2.7 Function Operations

Arithmetic Operations

Function Addition	$(f+g)(x)=f(x)+g(x)$ with domain $P\cap Q$.
Function Subtraction	$(f-g)(x)=f(x)-g(x)$ with domain $P\cap Q$.
Function Multiplication	$(fg)(x)=f(x)\cdot g(x)$ with domain $P\cap Q$.
Function Division	$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$ with domain $P\cap Q$ excluding values of x for which $g(x)=0$.

a) $f(x) + g(x)$

$$= \frac{(x-2)(x+2)}{(x-2)\,3} + \frac{3}{x-2}\cdot\frac{3}{3}$$

Your Turn #1

If $f(x) = \dfrac{x+2}{3}$ and $g(x) = \dfrac{3}{x-2}$, find each of the following:

(a) $(f+g)(x)$

(b) $(f-g)(x)$

(c) $(fg)(x)$

(d) $\left(\dfrac{f}{g}\right)(x)$

(e) the domain of $(fg)(x)$.

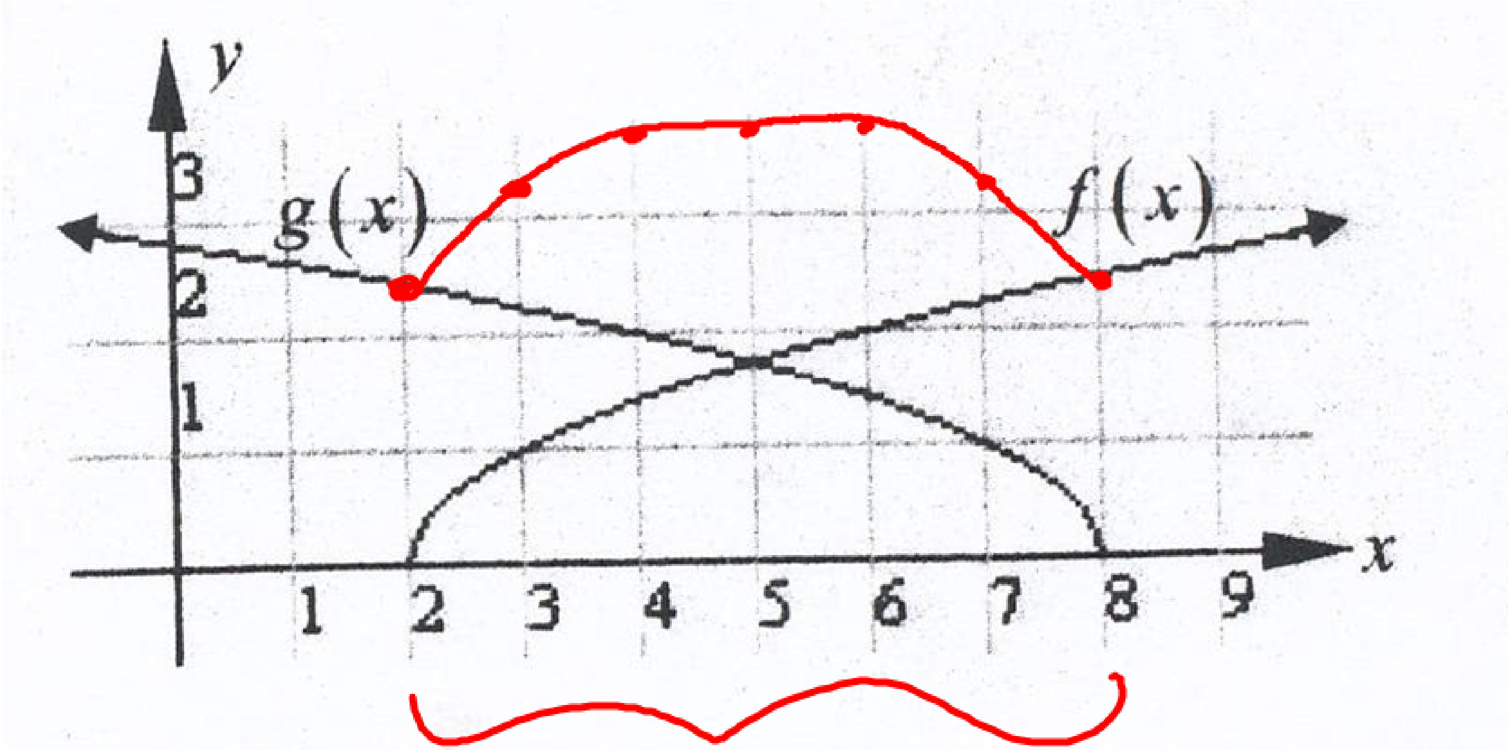
$$= \frac{x^2 - 4 + 9}{3(x-2)}$$

$$= \frac{x^2+5}{3(x-2)}$$

c) $f(x)\cdot g(x)$

$$= \left(\frac{x+2}{\cancel{3}}\right)\left(\frac{\cancel{3}}{x-2}\right) = \frac{x+2}{x-2} \qquad x \neq 2$$

Ex.1 Find (f+g)(x) or f(x)+g(x)



D: [2, 8]

Function Composition

Another way in which functions can be combined is known as **function composition**. This is when we develop a new function by creating a function within a function.

$$f(g(x)) \text{ or } g(f(x))$$

$$(f \circ g)(x) \text{ or } (g \circ f)(x)$$

Your Turn #3

If $f(x) = x^2 - 1$ and

$g(x) = \dfrac{4}{x-3}$, find:

(a) $(f \circ g)(1)$

(b) $(g \circ f)(0)$

(c) $(f \circ f)(-2)$

(d) $(g \circ g)(5)$

(e) $(f \circ g)(x)$

(f) $(g \circ f)(x)$

(g) $(f \circ f)(x)$

(h) $(g \circ g)(x)$

a) $\underline{f(g(1)) = 3}$

$g(1) = \dfrac{4}{1-3} = -2$

$f(-2) = (-2)^2 - 1$
$= 3$

b) $g(f(0))$

$g(-1) = -1$

e) $f(g(x))$

$$f\left(\frac{4}{x-3}\right) = \left(\frac{4}{x-3}\right)^2 - 1$$

$$= \frac{16}{(x-3)^2} - 1$$

$$= \frac{16}{x^2-6x+9} - 1\frac{(x^2-6x+9)}{(x^2-6x+9)}$$

$$= \frac{16-x^2+6x-9}{x^2-6x+9} = \frac{-x^2+6x+7}{(x-3)^2}$$

a) $1 + (-3) = -2$

c) $f(1)\,g(1)$

Ex.2 Find the following:

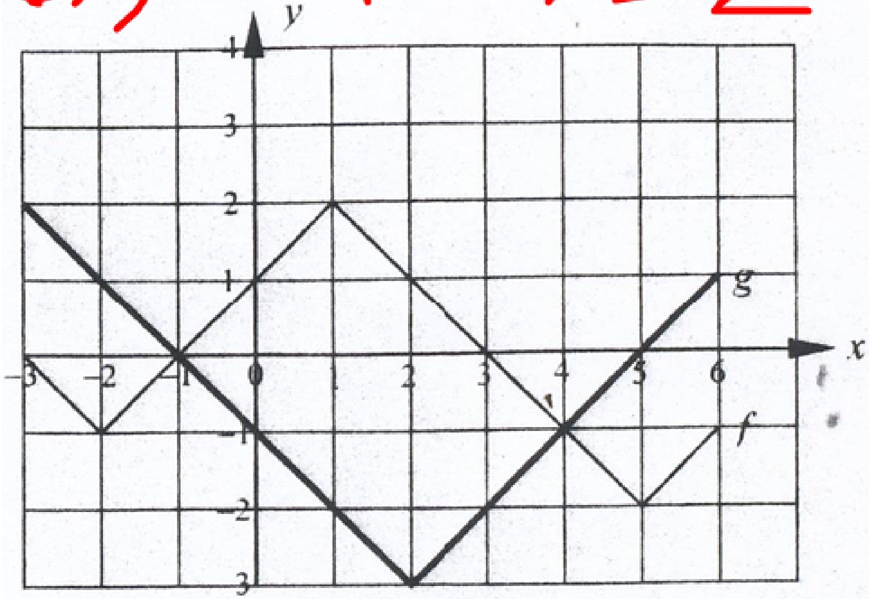
$f(2) + g(2)$

$(2)(-2) = -4$

(a) $(f+g)(2)$ (b) $(f-g)(5)$ (c) $(fg)(1)$ (d) $\left(\dfrac{f}{g}\right)(-2)$

(e) $(f \circ g)(6)$ (f) $(g \circ f)(-1)$ (g) $(f \circ f)(-2)$ (h) $(g \circ g)(1)$

e) $f(g(6)) = f(1) = 2$



x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

Domains of Composite Functions

$$f(g(x)) \text{ or } g(f(x))$$
$$(f \circ g)(x) \text{ or } (g \circ f)(x)$$

Ex Given that: $f(x)=x^2$ $= (-\infty,\infty)$

$g(x)=\sqrt{x-4}$ $= [4,\infty)$

find the domain of

a) $f(g(x))$

$f(\sqrt{x-4})$

$= \left(\sqrt{x-4}\right)^2$

$f(x) = x-4$

D: $[4,\infty)$

b) $g(f(x))$

$g(x^2) = \sqrt{x^2-4}$

$x^2-4 \geq 0$

$(x-2)(x+2) \geq 0$

$(-\infty,-2] \cup [2,\infty)$

Function Decomposition

The ability to compose and decompose functions is essential in order to fully understand that calculus that is to follow!

Your Turn #5

Find functions $f(x)$ and $g(x)$ so that $h(x) = (f \circ g)(x)$.

(a) $h(x) = \left(2x^2 + 3x\right)^4$

(b) $h(x) = \sqrt[5]{\sin 3x}$

(c) $h(x) = \dfrac{4}{2x+7}$

(d) $h(x) = 2^{\tan 3x}$

(e) $h(x) = \log_7\left(x^3 - 8\right)$

(f) $h(x) = \sin\left(3\cos x^3\right)$

a) $h(x) = \left(2x^2 + 3x\right)^4$

$g(x) = 2x^2 + 3x$

$f(x) = x^4$

b) $h(x) = \sqrt[5]{\sin 3x}$

$g(x) = \sin 3x$

$f(x) = \sqrt[5]{x}$

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Written Exercises

1. Ask your teacher for four copies of the grid below. On these grids draw the graphs of each of the following:

   (a) $(f+g)(x)$ (b) $(f-g)(x)$ (c) $(fg)(x)$ (d) $\left(\dfrac{f}{g}\right)(x)$