2.2 Verifying Solutions of Differential Equations

Verify $\underbrace{y = 4\cos(2x) + 6\ \sin(2x)}$ is a solution to $y'' + 4y = 0$.

$$y' = 4(-\sin 2x)\cdot 2 + 6(\cos 2x)2$$

$$y' = -8\sin 2x + 12\cos 2x$$

$$y'' = -8\cdot 2\cos 2x + 12(-2\sin 2x)$$

$$= -16\cos 2x - 24\sin 2x$$

$$-16\cos 2x - 24\sin 2x + 4(4\cos 2x + 6\sin 2x) \overset{?}{=} 0$$

$$-16\cancel{\cos 2x} - 24\cancel{\sin 2x} + 16\cancel{\cos 2x} + 24\cancel{\sin 2x} = 0$$

$$0 = 0$$

Practice 1

Determine whether the given function is a solution of the differential equation $f''(x) - f(x) = 0$

a) $f(x) = -\cos x$ b) $f(x) = 2e^{-x}$ ✓ c) $f(x) = Ke^{x}$ ✓

a) $\cancel{f' = \sin x}$

$\cancel{f'' = \cos x}$

$\cos x - (-\cos x) \neq 0$

b) $f' = -2e^{-x}$

$f'' = 2e^{-x}$

$2e^{-x} - 2e^{-x} = 0$

$0 = 0$

c) $f' = ke^{x}$

$f'' = ke^{x}$

$ke^{x} - ke^{x} = 0$

$0 = 0$

Practice 2

25. The equation $y = 2e^{6x} - 5$ is a particular solution to which of the following differential equations?

(A) $y' - 6y - 30 = 0$

(B) $2y' - 12y + 5 = 0$

(C) $y'' - 5y' - 6y = 0$

(D) $y'' - 2y' + y + 5 = 0$

$y' = 12e^{6x}$ $\quad$ $y'' = 72e^{6x}$

A) $12e^{6x} - 6(2e^{6x} - 5) - 30 \stackrel{?}{=} 0$

$12e^{6x} - 12e^{6x} + 30 - 30 = 0$

$0 = 0$

Verifying Solutions...

A solution of a differential equation can be checked by substituting the function and its derivative(s) into the original differential equation.

4. A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

a) $v(t) = e^{-t}\cos t + \sin t\,(-e^{-t}) = x'(t)$

$v(t) = e^{-t}(\cos t - \sin t)$

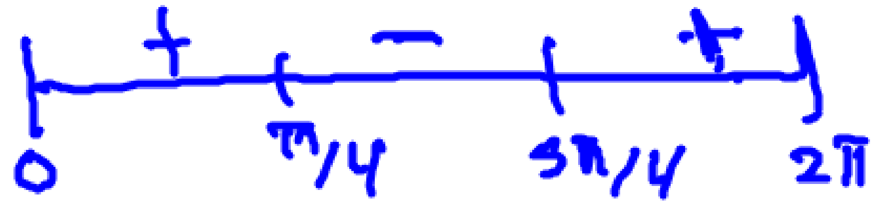
$v(t) = 0$

$e^{-t}(\cos t - \sin t) = 0$

$\cos t - \sin t = 0$

$\cos t = \sin t$

$t = \pi/4,\ 5\pi/4$



∴ particle farthest at $t = 5\pi/4$

$$x''(t) = e^{-t}(-\sin t - \cos t) + (\cos t - \sin t)(-e^{-t})$$

$$= e^{-t}(-\underline{\sin t} - \underbrace{\cos t - \cos t}_{} + \underline{\sin t})$$

$$= -2e^{-t}\cos t$$

$$A(-2e^{-t}\cos t) + e^{-t}(\cos t - \sin t) + e^{-t}\sin t = 0$$

$$-2Ae^{-t}\cos t + e^{-t}\cos t - \cancel{e^{-t}\sin t} + \cancel{e^{-t}\sin t} = 0$$

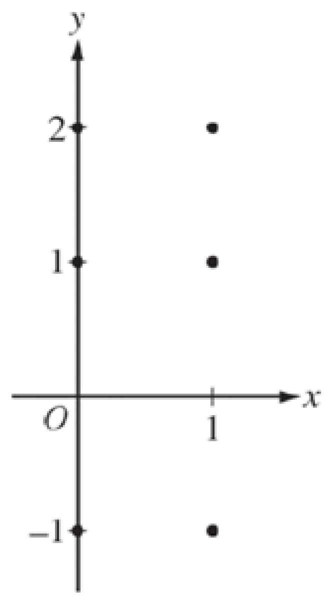
$$\underbrace{e^{-t}\cos t}_{}\,\underbrace{(-2A+1)}_{} = 0$$

$$-2A + 1 = 0$$

$$A = 1/2$$

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

$$y = mx + b$$

$$\frac{dy}{dx} = m$$

$$\frac{dy}{dx} = 2x - y$$

$$m = 2x - (mx + b)$$

$$m = 2x - mx - b$$

$$m = x\underbrace{(2-m)} - b$$

$$2 - m = 0$$

$$m = 2 \qquad b = -2$$

Assignment:
Handout

Assignment 2.2 Verifying Solutions to Differential Equations

1. Of the following, which are solutions to the differential equation $y'' - 6y' + 8y = 0$?

   I. $y = 2\sin(4x)$
   II. $y = 3e^{2x}$
   III. $y = Ce^{4x}$, where C is a constant.

2. For what value of k, if any, will $y = ke^{-2x} + 4\cos(3x)$ be a solution to the differential equation $y'' + 9y = 26e^{-2x}$?

3. For what value of k, if any, is $y = e^{-2x} + ke^{4x}$ a solution to the differential equation $y - \frac{y''}{4} = 5e^{4x}$?

4. Of the following, which are solutions to the differential equation $y'' - 10y' + 9y = 0$?

   I. $y = 2\sin(3x)$
   II. $y = 5e^{x}$
   III. $y = Ce^{9x}$, where C is a constant.

5. Which of the following is a solution to the differential equation $xy' - 3y = 6$?

   A $y = x^2 + 2$

   B $y = 3x^2 - 2$

   C $y = 5x^3 - 2$

   D $y = 7x^3 + 2$

6. Of the following, which is not a solution to the differential equation $y''' + 4y' = 0$?

   A $y = 10$

   B $y = 4e^{-2x}$

   C $y = 3\sin(2x)$

   D $y = 2\cos(2x) + 4$

Answers: 1. II, III 2. K=2 3. K=-5/3 4. II, III 5. C 6. B