

2.3 Piecewise Functions

We were introduced to **piecewise functions** last unit when we studied absolute value functions!

$$|x| = \left\{ \begin{array}{l} \underline{x}, \text{ if } \underline{x \geq 0} \\ -x, \text{ if } x < 0 \end{array} \right\}$$

In a piecewise function the function behaves differently in different parts of its domain!

A truck rental company charges a minimum \$75 rental fee or \$25 per hour.

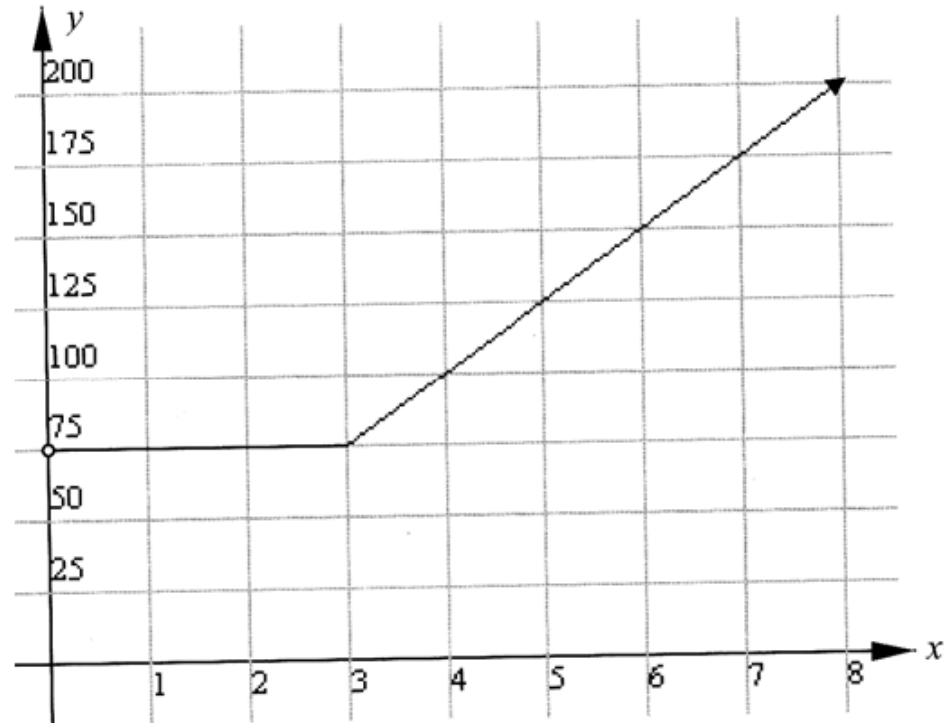
$$f(x) = \begin{cases} 75, & \text{if } 0 < x \leq 3 \\ 25x, & \text{if } x > 3 \end{cases}$$

Your Turn #1

By referring to the illustration at left, what is the rental charge for using the truck:

- (a) 1 hour?
- (b) 6 hours?
- (c) 7.25 hours?

75
150
183.75



Graphing Piecewise Functions

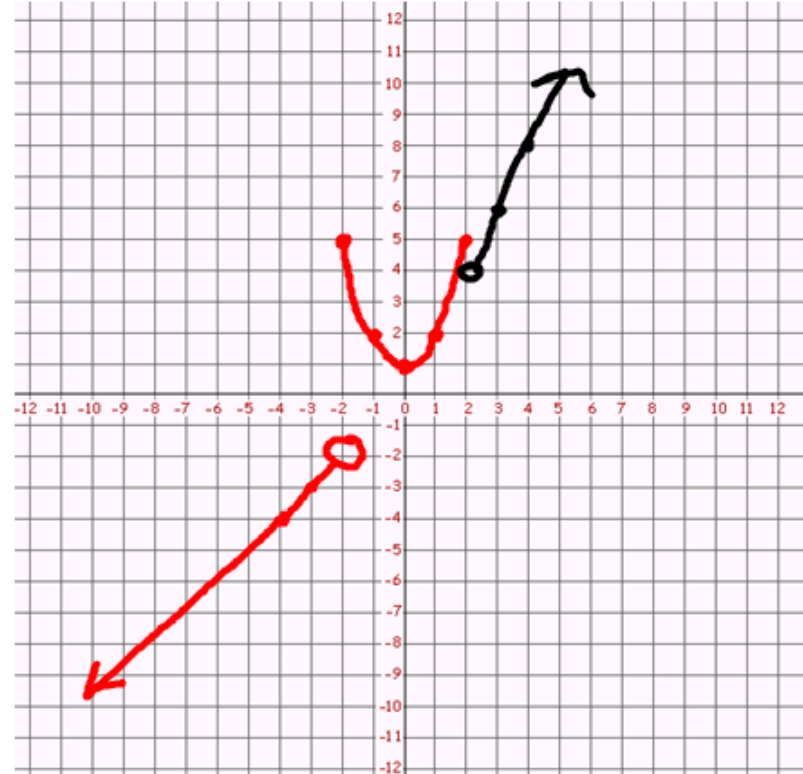
Ex.1 Graph the following:

$$f(x) = \begin{cases} x, & \text{if } x \in (-\infty, -2) \\ x^2 + 1, & \text{if } x \in [-2, 2] \\ 2x, & \text{if } x \in (2, \infty) \end{cases}$$

x	y = x
-2	-2
-3	-3
-4	-4

x	y = x ² + 1
-2	5
-1	2
0	1
1	2
2	5

x	y = 2x
2	4
3	6
4	8

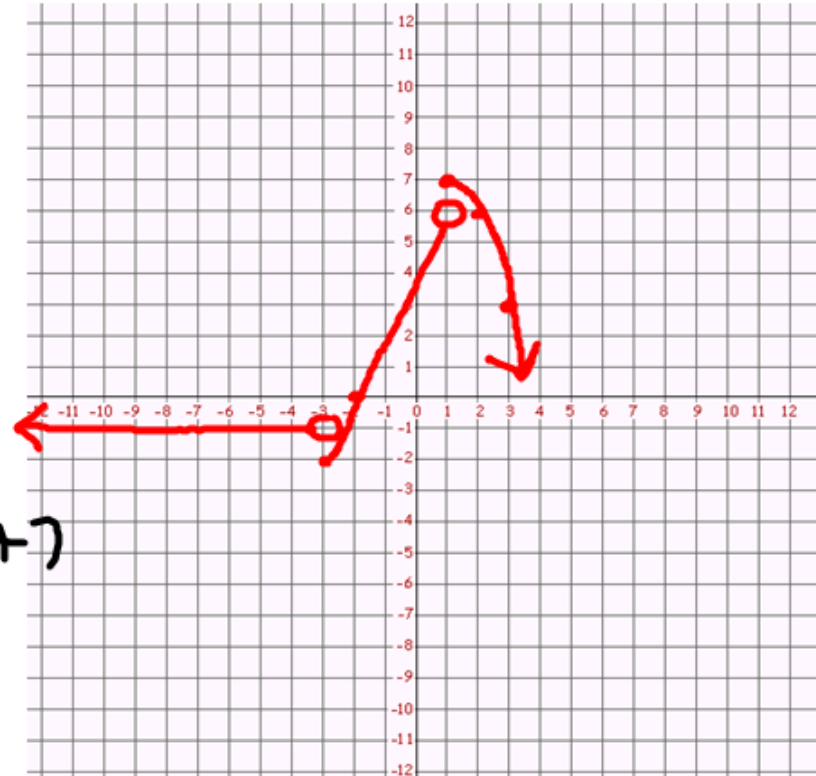


Ex.2 Graph the following:

$$g(x) = \begin{cases} -1, & \text{if } x \in (-\infty, -3) \\ 2x+4, & \text{if } x \in [-3, 1) \\ -(x-1)^2 + 7, & \text{if } x \in [1, \infty) \end{cases}$$

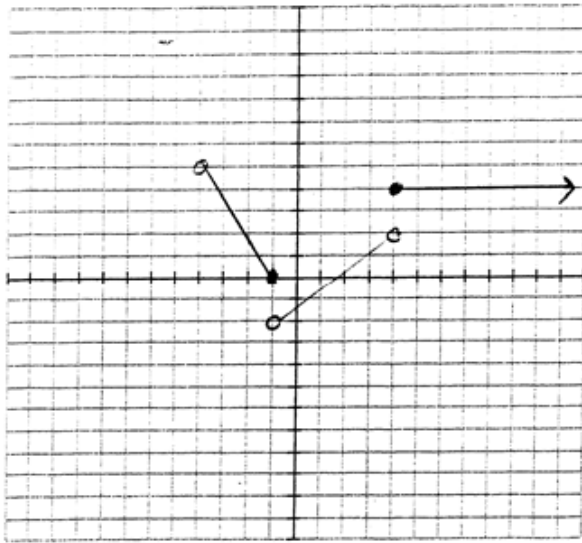
x	2x+4
-3	-2
-2	0
-1	2
0	4
1	6

x	$-(x-1)^2 + 7$
1	7
2	6
3	3



Developing Functions From Graphs of Piecewise Functions

Ex.3 Find the equation of the piecewise function whose graph is given below.



$$f(x) = \begin{cases} -\frac{5}{3}x - \frac{5}{3}, & x \in (-4, -1] \\ \frac{4}{5}x - \frac{6}{5}, & x \in (-1, 4) \\ 4, & \text{if } x \in [4, \infty) \end{cases}$$

$$m = -\frac{5}{3} \quad (-1, 0)$$

$$y - 0 = -\frac{5}{3}(x + 1)$$

$$y = -\frac{5}{3}x - \frac{5}{3}$$

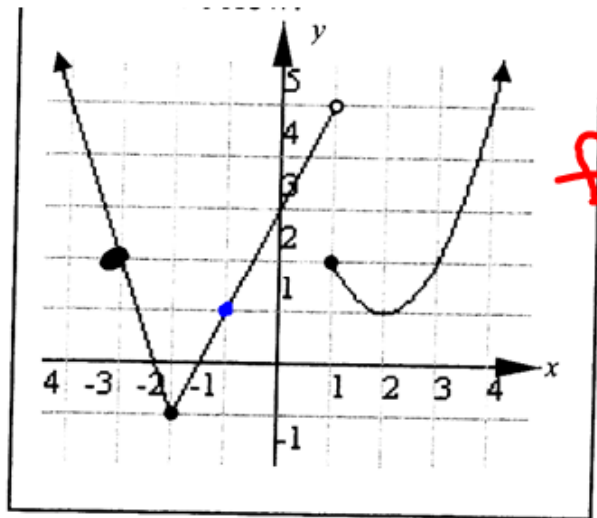
$$m = \frac{4}{5} \quad (4, 2)$$

$$y - 2 = \frac{4}{5}(x - 4)$$

$$y - 2 = \frac{4}{5}x - \frac{16}{5}$$

$$y = \frac{4}{5}x - \frac{6}{5}$$

Ex.4 Find the equation of the piecewise function whose graph is given below.



$$f(x) = \begin{cases} -3x - 7, & \text{if } x \in (-\infty, -1) \\ 2x + 3 & \text{if } x \in [-1, 1) \\ (x-2)^2 + 1 & \text{if } x \in [1, \infty) \end{cases}$$

$$m = -3 \quad (-2, -1)$$

$$y + 1 = -3(x + 2)$$

$$y + 1 = -3x - 6$$

$$y = -3x - 7$$

$$m = 2 \quad (-2, -1)$$

$$y + 1 = 2(x + 2)$$

$$y + 1 = 2x + 4$$

$$y = 2x + 3$$

$$y = a(x - p)^2 + q$$

$$y = a(x - 2)^2 + 1$$

(1, 2)

$$2 = a(1 - 2)^2 + 1$$

$$1 = a(-1)^2$$

$$1 = a$$

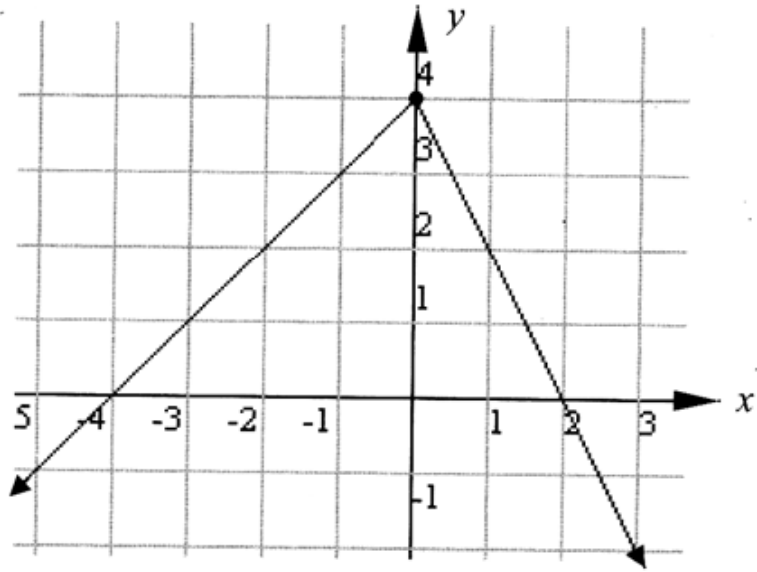
$$y = (x - 2)^2 + 1$$

Assignment

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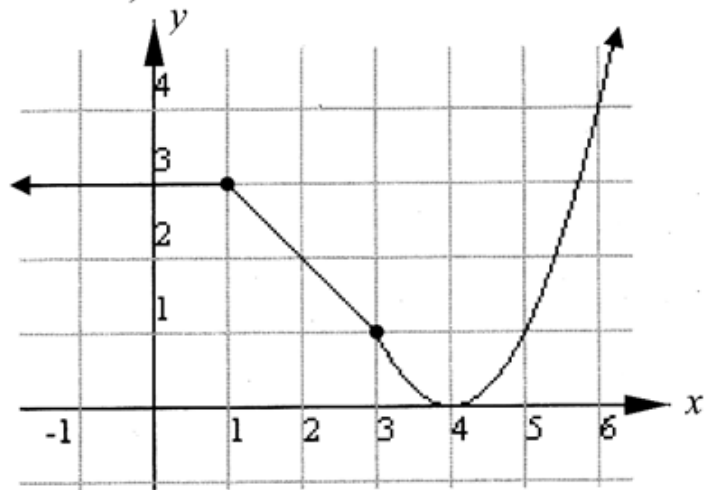
#'s 4,5,6

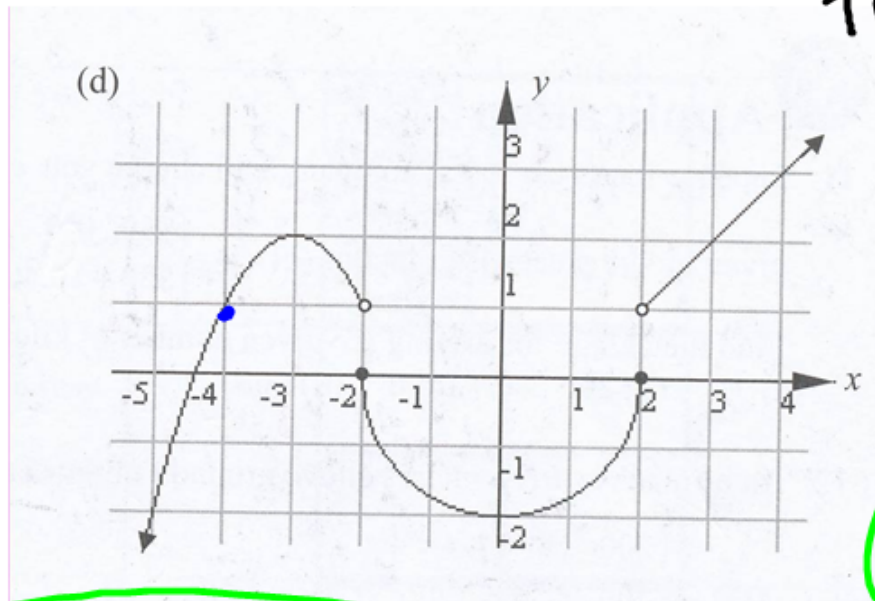
(b)



3. (Continued)

(c)





$$f(x) = \begin{cases} -(x+3)^2 + 2; & (-\infty, -2) \\ \sqrt{4-x^2} & [-2, 2] \\ x-1, & \text{if } x \in (2, \infty) \end{cases}$$

$$y = a(x-p)^2 + q$$

$$y = a(x+3)^2 + 2$$

$$1 = a(-4+3)^2 + 2$$

$$-1 = a$$

$$y = -(x+3)^2 + 2$$

$$y - y_1 = m(x - x_1)$$

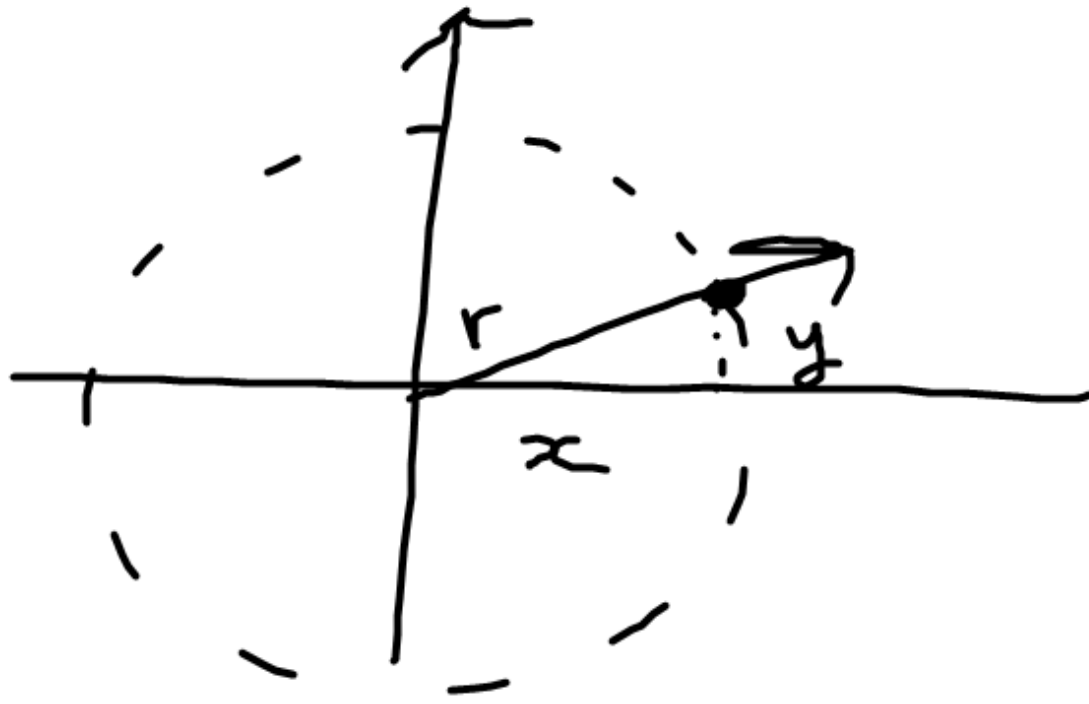
$$(y - 2) = 1(x - 3)$$

$$y - 2 = x - 3$$

$$y = x - 1$$

$$y = -\sqrt{r^2 - x^2}$$

$$y = -\sqrt{4 - x^2}$$



$$x^2 + y^2 = r^2$$

$$y^2 = r^2 - x^2$$

$$y = \pm \sqrt{r^2 - x^2}$$

Combining Function Notation and Piecewise Functions

Let $f(x)$ be defined as the following:

$$f(x) = \begin{cases} x - 3, & \text{if } x \in (-\infty, 2) \\ x^2 + 7, & \text{if } x \in [2, 8) \\ \sqrt{x}, & \text{if } x \in [8, \infty) \end{cases}$$

Ex.5 Evaluate the following:

a) $f(6)$

$$\begin{aligned} &= (6)^2 + 7 \\ &= 43 \end{aligned}$$

b) $f(-4)$

$$\begin{aligned} &= -4 - 3 \\ &= -7 \end{aligned}$$

$$\text{c) } f(11)$$

$$= \sqrt{11}$$

$$\text{d) } f(2)$$

$$= (2)^2 + 7$$

$$= 11$$

Let $h(x)$ be defined as the following:

$$h(x) = \begin{cases} x^2 + 1, & \text{if } x \in (-\infty, 0] \\ 2x - 3, & \text{if } x \in (0, 8) \\ 3x + 2, & \text{if } x \in [8, \infty) \end{cases}$$

$\{-5, 8\}$

Ex.6 Find values of x such that:

a) $h(x) = 26$

$$x^2 + 1 = 26$$
$$\sqrt{x^2} = \sqrt{25}$$

$$x = \pm 5$$

$$x = -5$$

$$2x - 3 = 26$$

$$2x = 29$$

$$x = 14.5$$

$$3x + 2 = 26$$

$$3x = 24$$

$$x = 8$$

$$\text{b) } h(x) = 0$$

$$x^2 + 1 = 0$$

~~$x^2 = -1$~~

$$2x - 3 = 0$$
$$2x = 3$$
$$x = \frac{3}{2}$$

$$3x + 2 = 0$$

~~$x = -\frac{2}{3}$~~

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6. $f(x) = \lfloor x \rfloor$

$x = 1.7 \rightarrow 1$

Step function
Greatest Int
Function

Assignment
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6.

$$f(x) = \lfloor x \rfloor$$

$$x = 1.7 \rightarrow 1$$

Step function
Greatest Int
Function

