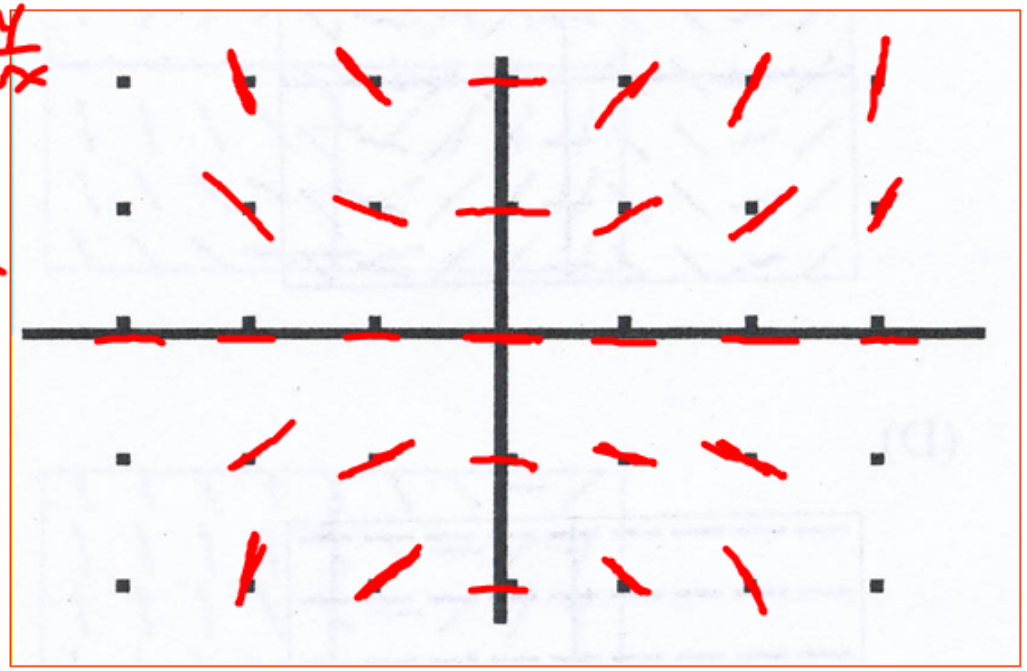
2.3 Slope Fields

Slope Fields

The idea of slope fields is to make a graphical representation of the slope of a function at various points in the plane.

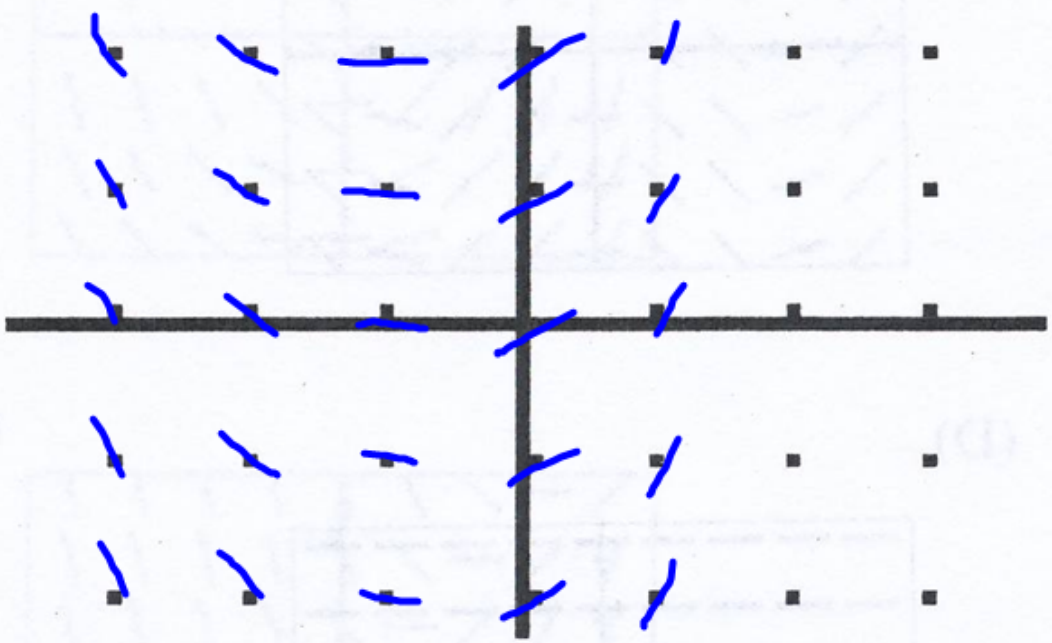
Example 1: Given $\frac{dy}{dx} = xy$, sketch the slope field of the function.

x	y	dy/dx
-1	-1	1
-1	-2	2
-2	-1	2
-2	-2	4
1	-1	-1
2	-1	-2
1	-2	-2
-2	-2	4

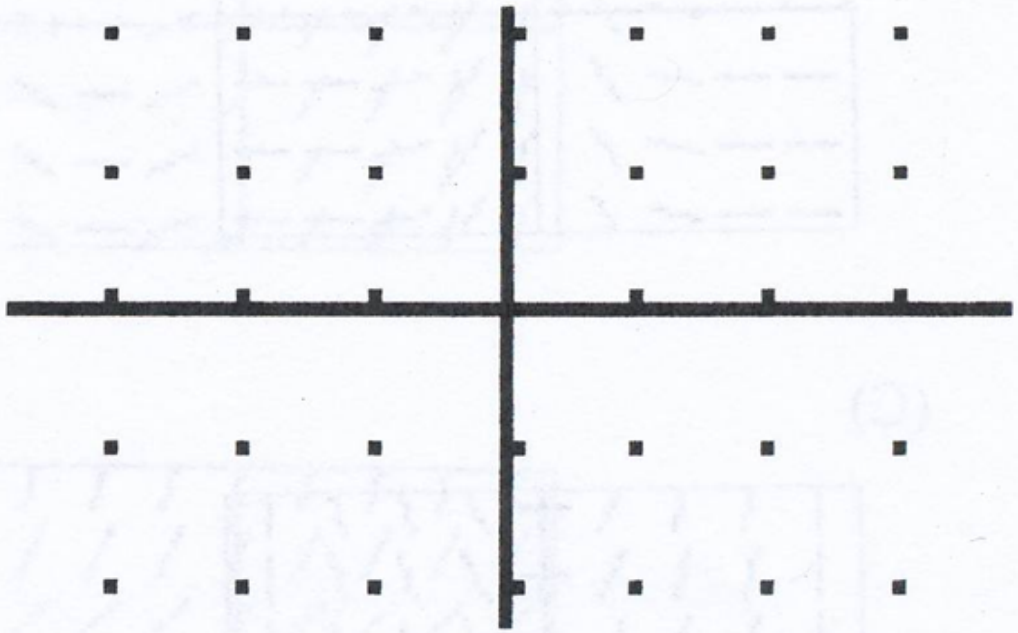


x	y	dy/dx
0	0	0
1	1	1
1	2	2
2	1	2
2	2	4
3	1	3
3	2	6
-1	1	-1
-1	2	-2
-2	2	-4
-2	3	-6

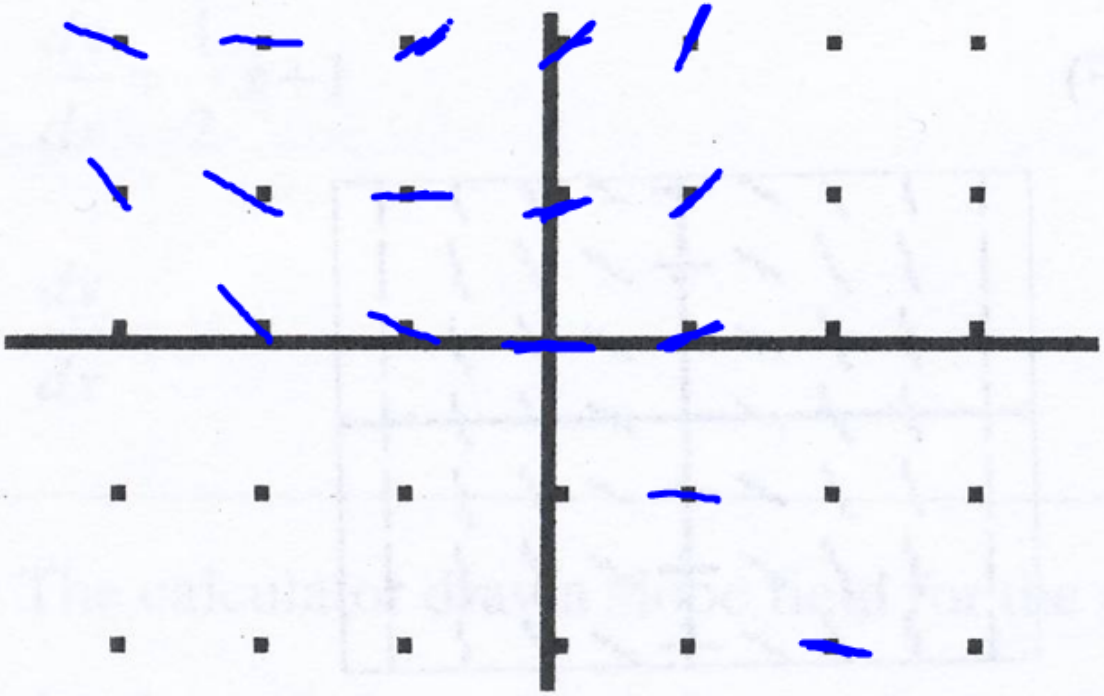
1. $\frac{dy}{dx} = x+1$



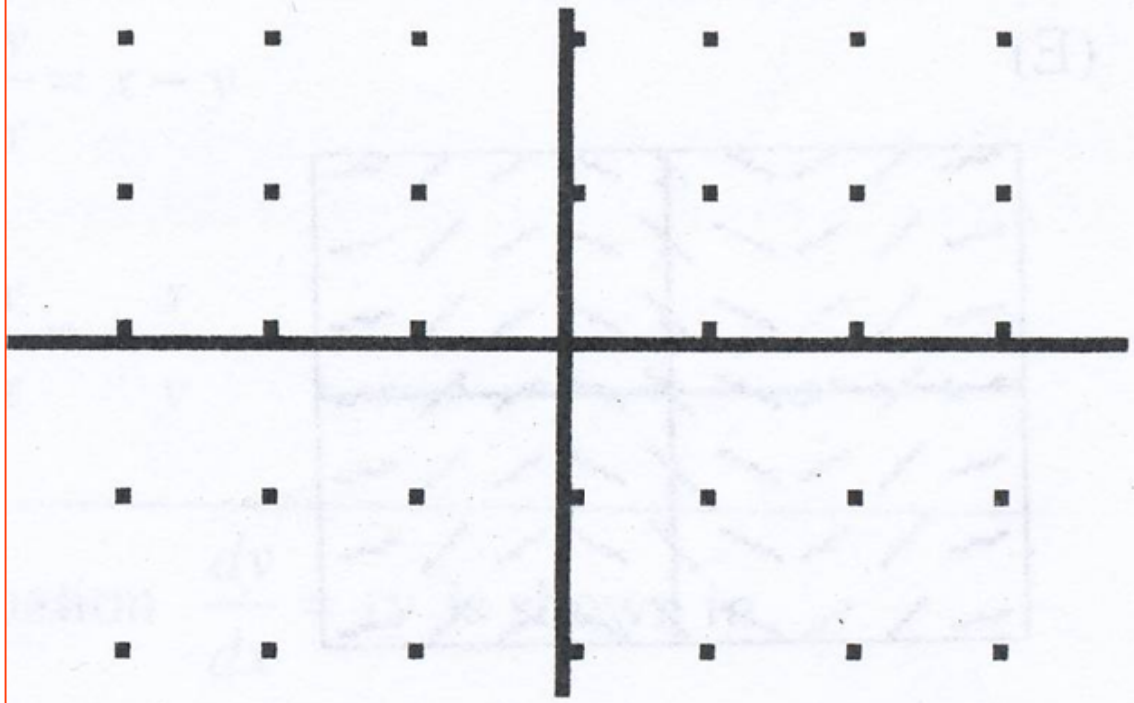
2. $\frac{dy}{dx} = 2y$



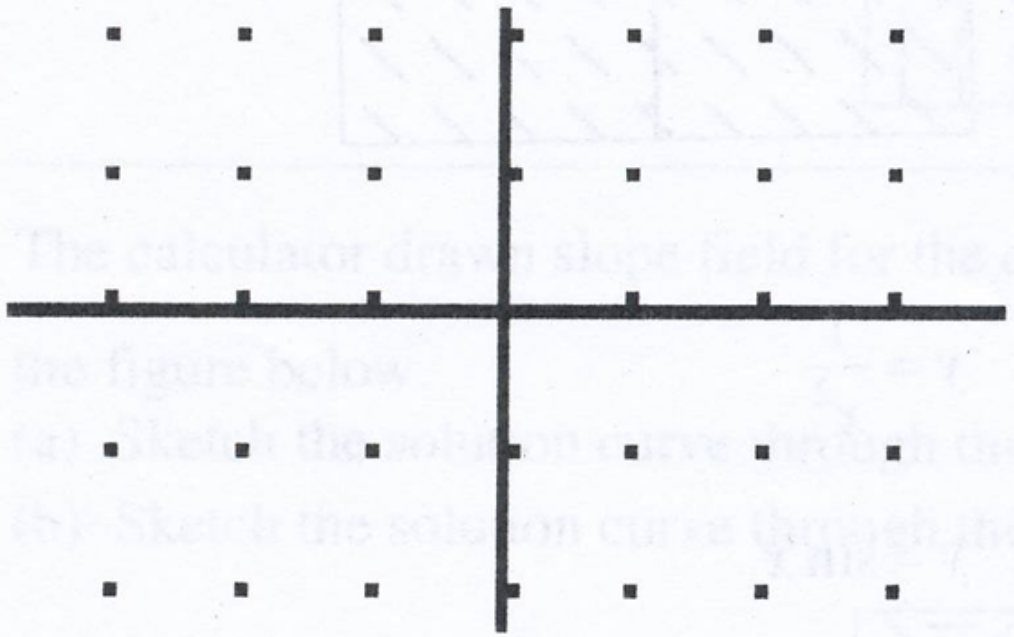
3. $\dfrac{dy}{dx} = x + y$



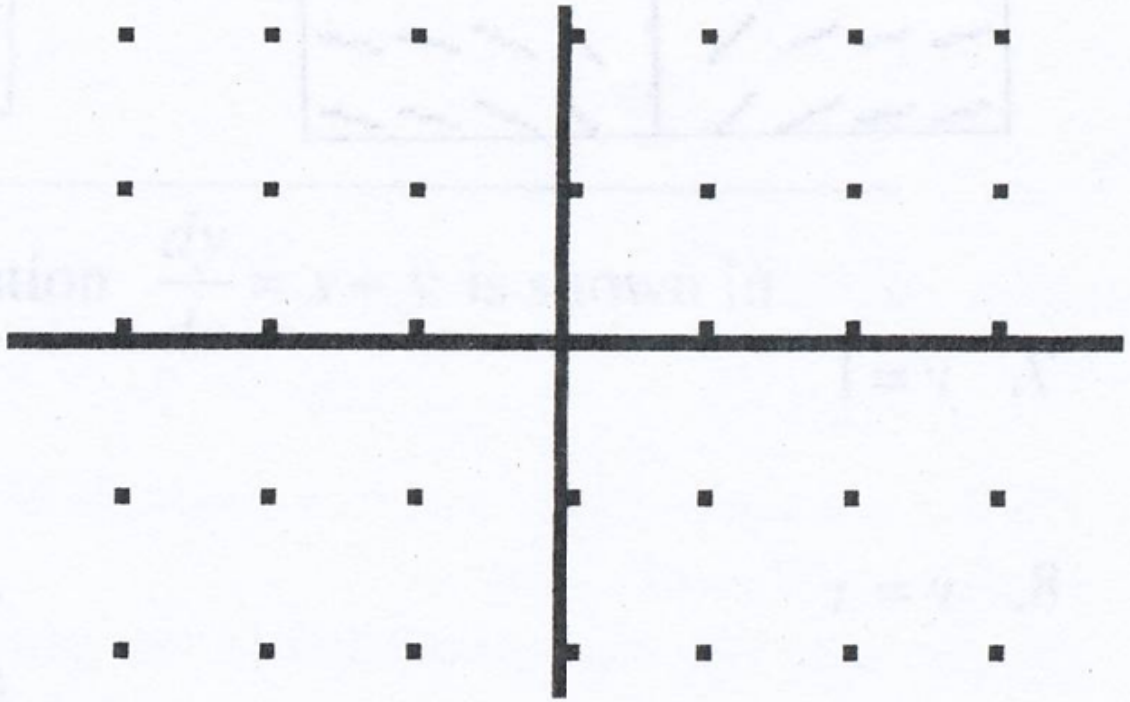
4. $\frac{dy}{dx} = 2x$



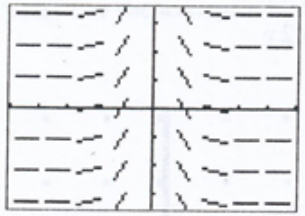
5. $\frac{dy}{dx} = y - 1$



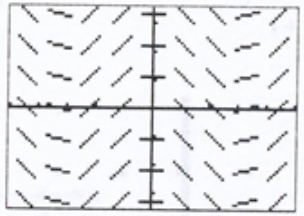
6. $\frac{dy}{dx} = -\frac{y}{x}$



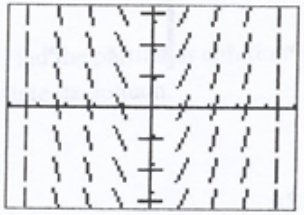
(A)



(B)



(C)



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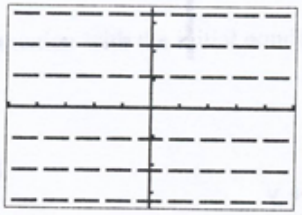
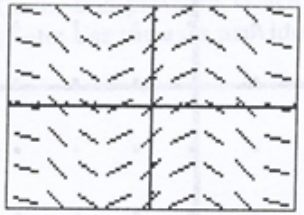


(E)



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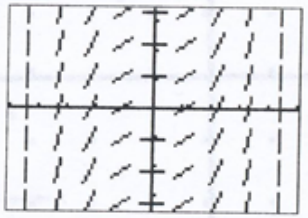
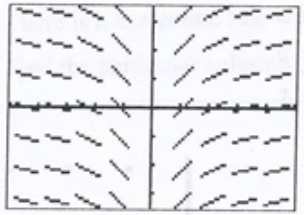
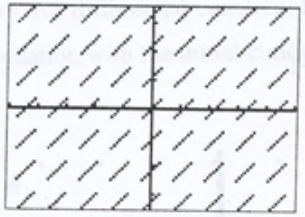


(G)



(H)



7. $y = 1$ D

8. $y = x$ H

9. $y = x^2$ C

10. $y = \frac{1}{6}x^3$ F

11. $y = \frac{1}{x^2}$ A

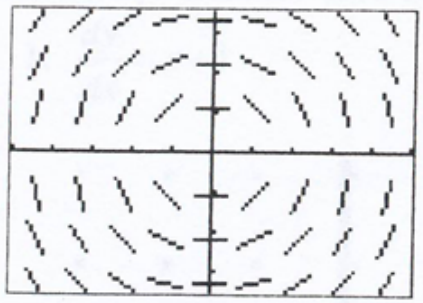
12. $y = \sin x$ E

13. $y = \cos x$ B

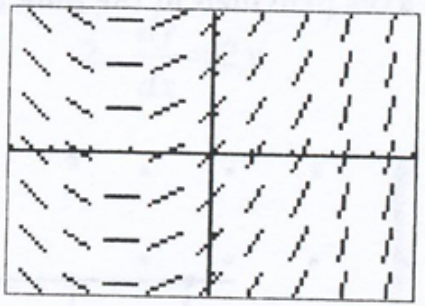
14. $y = \ln|x|$

Match the slope fields with their differential equations.

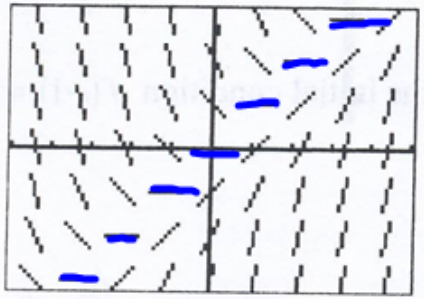
(A)



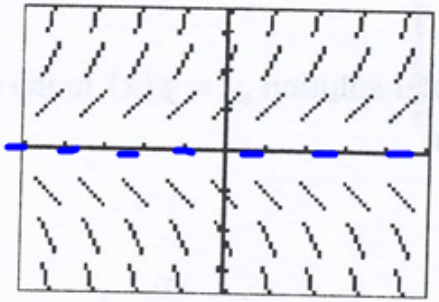
(B)



(C)



(D)



15. $\frac{dy}{dx} = \frac{1}{2}x + 1$ B

16. $\frac{dy}{dx} = y$ D

17. $\frac{dy}{dx} = x - y$ C

18. $\frac{dy}{dx} = -\frac{x}{y}$ A

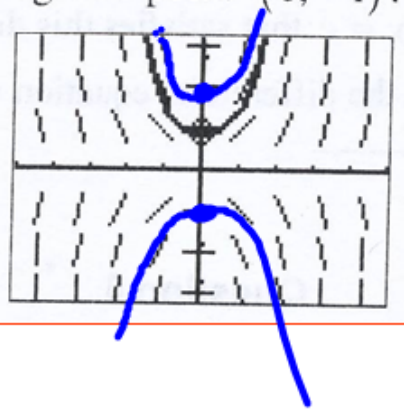
x	y	$\frac{dy}{dx}$
1	1	−1
−1	1	1

Sketching Possible Solutions On a Slope Field

19. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.

(a) Sketch the solution curve through the point (0, 2).

(b) Sketch the solution curve through the point $(0, -1)$.



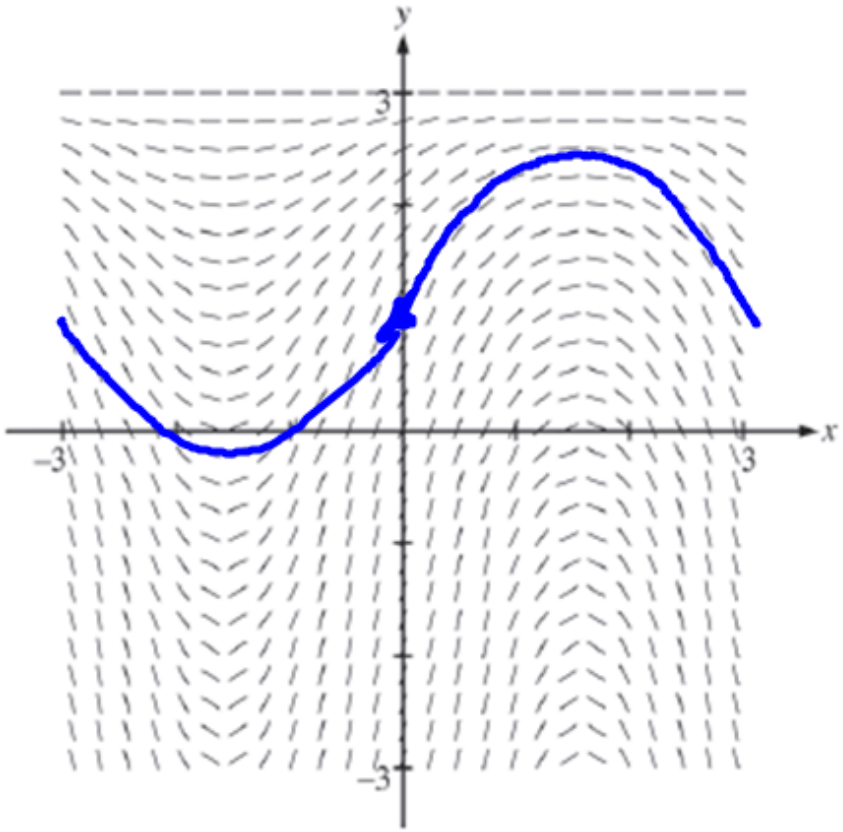
20. The calculator drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.

(a) Sketch the solution curve through the point $(0, 1)$.

(b) Sketch the solution curve through the point $(-3, 0)$.

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

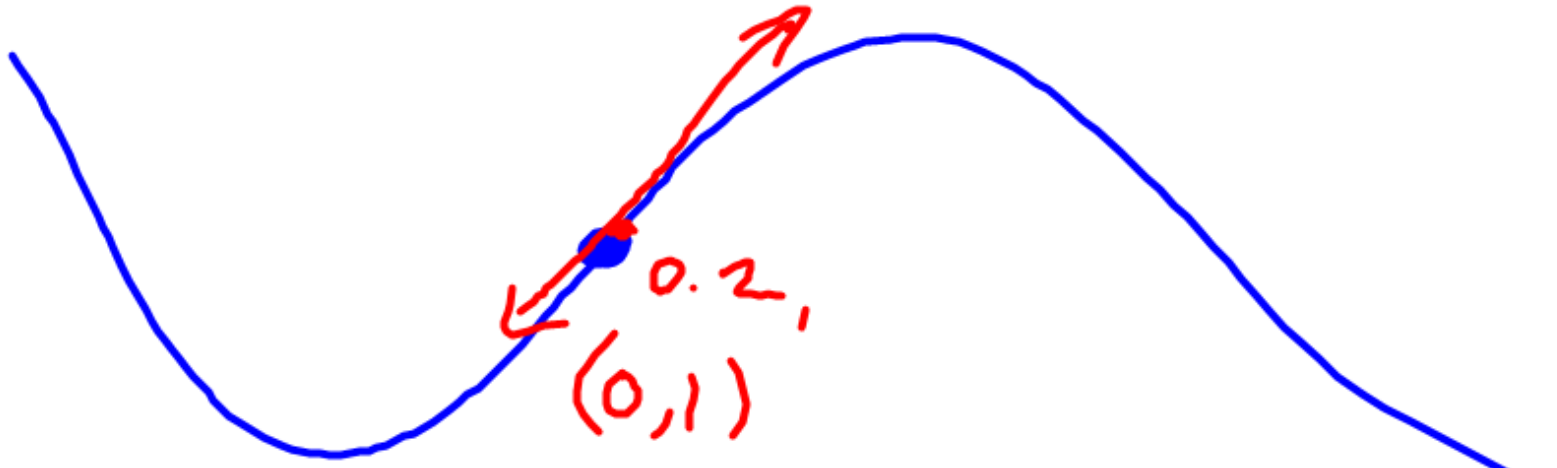
(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.

b) Tangent Line } approximation
Linear



Point $(0,1)$

$$\frac{dy}{dx} = (3-1)(\cos 0)$$
$$= 2$$

$$y - 1 = 2(x - 0)$$

$$\boxed{y = 2x + 1}$$

$$y = 2(0.2) + 1 = \boxed{1.4}$$

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(A) On the axes provided, sketch a slope field for the given differential equation.

(B) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point $(1, 1)$. Then use your tangent line equation to estimate the value of $f(1.2)$.

(C) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 1$. Use your solution to find $f(1.2)$.

(D) Compare your estimate of $f(1.2)$ found in part (b) to the actual value of $f(1.2)$ found in part

(E) Was your estimate from part (b) an underestimate or an overestimate? Use your slope field to explain why.

b)

$$\frac{dy}{dx} = \frac{(1)(1)}{2} = \frac{1}{2}$$

$$y - 1 = \frac{1}{2}(x - 1)$$

$$y - 1 = \frac{1}{2}(1.2 - 1)$$

$$y - 1 = \frac{1}{2}(0.2)$$

$$y = 1.1$$

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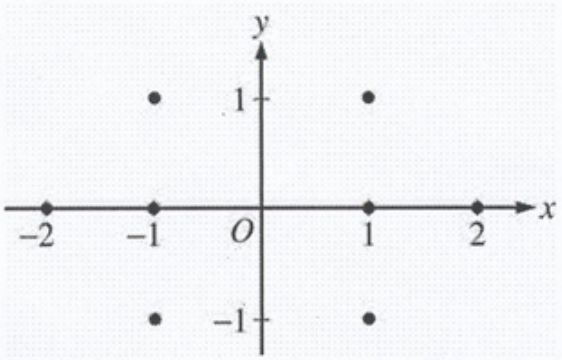
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated. **(Note: Use the axes provided in the pink exam booklet.)**



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

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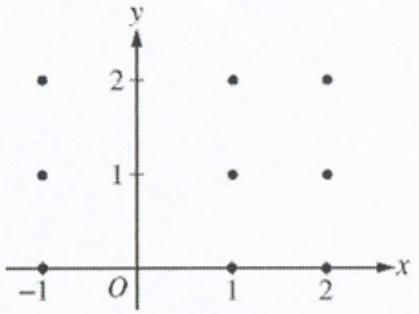
Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.



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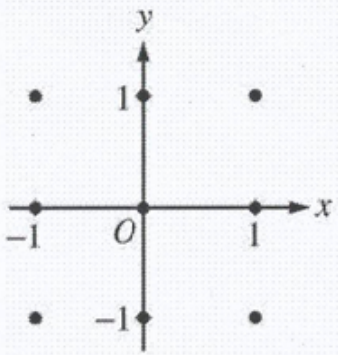
Question 5

Consider the differential equation $\frac{dy}{dx} = (y-1)^2 \cos(\pi x)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated. **(Note: Use the axes provided in the exam booklet.)**



(b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c.

(c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

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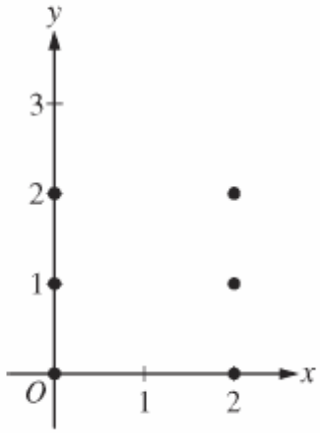
4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.