2012 AP Mutliple Choice Exam
No Calculator Allowed

1. If $y = x\sin x$, then $\frac{dy}{dx} =$

(A) $\sin x + \cos x$

(B) $\sin x + x\cos x$

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

3. $\int \sec x \tan x \, dx =$

(A) $\sec x + C$

(B) $\tan x + C$

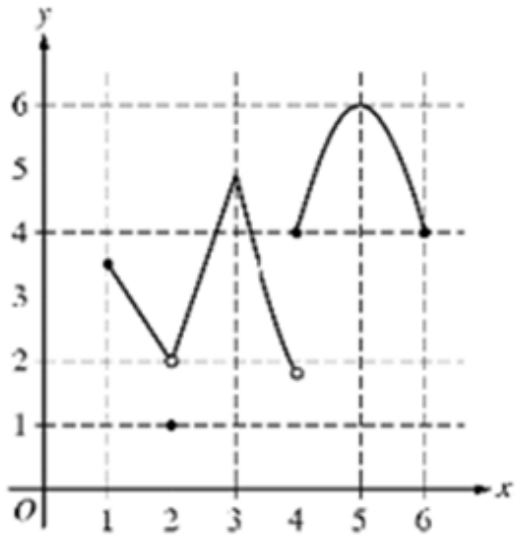
(C) $\frac{\sec^2 x}{2} + C$

(D) $\frac{\tan^2 x}{2} + C$

(E) $\frac{\sec^2 x \tan^2 x}{2} + C$

4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8



Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists.

(B) $\lim_{x \to 3} f(x)$ exists.

(C) $\lim_{x \to 4} f(x)$ exists.

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function f is continuous at $x = 3$.

A B C D E

6. A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

(A) 3 (B) 6 (C) 9 (D) 18 (E) 27

7. If $y = \left(x^3 - \cos x\right)^5$, then $y' =$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

$$f(x) = \begin{cases} \dfrac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e \quad 1}{2}$ (E) $e - 1$

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

12. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

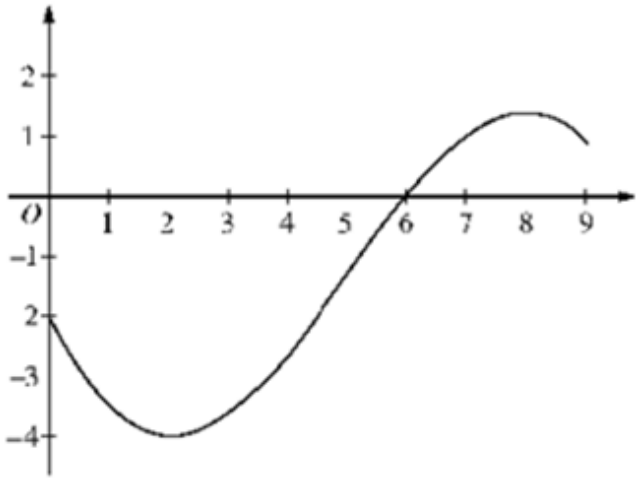
(A) $2\int_1^{16} e^u\,du$ (B) $2\int_1^4 e^u\,du$ (C) $2\int_1^2 e^u\,du$ (D) $\frac{1}{2}\int_1^2 e^u\,du$ (E) $\int_1^4 e^u\,du$

13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\,dx$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12

14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

16. A particle moves along the x-axis with its position at time t given by $x(t) = (t - a)(t - b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?

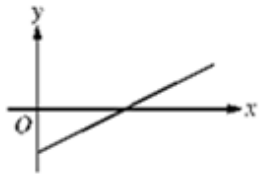
(A) $t = ab$

(B) $t = \dfrac{a + b}{2}$

(C) $t = a + b$

(D) $t = 2(a + b)$

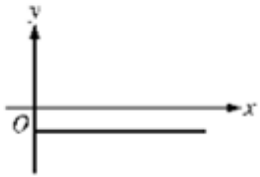
(E) $t = a$ and $t = b$

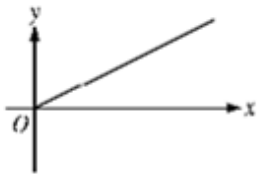


Graph of f

17. The figure above shows the graph of f. If $f(x) = \int_2^x g(t)\,dt$, which of the following could be the graph of $y = g(x)$?

(A)


(B)


(C)


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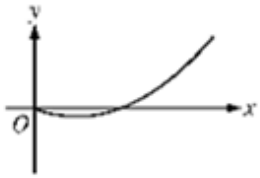
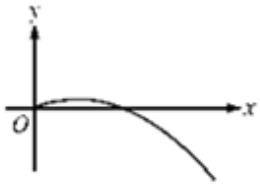


(E)


18. $\lim_{h \to 0} \dfrac{\ln(4+h) - \ln(4)}{h}$ is

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

20. Let $f(x) = (2x + 1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \dfrac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \dfrac{1}{x \quad 5}$ (D) $y = \dfrac{5x}{1 \quad x}$ (E) $y = \dfrac{20x^2 - x}{1 + 4x^2}$

22. Let f be the function defined by $f(x) = \frac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\frac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

24. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

(A) -3 (B) $-\frac{3}{2}$ (C) $-\frac{1}{3}$ (D) 0 (E) There is no such k.

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$

26. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\,dt$. Which of the following must be true on the interval $0 < x < 2$?

(A) g is increasing, and the graph of g is concave up.

(B) g is increasing, and the graph of g is concave down.

(C) g is decreasing, and the graph of g is concave up.

(D) g is decreasing, and the graph of g is concave down.

(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

27. If $(x + 2y) \cdot \frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$