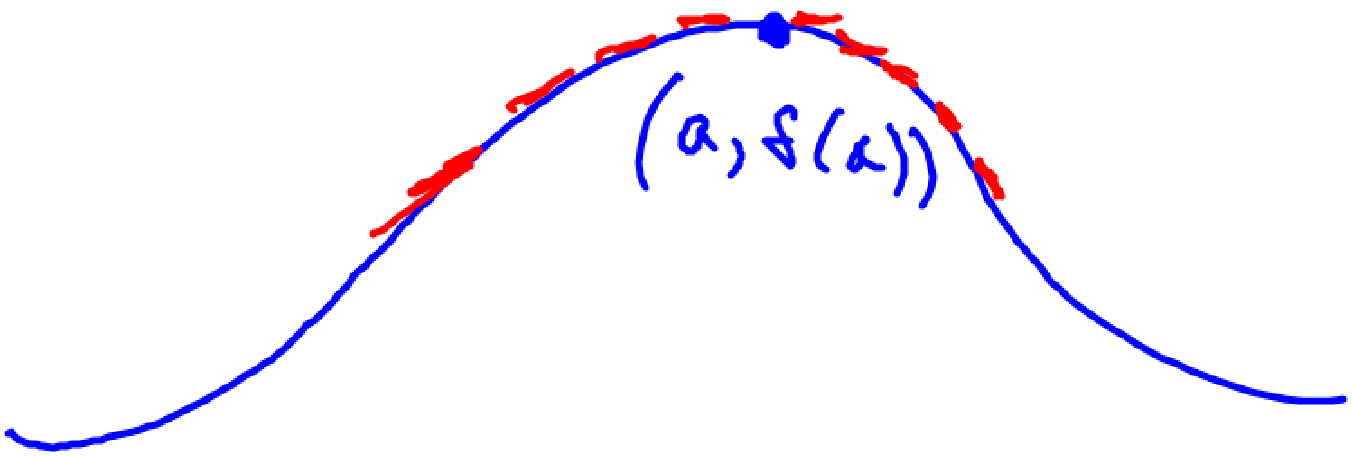
3.2 Differentiability

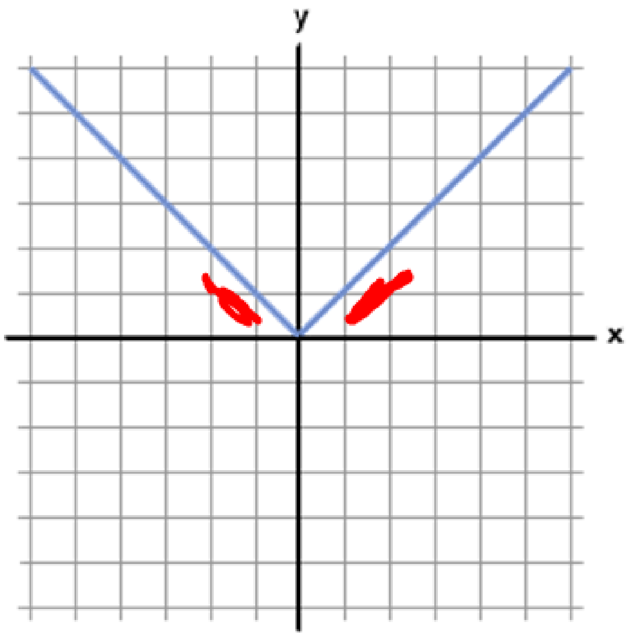
A function will **not have a derivative** at a point P*(a,f(a))* where the slopes of the secant lines,

$\frac{f(x)-f(a)}{x-a}$ fail to approach a limit as x approaches a.

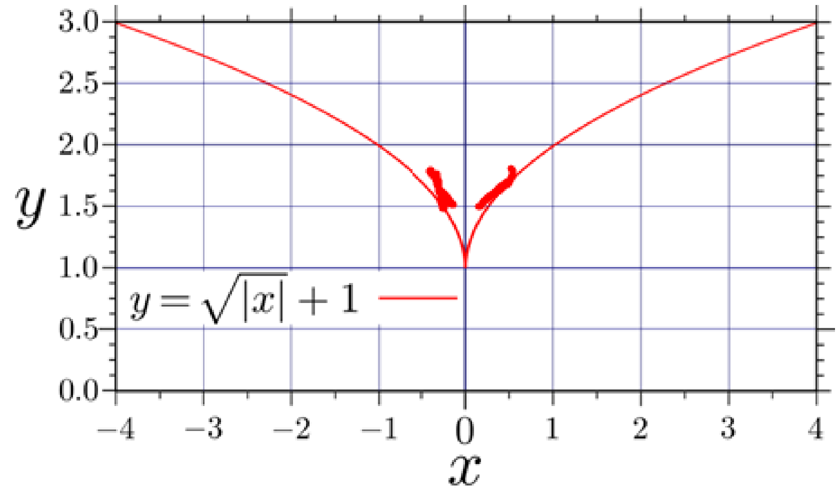


A function will **not have a derivative** at the following:

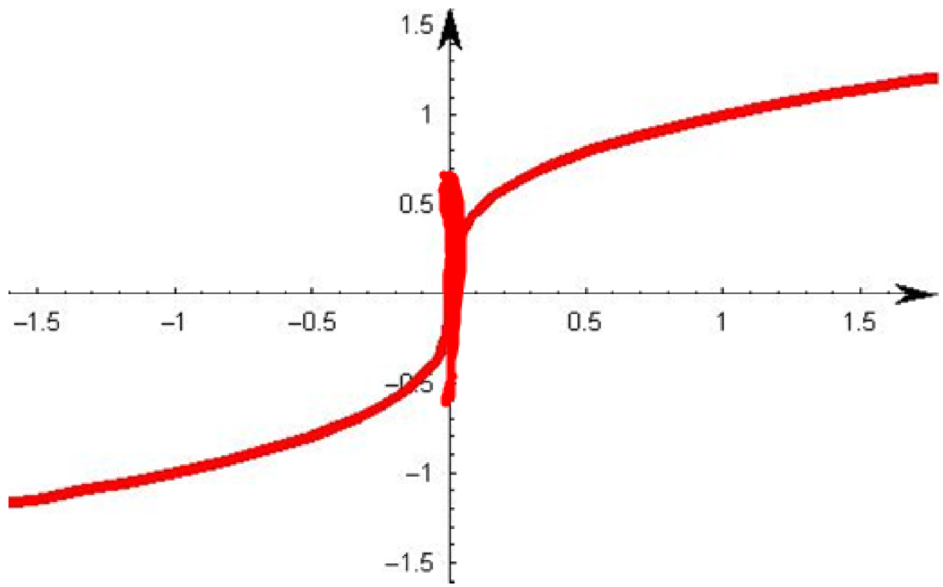
1. A **corner**, where the one sided derivatives differ.



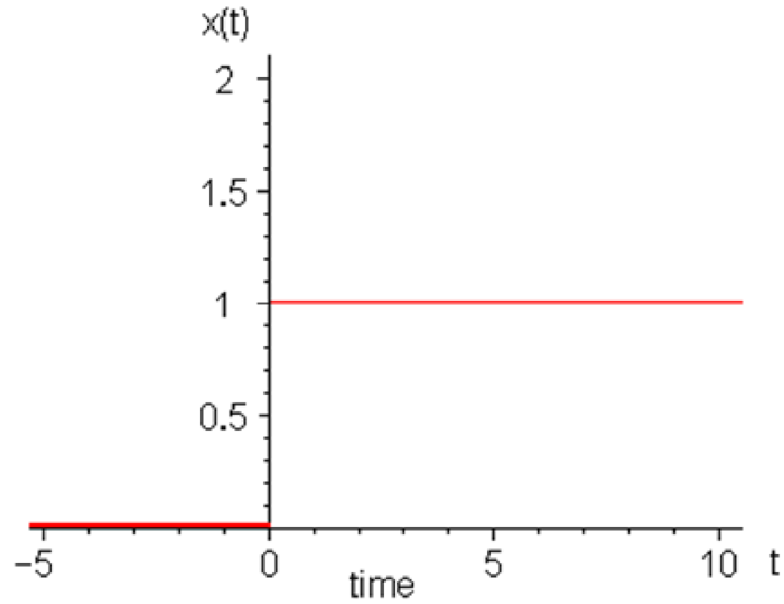
2. A **cusp**, where the slopes of the secants approach infinity from one side and negative infinity from the other side.



3. A **vertical tangent**, where the slopes of the tangents approach either infinity of negative infinity from both sides.



4. A **discontinuity**.



Differentiability Implies Continuity

AP Classroom

Is $g(x) = x^2$ diff at $x = 1$?

$$\lim_{x \to 1^+} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1^+} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = 1+1 = 2$$

$$\lim_{x \to 1^-} \frac{x^2 - 1}{x - 1} = 2$$

$\therefore$ Diff at $x = 1$

$$h(x) = \begin{cases} 2x & x < 1 \\ x^2 + 1 & x \geq 1 \end{cases}$$

Diff $x = 1$ ✓

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1}$$

$$\lim_{x \to 1^-} \frac{2x - 2}{x - 1}$$

$$\lim_{x \to 1^-} \frac{2(x-1)}{(x-1)} = 2$$

$$\lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\lim_{x \to 1^+} \frac{x^2 + 1 - 2}{x - 1}$$

$$\lim_{x \to 1^+} \frac{x^2 - 1}{x - 1}$$

$$\lim_{x \to 1^+} \frac{(x-1)(x+1)}{(x-1)} = 2$$

Just because a function is continuous does not imply differentiability

AP Classroom #2

$f(x) = |x|$

diff $x = 0$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

cont $x = 0$

$$\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0}$$

$$\lim_{x \to 0^+} \frac{x - 0}{x - 0} = 1$$

$$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0}$$

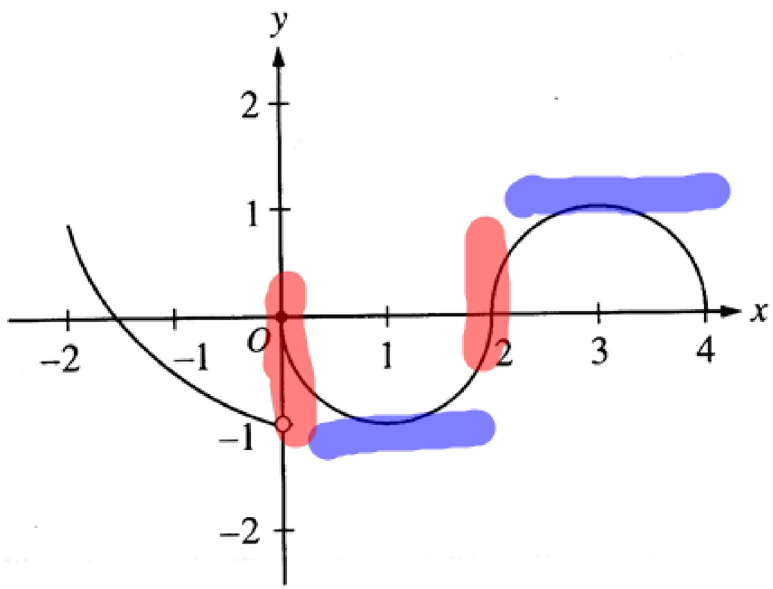
$$\lim_{x \to 0^-} \frac{-x - 0}{x - 0} = -1$$

$\therefore$ Not Diff at $x = 0$.

AP Classroom #3

$$h(x) = \begin{cases} 2x, & x < 1 \\ x^2+1, & x > 1 \end{cases}$$

Not cont at $x = 1$



13. The graph of the function f shown in the figure above has a vertical tangent at the point (2, 0) and horizontal tangents at the points (1, −1) and (3, 1). For what values of x, $-2 < x < 4$, is f not differentiable?

(A) 0 only (B) 0 and 2 only (C) 1 and 3 only (D) 0, 1, and 3 only (E) 0, 1, 2, and 3

Differentiability Implies Local Linearity

A differentiable function is sometimes referred to as locally linear. That is the graph of the function will "**straighten out**" if we zoom in to a point where the graph has a derivative.

Demonstrate with a quadratic and an absolute value.

Derivatives on a Calculator

Calculate the derivative of the following functions on a calculator at the point indicated:

$y = x^3$ at $x = 2$

$y = 4\sqrt{x}$ at $x = 6$

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#'s 1- 10,