2.8 Chapter Review

1. $f(x) = 2x^2 - 3x \qquad g(x) = \frac{4}{x-1}$

a) $f(5) = 2(5)^2 - 3(5)$
$= \boxed{35}$

b) $g(-3) = \frac{4}{-3-1} = \boxed{-1}$

c) $\frac{f(2+h) - f(2)}{h}$
$= \frac{(2(2+h)^2 - 3(2+h)) - (2(2)^2 - 3(2))}{h}$
$= \frac{2(4+4h+h^2) - 6 - 3h - 8 + 6}{h}$
$= \frac{8 + 8h + 2h^2 - 6 - 3h - 8 + 6}{h}$
$= \frac{2h^2 + 5h}{h} = \boxed{2h+5}$

d) $\frac{g(-1+h) - g(-1)}{h}$
$= \frac{\frac{4}{-1+h-1} - \frac{4}{-1-1}}{h}$
$= \frac{\frac{4}{h-2} + 2}{h}$
$= \frac{4 + 2(h-2)}{h-2} \cdot \frac{1}{h}$
$= \frac{4 + 2h - 4}{h(h-2)}$
$= \frac{2}{h-2}$

e) $(f+g)(3)$
$= \frac{(2x^2 - 3x)}{1} + \frac{4}{x-1}$
$= \frac{(2x^2 - 3x)(x-1) + 4}{x-1}$
$= \frac{2x^3 - 2x^2 - 3x^2 + 3x + 4}{x-1}$
$= \frac{2x^3 - 5x^2 + 3x + 4}{x-1}$
$= \frac{2(3)^3 - 5(3)^2 + 3(3) + 4}{3-1}$
$= \frac{54 - 45 + 9 + 4}{2}$
$= \frac{22}{2} = \boxed{11}$

f) $(f-g)(-1)$
$= (2x^2 - 3x) - \left(\frac{4}{x-1}\right)$
$= \frac{(2x^2 - 3x)(x-1) - 4}{x-1}$
$= \frac{2x^3 - 2x^2 - 3x^2 + 3x - 4}{x-1}$
$= \frac{2x^3 - 5x^2 + 3x - 4}{x-1}$
$= \frac{2(-1)^3 - 5(-1)^2 + 3(-1) - 4}{-1-1}$
$= \frac{-2 - 5 - 3 - 4}{-2}$
$= \frac{-14}{-2} = \boxed{7}$

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1. g) $(Fg)(0)$

$(2x^2-3x)\left(\frac{4}{x-1}\right)$

$= \frac{8x^2-12x}{x-1}$

$= \frac{8(0)^2-12(0)}{0-1}$

$= \boxed{0}$

h) $\left(\frac{f}{g}\right)(1)$

$= (2x^2-3x) \div \left(\frac{4}{x-1}\right)$ undefined at $x=1$

i) $(Fog)(2)$

$= 2\left(\frac{4}{x-1}\right)^2 - 3\left(\frac{4}{x-1}\right)$

$= 2\left(\frac{4}{2-1}\right)^2 - 3\left(\frac{4}{2-1}\right)$

$= 2(16) - 3(4)$

$= \boxed{20}$

j) $(g \circ f)(1)$

$= \frac{4}{2x^2-3x-1}$

$= \frac{4}{2(1)^2-3(1)-1}$

$= \frac{4}{-2} = \boxed{-2}$

k) $(f \circ f)(3)$

$= 2(2x^2-3x)^2 - 3(2x^2-3x)$

$= 2(2(3)^2-3(3))^2 - 3(2(3)^2-3(3))$

$= 2(9)^2 - 3(9)$

$= \boxed{135}$

l) $(g \circ g)\left(\frac{1}{2}\right)$

$= \frac{4}{\left(\frac{4}{x-1}\right)-1}$

$= \frac{4}{\frac{4}{\left(\frac{1}{2}-1\right)}-1}$

$= \boxed{\frac{-4}{9}}$

$\frac{4}{\frac{-1}{2}} = 4\left(\frac{-2}{1}\right)$

m) $(Fog)(x)$

$= 2\left(\frac{4}{x-1}\right)^2 - 3\left(\frac{4}{x-1}\right)$

$= \frac{32}{(x-1)^2} - \frac{12(x-1)}{(x-1)^2}$

$= \frac{32-12x+12}{(x-1)^2}$

$= \boxed{\frac{-12x+44}{(x-1)^2}}$

n) $(g \circ f)(x)$

$= \boxed{\frac{4}{2x^2-3x-1}}$

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1. o) $g \circ g(x)$

$= \dfrac{4}{\frac{4}{x-1} - 1}$

$= \dfrac{4}{\frac{4-(x-1)}{x-1}}$

$= \dfrac{4(x-1)}{5-x}$

$= \boxed{\dfrac{-4x+4}{x-5}}$

p) $fog(2x+1)$

form m) $\rightarrow = \dfrac{-12x+44}{(x-1)^2}$

$= \dfrac{-12(2x+1)+44}{(2x+1-1)^2}$

$= \dfrac{-24x-12+44}{4x^2}$

$= \dfrac{-24x+32}{4x^2}$

$= \boxed{\dfrac{-6x+8}{x^2}}$

2. a) $f(3) \rightarrow -2$
b) $g(-1) \rightarrow -6$
c) $f(g(3)) \rightarrow -3$
d) $g(F(5)) \rightarrow -1$
e) $(g-f)(3) \rightarrow 4$
f) $(fg)(5) \rightarrow 0$

$\Rightarrow$ g) $(F \circ F)(3) \rightarrow 1$

$\Rightarrow$ h) $(g \circ g)(1) \rightarrow 3$

i) values where $f(x) = g(x) \rightarrow x \in \{0, 4\}$

j) values where $g(x) > f(x)$ $x \in (0, 4)$

k) open interval where $g(x)$ is increasing $(-\infty,$

l) open interval where $f(x)$ is decreasing $(-\infty,$

3. $f(x) = x^{1/3}$, $g(x) = x^{1/5}$, $h(x) = x^{1/7}$: for $|x| > 1$, $f(x) > g(x) > h(x$
for $|x| < 1$, $f(x) > g(x) > h(x)$

4. $y = \log_{\frac{1}{2}} x \rightarrow$ decreasing function for $|x| = 1$, $f(x) = g(x) = h(x$

5. a) $f(x) = x^3 - 5x^2$ polynomial
b) $f(x) = \cos x$ trigonometric
c) $F(x) = x$ identity
d) $f(x) = x^{1/4}$ root with doma
e) $F(x) = x^{12}$ power
f) $F(x) = \log_4 x$ logarithmic
g) $f(x) = 5^x$ increasing exponential
h) $f(x) = x^{1/5}$ $(0, \infty)$?alwa
root with doma $(-\infty, \infty)$
i) $f(x) = 11$ constant
j) $F(x) = \frac{1}{x}$ reciprocal
k) $f(x) = \left(\frac{1}{3}\right)^x$ decreasing exponential
l) $f(x) = |x|$ absolute value

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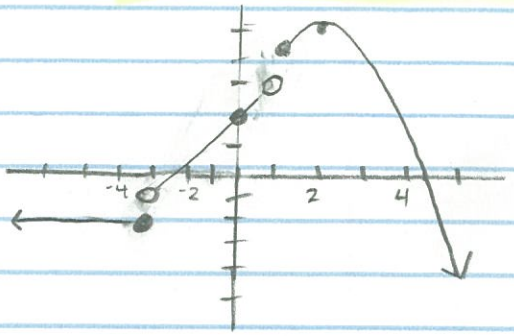
6. $g(x) = \begin{cases} -2, & x \in (-\infty, -3] \\ x+2, & x \in (-3, 1) \\ -x^2+4x+1, & x \in [1, \infty) \end{cases}$

$-(x^2-4x+4-4)+1$

$-(x-2)^2+5 \quad \vee(2,5)$



7. $f(x) = \begin{cases} 2^x, & x \in (-2, 2] \\ -x+5, & x \in (2, \infty) \end{cases}$

8. a) $f(x) = 2x^3 - 3x$

$f(-x) = 2(-x)^3 - 3(-x)$

$= -2x^3 + 3x$

$= -(2x^3 - 3x)$ odd

b) $f(x) = -4x^2 - 6$

$f(-x) = -4(-x)^2 - 6$

$= -4x^2 - 6$

even

c) $f(x) = \dfrac{2x^2+1}{x^3-x}$

$f(-x) = \dfrac{2(-x)^2+1}{(-x)^3-(-x)}$

$= \dfrac{2x^2+1}{-x^3+x}$

$= -1\left(\dfrac{2x^2+1}{x^3-x}\right)$ odd

d) $f(x) = \sin x$

$f(-x) = \sin(-x)$

$= -\sin x$

odd

e) $f(x) = \cos x$

$f(-x) = \cos(-x)$

$= \cos x$

even

f) $f(x) = \sin^2 x$

$f(-x) = (\sin(-x))^2$

$= (-\sin x)^2$

$= \sin^2 x$

even

g) $f(x) = \sin(x^2)$

$f(-x) = \sin(-x)^2$

$= \sin x^2$

even

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8 h) $f(x) = \sin x + \cos x$

$f(-x) = \sin(-x) + \cos(-x)$

$= -\sin x + \cos x$

neither

i) $f(x) = \sin x \cos x$

$f(-x) = \sin(-x)\cos(-x)$

$= (-\sin x)(\cos x)$

$= -(\sin x \cos x)$

odd

odd function → symmetric at origin

j) $f(x) = \tan(x^2)$

$f(-x) = \tan(-x)^2$

$= \tan x^2$

even

k) $f(x) = \sqrt{x^2+y}$

$f(-x) = \sqrt{(-x)^2+y}$

$= \sqrt{x^2+y}$

even

even function → symmetric about y-axis

l) $f(x) = \sqrt{x^2+x}$

$f(-x) = \sqrt{(-x)^2+(-x)}$

$= \sqrt{x^2-x}$

neither

9. Symmetric about y-axis? (Even function)

8befgjk

10. Symmetric about origin? (odd function)

8acdi

11. One to one → all y-values are different $y = x$

Many to one → repeated y-values $y = x^2$

12 a) $F(x+1)$ → vii

b) $F(x)+1$ → vi

c) $F(x-1)$ → v

d) $F(x)-1$ → i

e) $-f(x)$ → xi

f) $f(-x)$ → ix

g) $f(2x)$ → viii

h) $2F(x)$ → iii

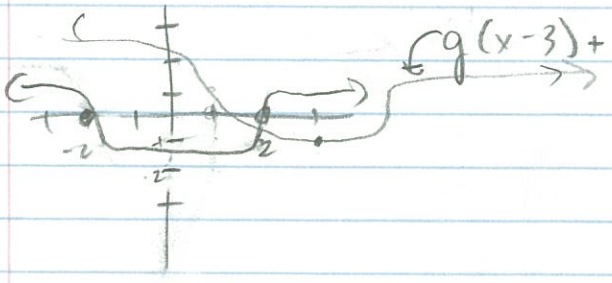
i) $f(\frac{x}{2})$ → x

j) $\frac{f(x)}{2}$ → iv

k) $f(x+1)-1$ → ii

l) $-f(x-1)-1$ → xii

13



b) g(x) is even because symmetric about y-axis

c) $D = \{x : x \in (-\infty, \infty)$

$R = \{y : y \in (-1.3, \infty)$

approx

Hilroy

14. a) $f(x) = 19x^5 + \sqrt{3}x^4 - \pi x^3 + 5.1$

DOMAIN $D = \{x : x \in (-\infty, \infty)\}$

b) $f(x) = \dfrac{2x-5}{2x+5}$ $D = \{x : x \in (-\infty, -\frac{5}{2}) \cup (-\frac{5}{2}, \infty)$

c) $f(x) = \dfrac{\cos x}{x^3 - 25x} = \dfrac{\cos x}{x(x^2-25)} = \dfrac{\cos x}{x(x+5)(x-5)}$

$\{x : x \in (-\infty, \infty),\ x \neq \pm 5, 0$

d) $f(x) = \sqrt{x^2 - 2x - 15}$

$f(x) = \sqrt{(x+3)(x-5)}$ $\{x : x \in (-\infty, -3] \cup [5, \infty)\}$

e) $f(x) = \dfrac{3}{\sqrt{6 - 5x - x^2}}$

$= \dfrac{3}{\sqrt{(6+x)(1-x)}}$ $\{x : x \in (-6, 1)\}$

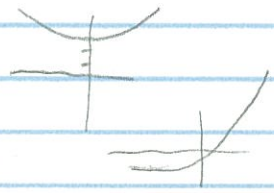
f) $f(x) = \dfrac{12}{\sqrt{25x^4 + 9x^2 + 4}}$ $\{x : x \in (-\infty, \infty)\}$

g) $f(x) = \csc(x-1)$ $\{x : x \in (-\infty, \infty)\ x \neq k\pi + 1$ where $k \in I$

h) $f(x) = \sqrt{-|x-3|}$ $\{x : x = 3\}$

15. a) $f(x) = \sqrt{x^2 + 9}$ $\{y : y \in [3, \infty)\}$

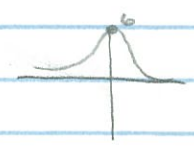
RANGE



b) $f(x) = 2^x - 5$ $\{y : y \in (-5, \infty)\}$

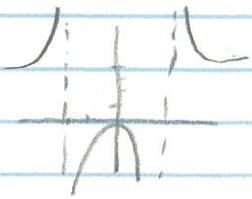
c) $f(x) = \log_3(x^2 + 9)$ $\{y : y \in [2, \infty)\}$

d) $f(x) = \dfrac{12}{\sqrt{25x^4 + 9x^4 + 4}}$ $\{y : y \in (0, 6]\}$



e) $f(x) = \sin^2 x + \cos^2 x$ $\{y : y = 1\}$

f) $f(x) = \dfrac{4x^2}{x^2 - 4}$ $\{y : y (-\infty, 0] \cup (4, \infty)\}$



16. $A = 100\ m^2$ x = width y = length

a) $P = 2x + 2y$ $\quad P = 2x + 2\left(\frac{100}{x}\right)$

$A = xy$

$100 = xy$

$\frac{100}{x} = y$ $\quad P = 2x + \frac{200}{x} = \frac{2x^2 + 200}{x}$

b) $D: \{x : x \in (0, 10]\}$ $\quad$ c) $R = \{y : y \in [40, \infty)\}$ ← work on.

17. $(x-3)^2 + (y+2)^2 = 25$

$x^2 - 6x + 9 + (y+2)^2 = 25$

$(y+2)^2 = 16 + 6x - x^2$

$y + 2 = \sqrt{16 + 6x - x^2}$

$y = -\sqrt{16 + 6x - x^2} - 2$ ← bottom half of circle

18. $f(x) = \frac{x^5 - 32}{\sqrt{2x+3}}$ (B) $(-\frac{3}{2}, \infty)$ ← domain

19. $f(x) = \frac{1}{\log_2(\log_2 x)}$ (E) $(1,2) \cup (2, \infty)$

$\log_2 1 = 0$

$\log_2 2 = 1$

$\log_2 0 =$ und.

$\log_2 3 = 1.58$

20. $f(x) = \log_2(1 - \log_2 x)$ (D) $(0, 2)$

21. $f(x) = \frac{1}{x^2 + 4x + 5} = \frac{1}{(x+2)^2 + 1}$ (H) $(0, 1]$ ← range

22. $f(x) = x^2 + 1$ $\quad g(x) = \sqrt{x} - 4$ $\quad (fog)x = (\sqrt{x} - 4)^2 + 1$ (B) $[0, \infty)$

$= x - 8\sqrt{x} + 16 + 1$

$= x - 8\sqrt{x} + 17$ ← domain?

23. $f(x) = \frac{1}{x^4 - 4}$ $\quad g(x) = \sqrt{x+3}$ $\quad (fog)x = \frac{1}{(\sqrt{x+3})^4 - 4} = \frac{1}{(x+3)^2 - 4}$ (A) $[-3, -1) \cup (-1, \infty$

24. $y = \sqrt{x}$ 5 units right + 2 units up. (G) $y = \sqrt{x-5} + 2$

25. $y = 2^x$ reflecting y-axis, 3 units right, 2 up (D) $y = 2^{-x+3} + 2$

Hilbert