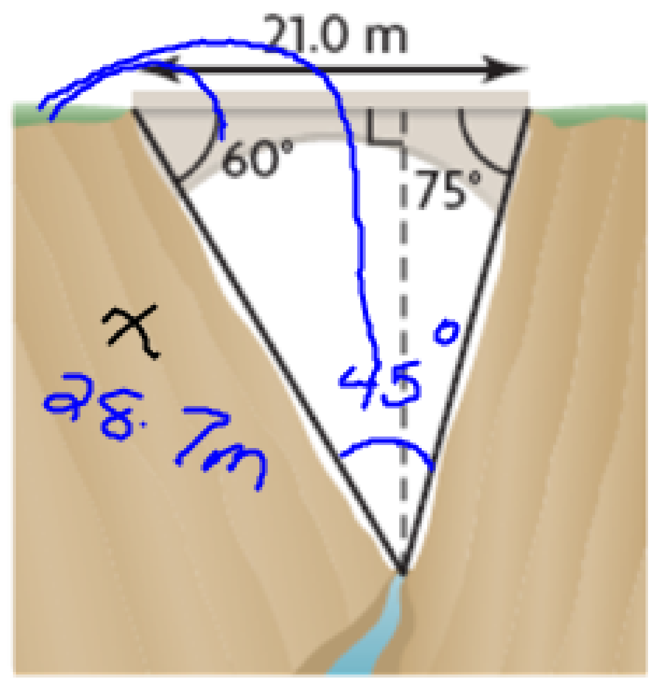
$$\sin B = 0.7386$$

$$B = \sin^{-1}(0.7386)$$

$$B = 47.6^\circ \simeq 48^\circ$$

Example 13 (p. 127)

13. A bridge has been built across a gorge. Jordan wants to bungee jump from the bridge. One of the things she must know, to make the jump safely, is the depth of the gorge. She measured the gorge as shown. Determine the depth of the gorge to the nearest tenth of a metre.



$$\frac{21}{\sin 45^\circ} = \frac{x}{\sin 75^\circ}$$

$$\frac{(\sin 75)\,21}{\sin 45} = x = 28.7\text{m}$$

$$\sin 60^\circ = \frac{h}{28.7}$$

$$(28.7)\sin 60^\circ = h$$

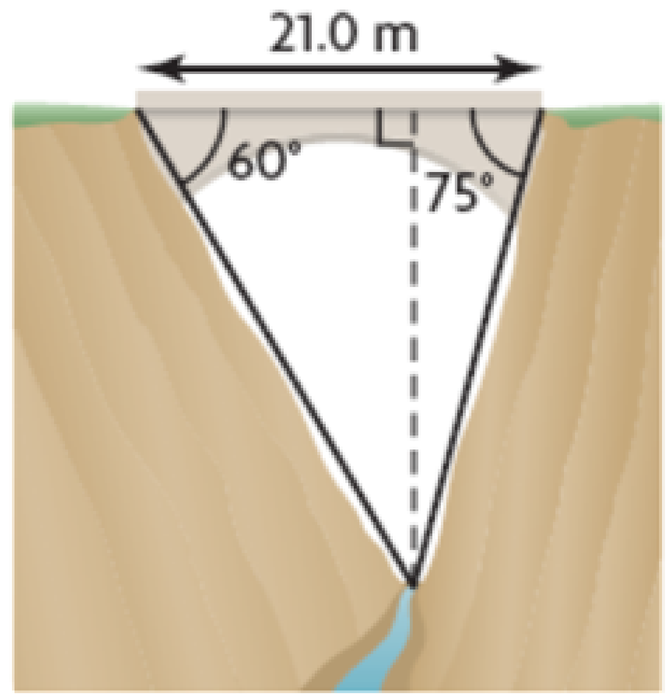
$$24.8\text{ m} = h$$

Example 13 (p. 127)

What information do we know?

What can we find with it?

What do we need?



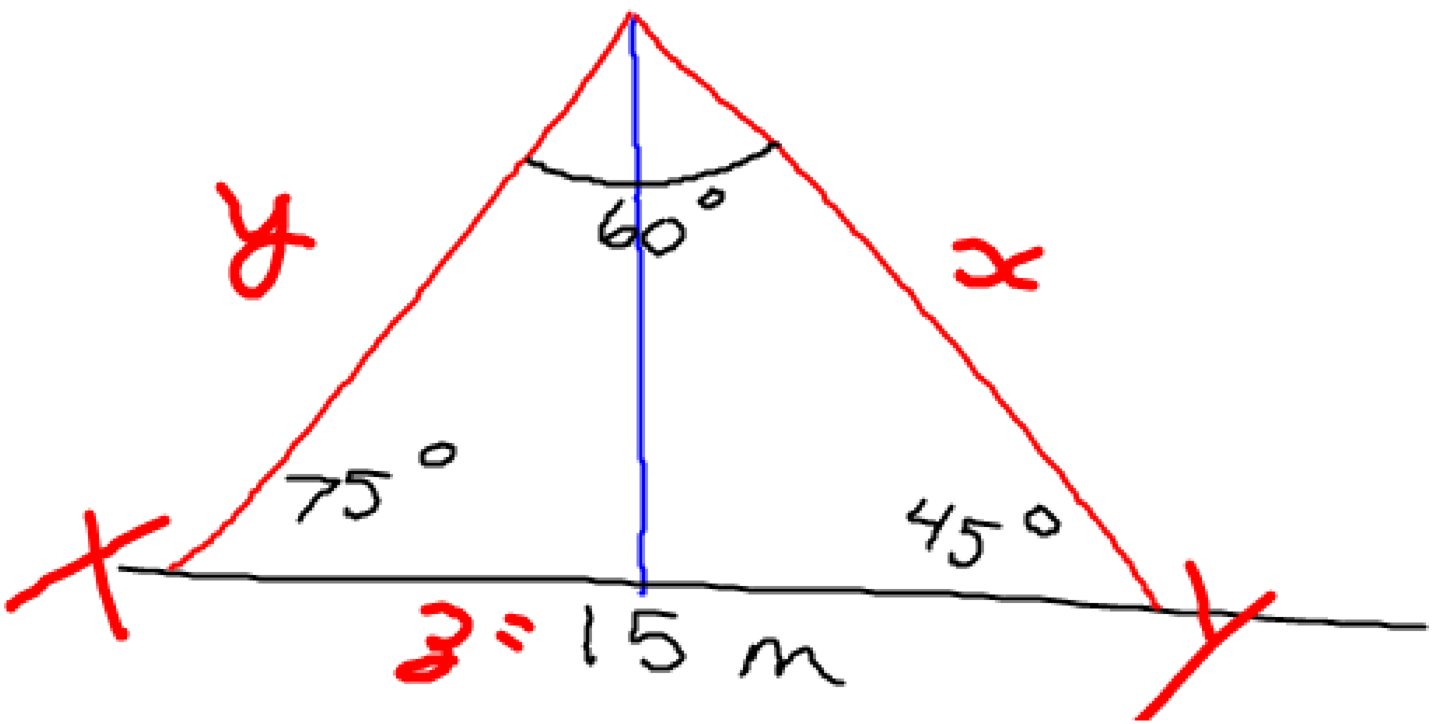
Example 10 (p. 126)

A telephone pole is supported by two wires on opposite sides. At the top of the pole, the wires form an angle of 60°. On the ground, the ends of the wires are 15.0 m apart. One wire makes a 45° angle with the ground.

a) Draw a diagram of this situation, then compare it with a classmate's.

b) How long are the wires, and how tall is the pole? Express your answers to the nearest tenth of a metre.

a)



b)

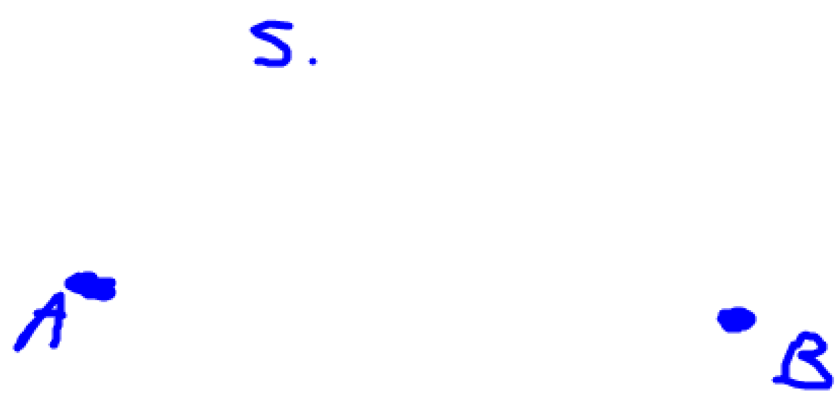
$$\frac{x}{\sin 75} = \frac{15}{\sin 60}(\sin 75)$$

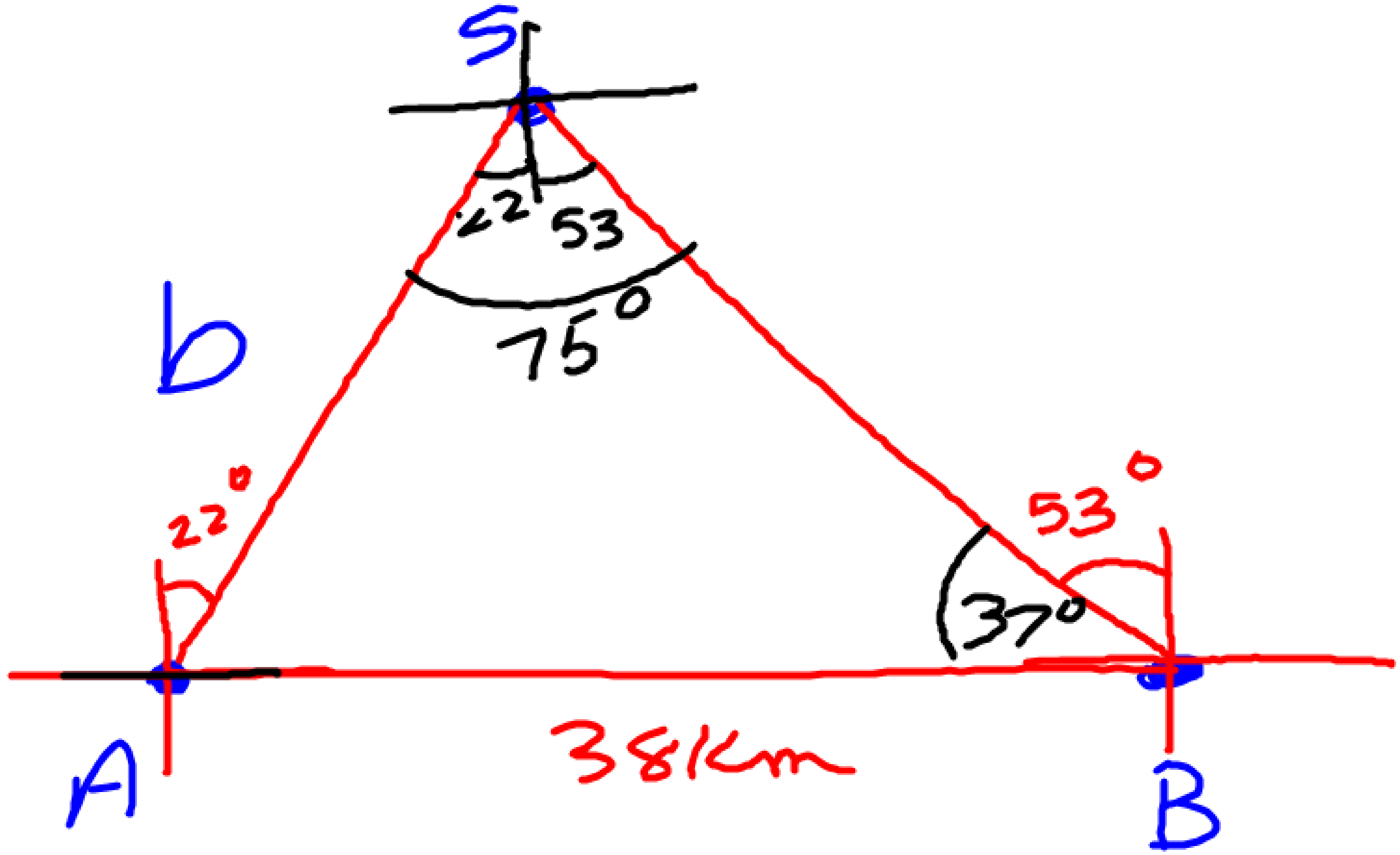
$x = 16.73$ m

$$\frac{y}{\sin 45} = \frac{15}{\sin 60^\circ}(\sin 45^\circ)$$

$y = 12.25$ m

Ex. Tracking stations A and B are 38 km apart. From A the direction of a ship is N 22° E. From B the direction of the same ship is N 53° W. How far is the ship from station A.





$$\frac{b}{\sin 37^\circ} = \frac{38}{\sin 75^\circ}(\sin 37^\circ)$$

$$b = 23.7\ \text{Km}$$

Homework

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 # 2, 3, 6, 7, 8, 11, 15

4, 5, 9, 12, 18

Bisect – to cut an angle in half (i.e. two equal pieces)