Unit 3 Polynomial Functions

3.1 Characteristics of Polynomial Functions

Polynomial Function- is a function in the form:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

where:

n is a whole number

x- variable

the coefficients are real numbers.

The **degree of a polynomial** is determined by the largest **exponent** of an individual term.

$f(x) = 5x^4 + 3x^2 - 2x$ degree 4

$f(x) = (x + 2)(x + 3)$ degree 2

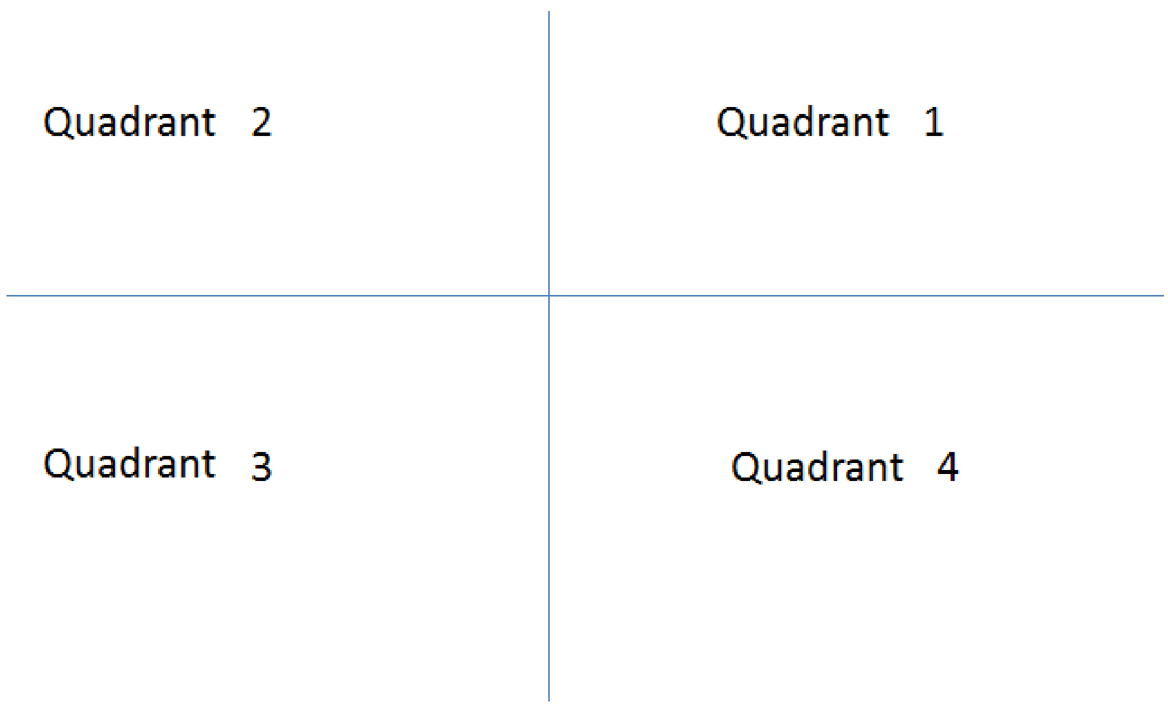
You try!

$$f(x) = (x^2 + 2)(x^3 + 2x - 1)(x - 2)$$

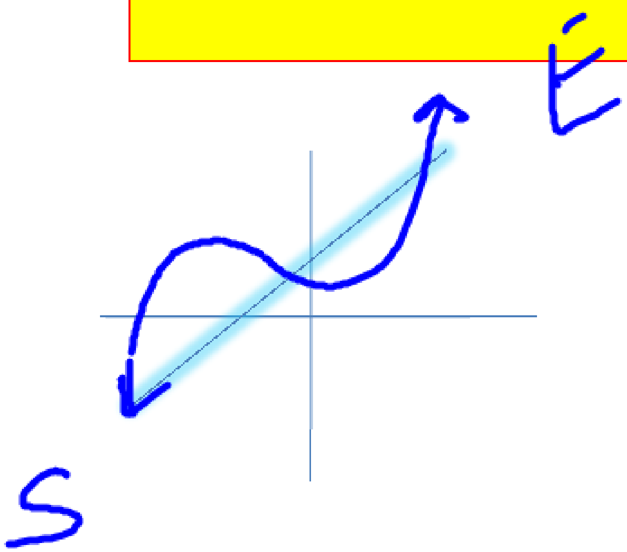
What do you think the degree is? 6

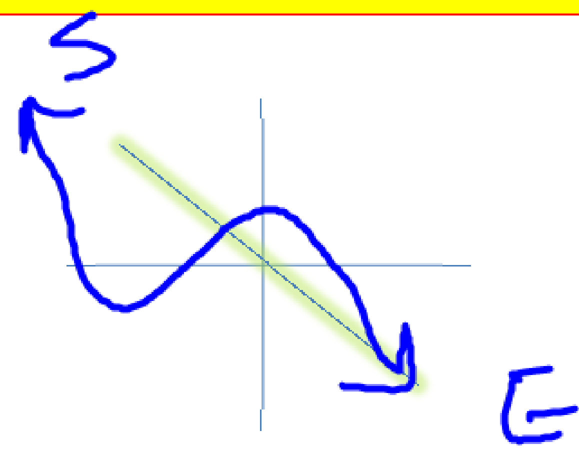
The **degree** also determines the maximum number of **x-intercepts.**

Degree 0- no x intercepts
Degree 1- max one x intercept
Degree 2- max 2 intercepts
Degree 3- max 3 intercepts and so forth.



Any polynomial with an odd (1,3,5...) degree will have a similar end behavior as a line.

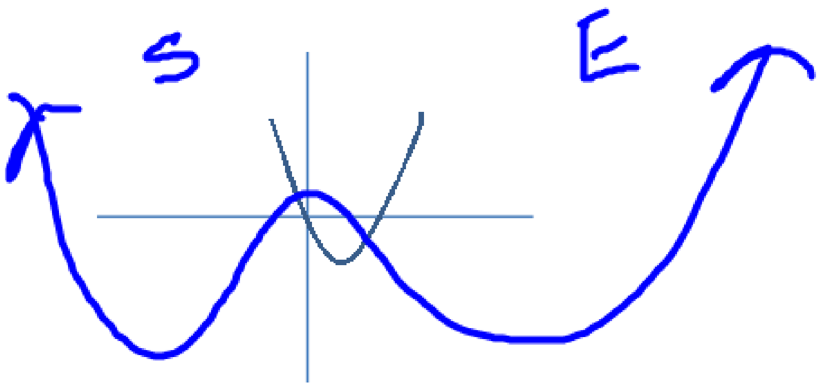


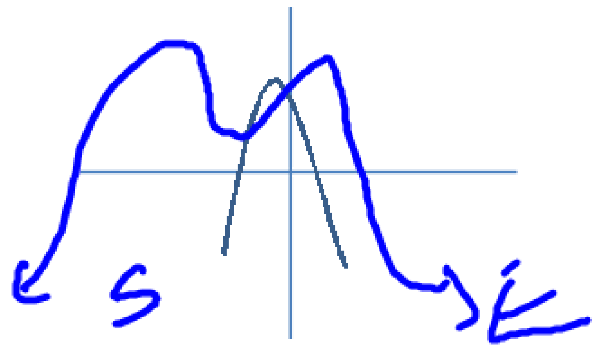


If the polynomial has an odd degree and a positive leading coefficient, the graph is moving up, starting in quadrant 3 and ending in quadrant 1.

If the polynomial has an odd degree and a negative leading coefficient, the graph is moving down, starting in quadrant 2 and ending in quadrant 4.

Any polynomial with and even degree(2,,4,6,8…) will have the same end behavior as a parabola. Parabola has a degree of 2





If the polynomial has an even degree and a positive leading coefficient, the graph starts in quadrant 2 and ends in quadrant 1.

If the polynomial has an even degree and a negative leading coefficient, the graph starts in quadrant 3 and ends in quadrant 4.

Any polynomial where the leading coefficient is (+) will extend up into quadrant 1. Any polynomial where the leading coefficient is (-) will extend down into quadrant 4.

Identifying Polynomial Functions

State whether each function is a polynomial? If so state the degree, the leading coefficient, and the constant term.

a) $y = 2x^2 - 3x + 2$ ✓ Deg: 2 LC: 2 C: 2

b) $y = 3^x + 5$ ✗

c) $g(x) = (3x + 2)(x - 6)$ ✓

$3x^2 - 18x + 2x - 12$

$3x^2 - 16x - 12$

D: 2

LC: 3

C: −12

d) $g(x) = x^{-2} + 7x^3$

e) $y = \sin x$

You try!

Which functions are polynomials? Justify your answer. State the degree, the leading coefficient, and the constant term of each polynomial function.

a) $g(x) = \sqrt{x} + 5$ $\quad x^{1/2} + 5$

b) $f(x) = 3x^4$ $\quad$ D: 4, L.C 3, C: 0

c) $y = |x|$

d) $y = 2x^3 + 3x^2 - 4x - 1$

D: 3

LC: 2

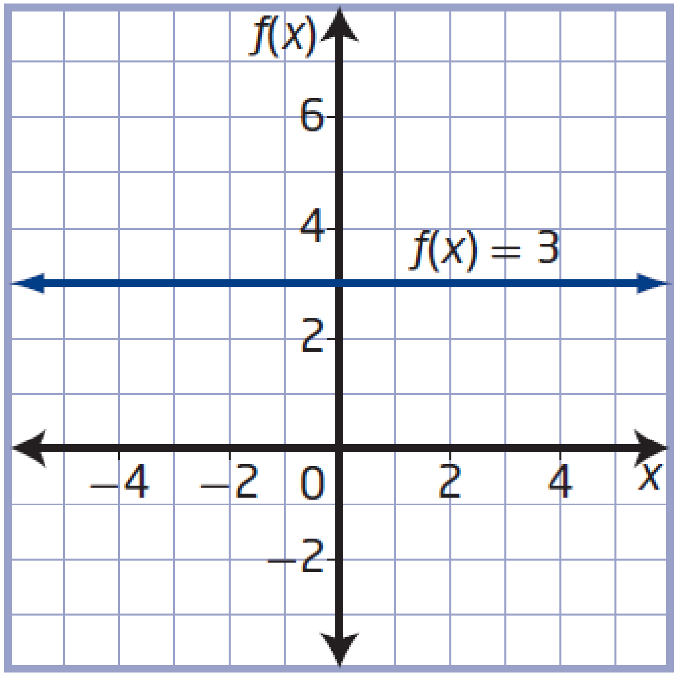
C: -1

See page 109 textbook

Degree 0: Constant Function

Even degree

Number of x-intercepts: 0 (for $f(x) \neq 0$)



Example: $f(x) = 3$

End behaviour: extends horizontally

Domain: $\{x \mid x \in R\}$

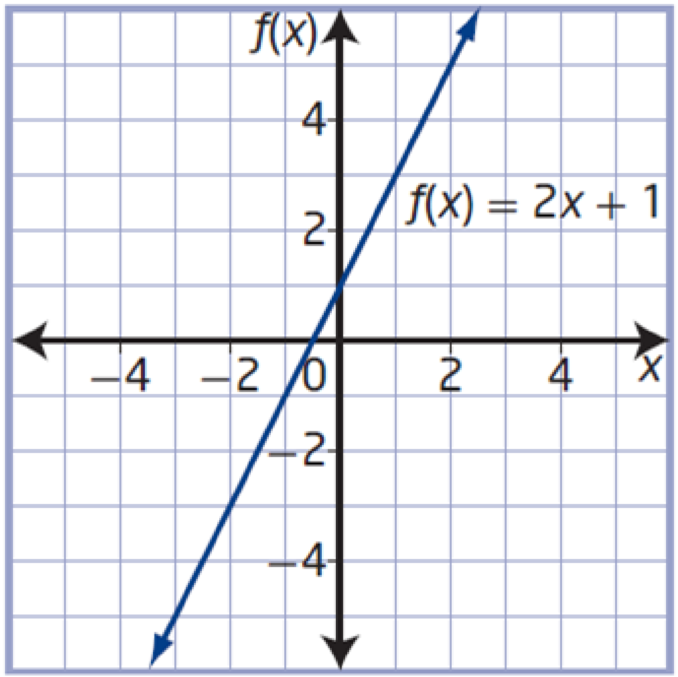
Range: $\{3\}$

Number of x-intercepts: 0

Degree 1: Linear Function

Odd degree

Number of x-intercepts: 1



Example: $f(x) = 2x + 1$

End behaviour: line extends down into quadrant III and up into quadrant I

Domain: $\{x \mid x \in R\}$

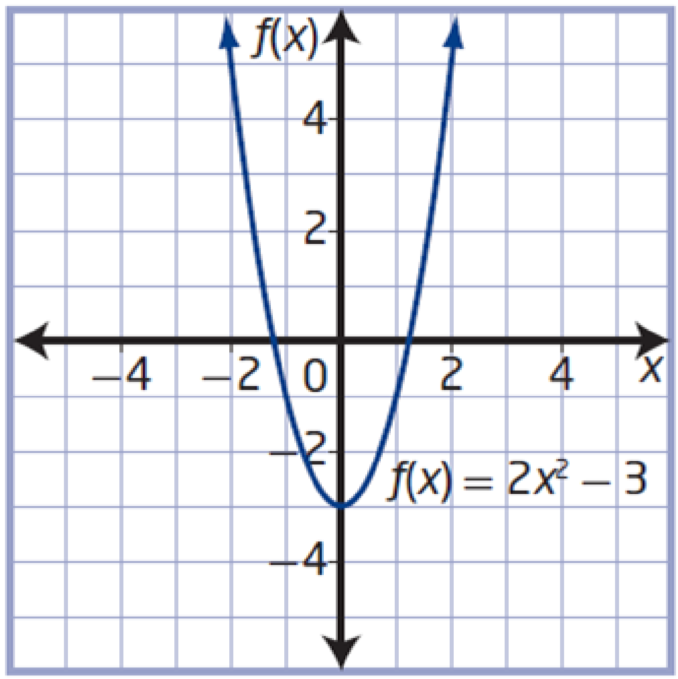
Range: $\{y \mid y \in R\}$

Number of x-intercepts: 1

Degree 2: Quadratic Function

Even degree

Number of x-intercepts: 0, 1, or 2



Example: $f(x) = 2x^2 - 3$

End behaviour: curve extends up into quadrant II and up into quadrant I

Domain: $\{x \mid x \in R\}$

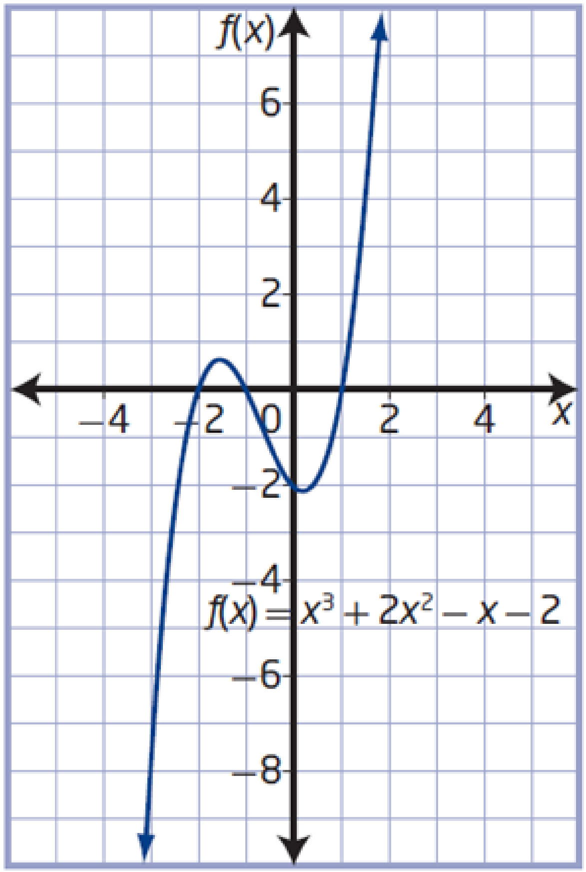
Range: $\{y \mid y \geq -2, y \in R\}$

Number of x-intercepts: 2

Degree 3: Cubic Function

Odd degree

Number of x-intercepts: 1, 2, or 3



Example:

$f(x) = x^3 + 2x^2 - x - 2$

End behaviour: curve extends down into quadrant III and up into quadrant I

Domain: $\{x \mid x \in \mathrm{R}\}$

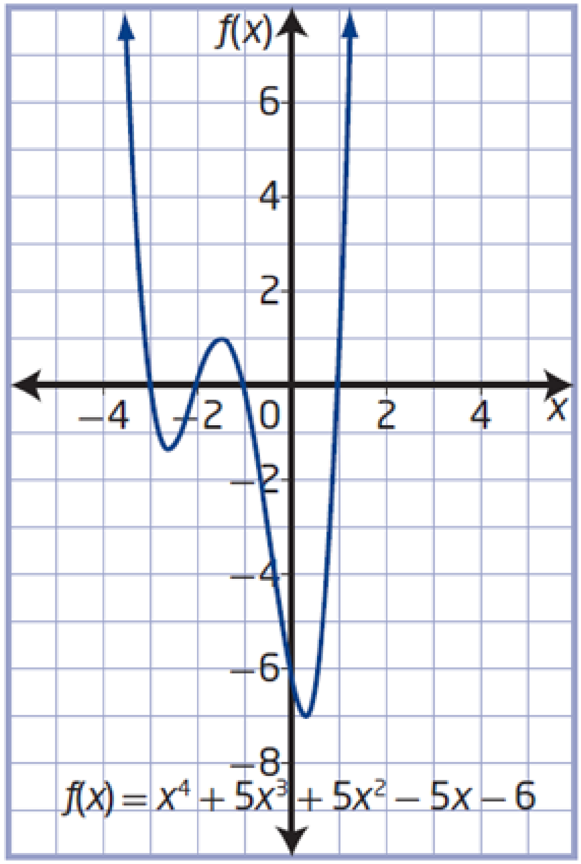
Range: $\{y \mid y \in \mathrm{R}\}$

Number of x-intercepts: 3

Degree 4: Quartic Function

Even degree

Number of *x*-intercepts: 0, 1, 2, 3, or 4



Example:

$f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$

End behaviour: curve extends up into quadrant II and up into quadrant I

Domain: $\{x \mid x \in R\}$

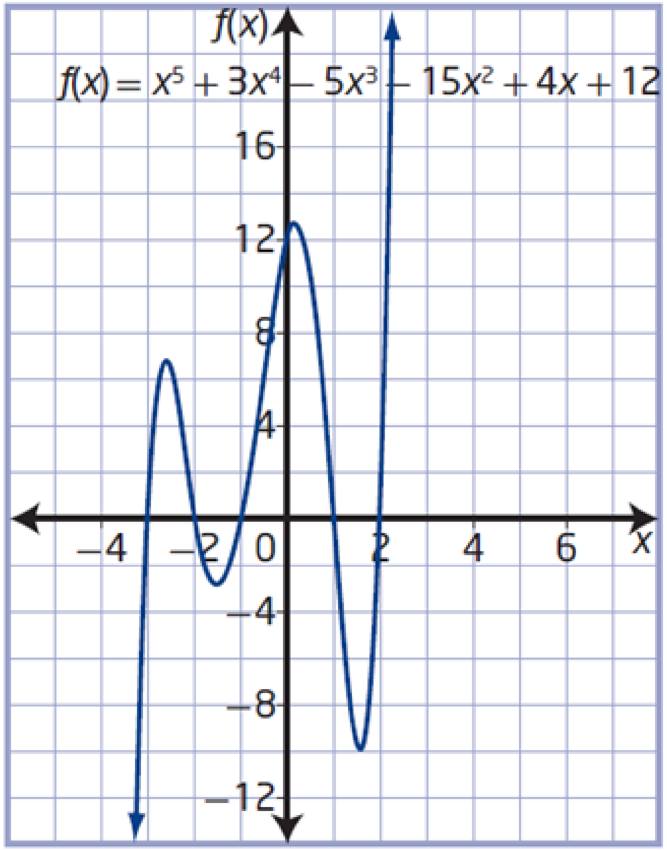
Range: $\{y \mid y \geq -6.91, y \in R\}$

Number of *x*-intercepts: 4

Degree 5: Quintic Function

Odd degree

Number of x-intercepts: 1, 2, 3, 4, or 5



Example:

$f(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

End behaviour: curve extends down into quadrant III and up into quadrant I

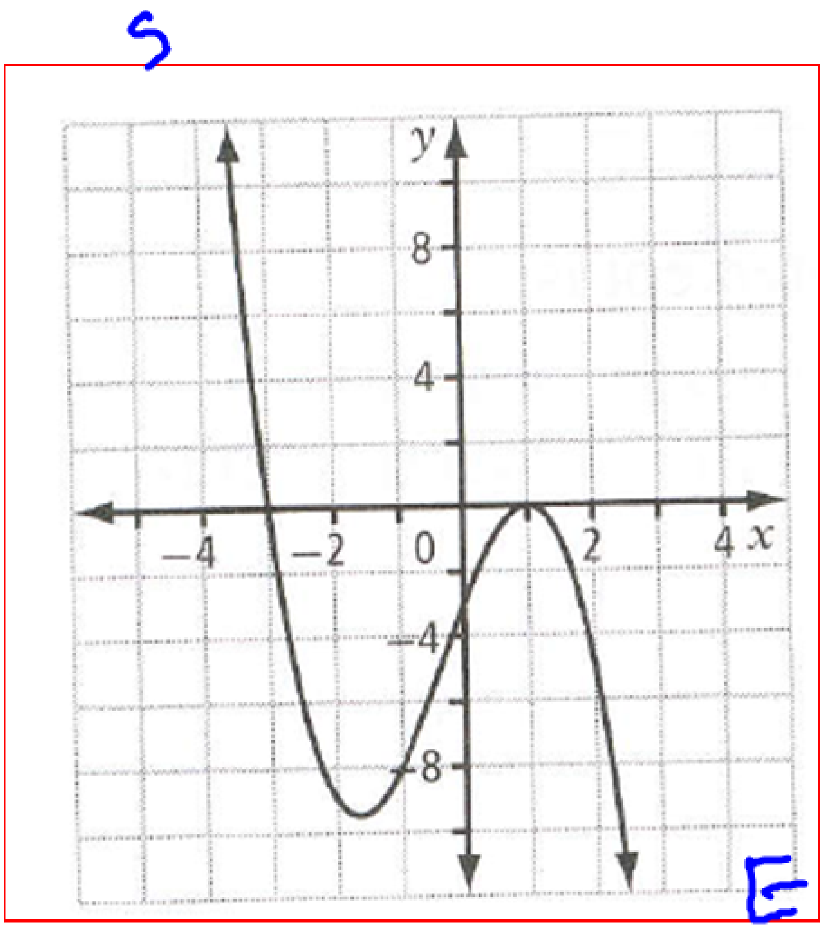
Domain: $\{x \mid x \in R\}$

Range: $\{y \mid y \in R\}$

Number of x-intercepts: 5

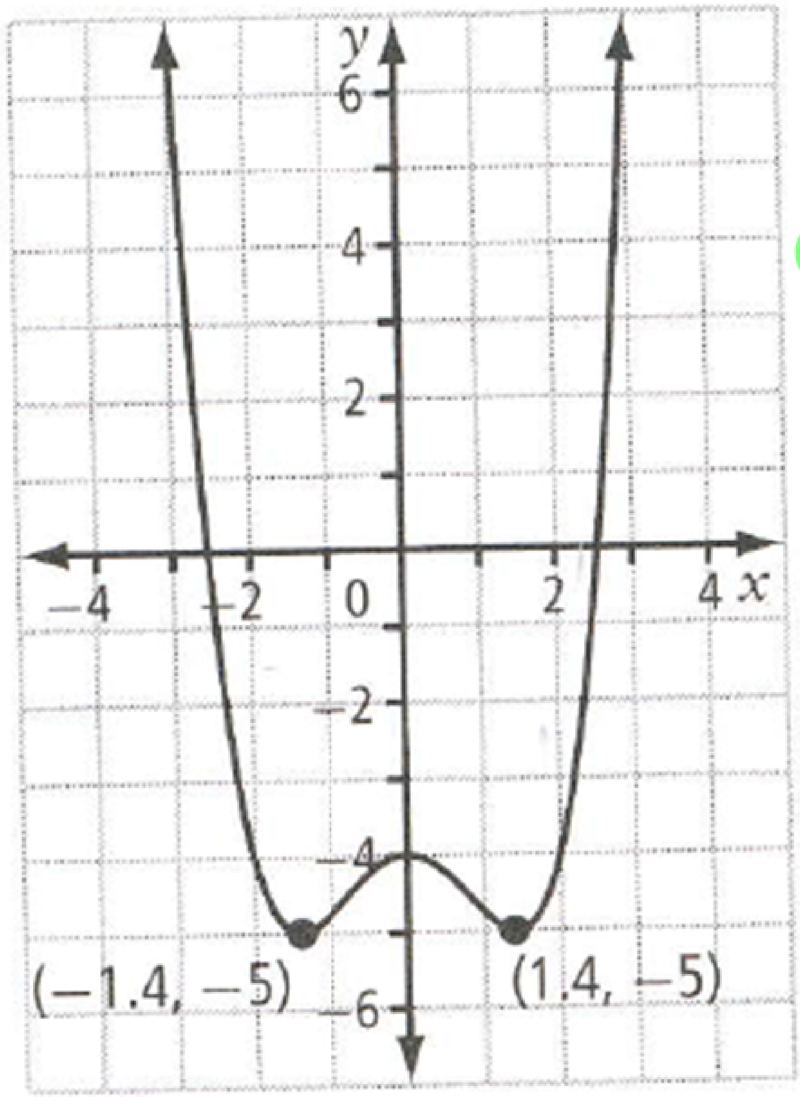
For the following graphs determine whether the function is odd or even degree
Leading coefficient is positive or negative
State the number of x intercepts
State the domain and Range

a)



ODD
LC −
2 x int
D: $x \in R$
R: $y \in R$

b)



Even

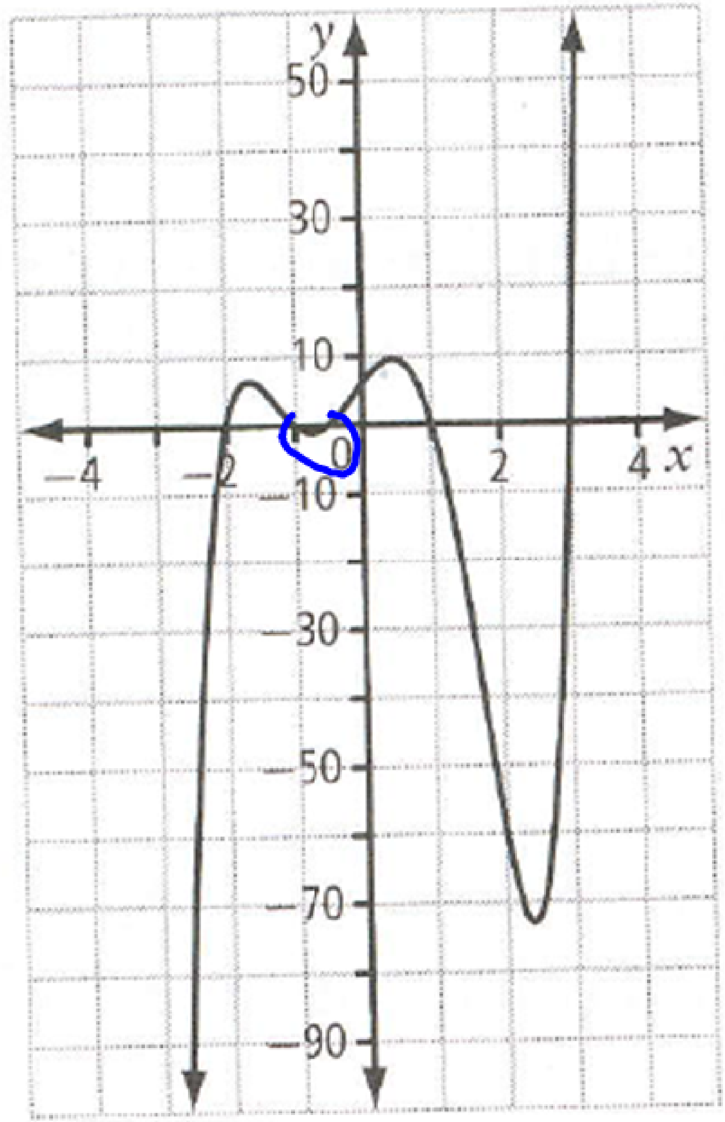
LC +

2 x int

D: $x \in R$

R: $y \geq -5$

c)



ODD

LC +

5 x int

D: x ∈ R

R: y ∈ R

For the following:
a) Identify the type of polynomial
b) Identify the degree
c) Identify the end behavior
d) Identify the maximum number of x intercepts
e) Identify whether the polynomial has a max or min
f) Identify the y intercept

$a)\ g(x) = -x^4 + 10x^2 + 5x - 4$

a) quartic
b) 4
c) S: 3 E: 4
d) 4
e) Max
f) y int
-4
(0, -4)

$b)\ h(x) = x^3 + x^2 - 5x + 3$

a) cubic

b) 3

c) S: 3 E: 1

d) 3

e) Neither

f) $\frac{y\text{ int}}{3}$

(0, 3)

$$c)\ h(x) = -2x^5 + 5x^3 - x$$

$d)\ h(x) = x^4 + 4x^3 - x^2 - 16x - 12$

Now lets match each polynomial to its graph.

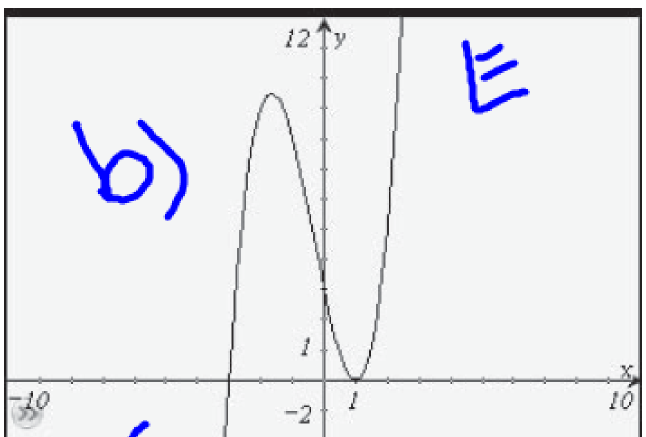
a) $g(x) = -x^4 + 10x^2 + 5x - 4$

b) $f(x) = x^3 + x^2 - 5x + 3$

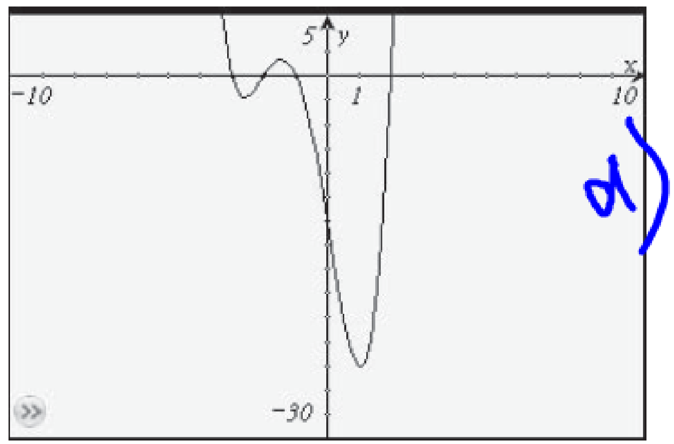
c) $p(x) = -2x^5 + 5x^3 - x$

d) $h(x) = x^4 + 4x^3 - x^2 - 16x - 12$

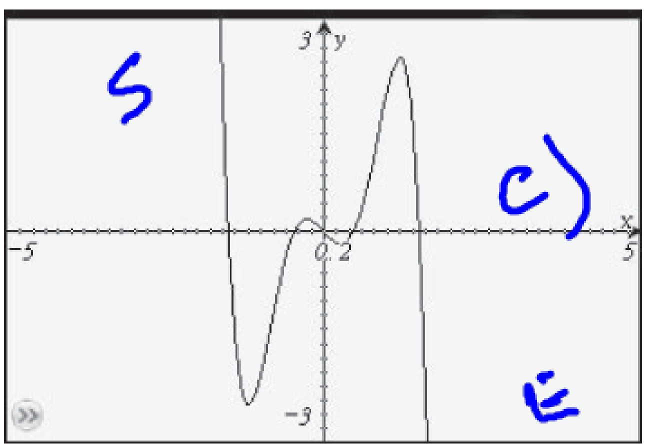
A



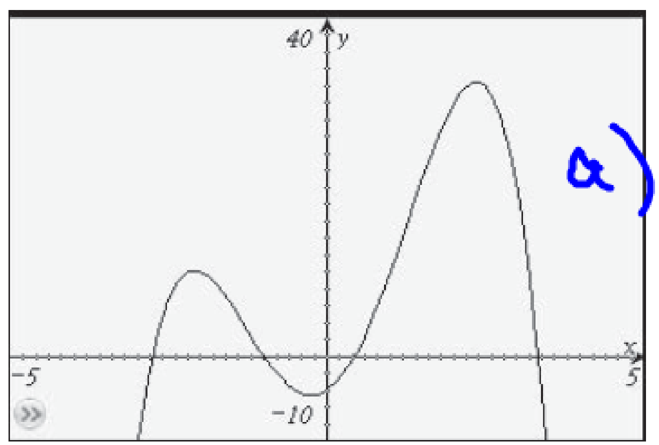
B



C



D



Application Example:
A skateboard manufacturer determines that its profit, P in dollars, can be modelled by the function $P(x)=1000x+1.25x^4-3200$ where x represents the number, in hundreds of skateboards sold.

$ = 34720

a) What is the degree of the function

4

b) What is the LC and the constant(y-intercept)

1.25 -3200

c) Describe the end behavior

S:2 E:1

d) What are the restrictions on the domain

$x \geq 0$

e) What do x intercepts mean

break even points

f) Profit from 1200 skateboards

$P(12) = 1000(12) + 1.25(12)^4 - 3200$

$= 12000 + 25920 - 3200$

Assignment

Page 114

1-5,7,9,10,c1,c2