Unit #3 DERIVATIVES

3.1 Derivative of a Function

Last unit we learned how to find the slope of a tangent line using the formula:

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This slope is known as a **instantaneous rate change.**

It can also be referred to as the **definition of a derivative.**

Definition of a derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

A function that is differentiable at every point of its domain is called a **differentiable function.**

Ex.1 Find the derivative of $f(x) = x^2$

$$f' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = 2x + 0$$

$$= 2x$$

$$f'(x) = \underline{2x}$$

$$f'(3) = 2(3) = 6$$

There is an alternative formula for finding the derivative of a function at a **specific point**, say at $x=a$:

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

Ex. 2 Find the derivative of $f(x) = x^2 - 3x$ at the point $x = 4$, using the alternative formula.

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$$f'(4) = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4}$$

$$= \lim_{x \to 4} \frac{x^2 - 3x - 4}{x - 4}$$

$$= \lim_{x \to 4} \frac{(x-4)(x+1)}{(x-4)} = 4+1$$

$$= 5$$

There are **many notations** used for the derivative:

y' "read as y prime"

$\frac{dy}{dx}$ "read as the derivative of y with respect to x"

$\frac{df}{dx}$ "read as the derivative of f with respect to x"

$\frac{d}{dx}f(x)$ "read as the derivative of f at x"

One-Sided Derivatives

A function $y = f(x)$ is differentiable on a closed interval $[a, b]$ if it has a derivative at every interior point on the interval, and if the limits

$\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$ (Right hand derivative of a)

$\lim\limits_{h \to 0^-} \frac{f(b+h) - f(b)}{h}$ (Left hand derivative at b) exist at the endpoints.

Example of One Sided Derivatives

Determine if the following function has a derivative at $x = 0$.

$$f(x) = \begin{cases} x^2, if\ x \leq 0 \\ 2x, if\ x > 0 \end{cases}$$

$$\lim_{x \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$\lim_{x \to 0^+} \frac{2(0+h) - 0}{h}$$

$$\lim_{x \to 0^+} \frac{2h}{h} = 2$$

i) $f(0) = (0)^2 = 0$

ii) $\lim_{x \to 0^+} 2x = 0$ $\quad \lim_{x \to 0^-} x^2 = 0$

iii) $f(0) = \lim_{x \to 0} f(x) = 0$

$\therefore$ function cont. at $x = 0$

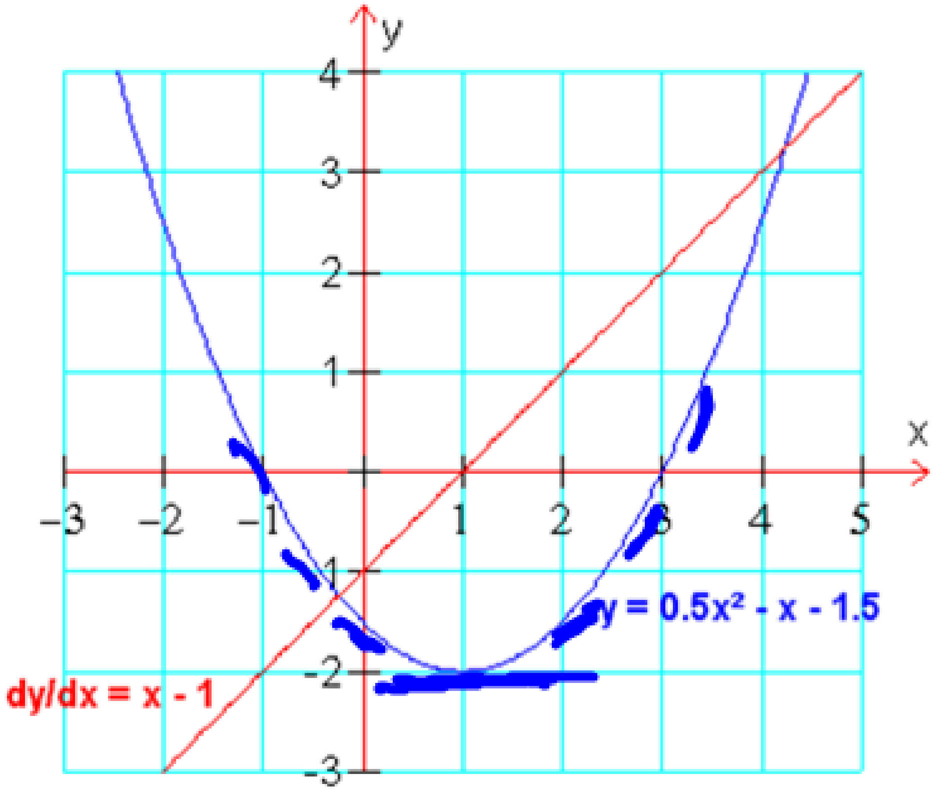
$$\lim_{x \to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{x \to 0^-} \frac{(0+h)^2 - 0}{h}$$

$$= \lim_{x \to 0^-} \frac{h^2}{h} = h = 0$$

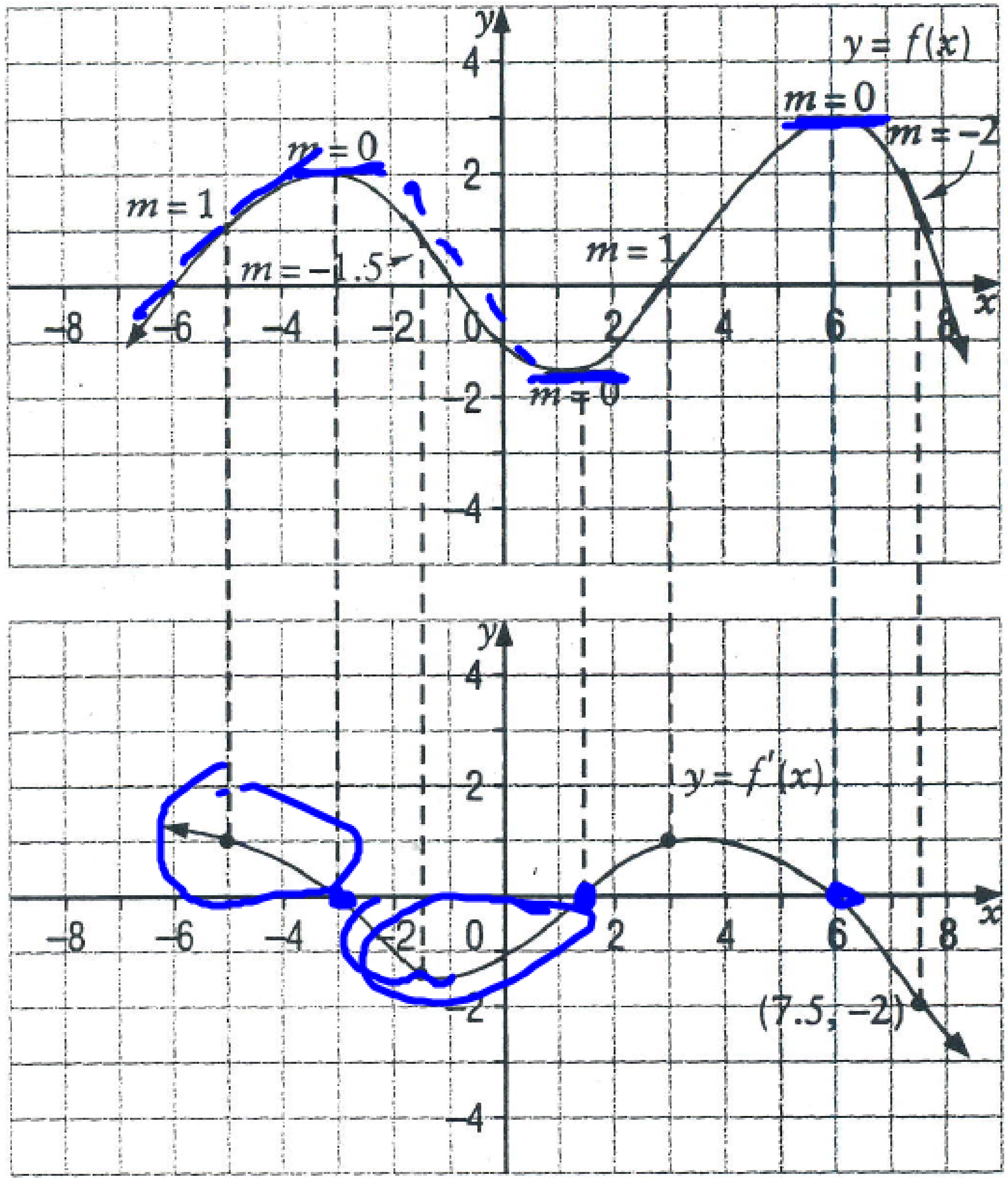
$\therefore$ Not Differentiable at $x = 0$.

Relationship Between the Graph of $f(x)$ and $f'(x)$



Blue graph is the graph of the function.

Red graph is the graph of the derivative.


y = f(x)
m = 0
m = −2
m = 1
m = −1.5
m = 1
m = 0
m = 0
y = f′(x)
(7.5, −2)

http://www.univie.ac.at/future.media/moe/tests/diff1/ablerkennen.html

Assignment AP Text
Page 101
#'s 1,3, 7-10, 11, 22, 28

Calc 30 Text
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#'s 1 a,c,f, 2,a,b,3,