

2.2 Square Root of a Function

Today we will be examining how the square root operation affects the graph of a function.

$$y = \sqrt{x}$$

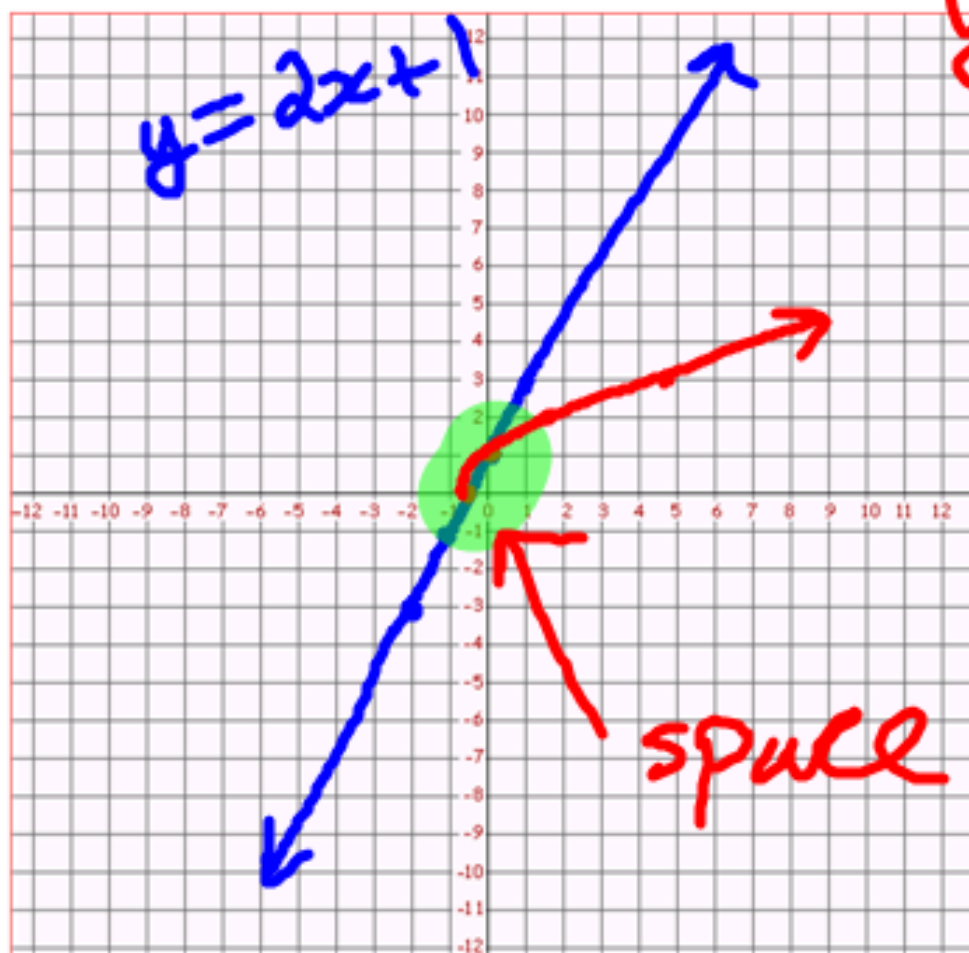
You can determine how two functions, $y = f(x)$ and $y = \sqrt{f(x)}$, are related by comparing how the values of y are calculated:

x	y
0	0
1	1
4	2
9	3

- For $y = 2x + 1$, multiply x by 2 and add 1.
- For $y = \sqrt{2x + 1}$, multiply x by 2, add 1, and take the square root.

$$y = 2x + 1$$

x	y
-2	-3
-1	-1
0	1
1	3



$$y = \sqrt{2x + 1}$$

The function $y = \sqrt{2x + 1}$ represents the **square root of the function** $y = 2x + 1$.

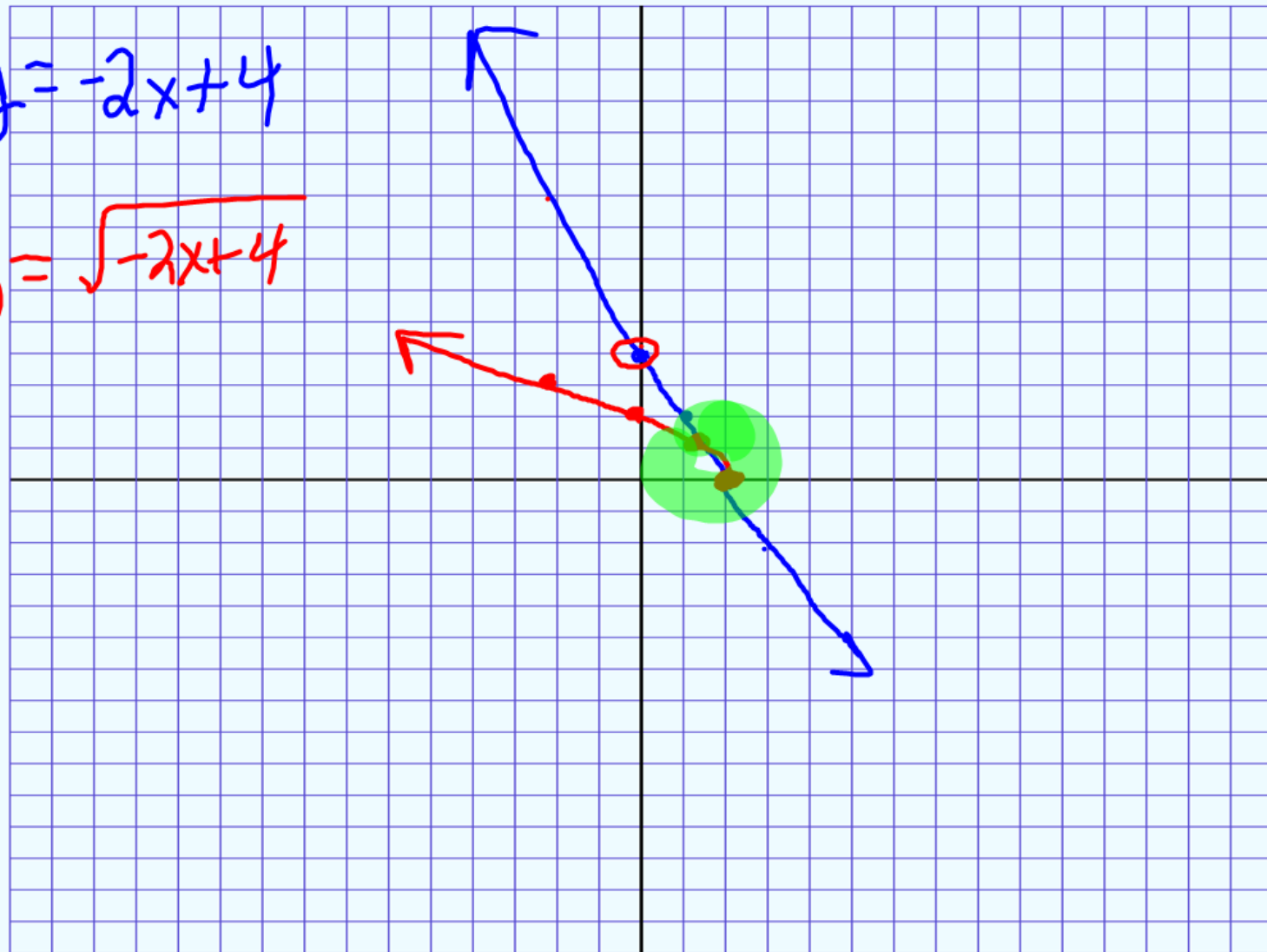
$$y = \sqrt{-2x + 4}$$

$$y = -2x + 4$$

x	y
2	0
3	-2
1	2

$$y = -2x + 4$$

$$y = \sqrt{-2x + 4}$$



It is important to remember:

square root of a function

- the function $y = \sqrt{f(x)}$ is the square root of the function $y = f(x)$
- $y = \sqrt{f(x)}$ is only defined for $f(x) \geq 0$

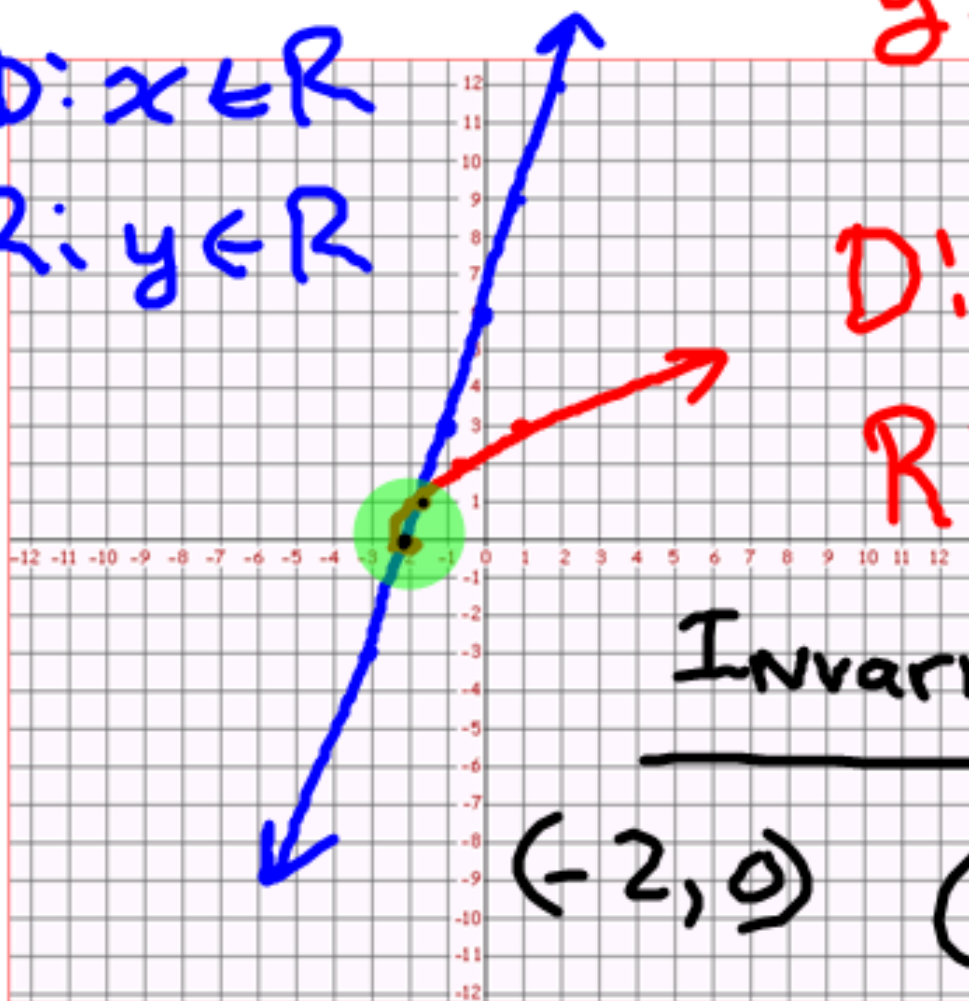
Your Turn

- a) Given $g(x) = 3x + 6$, graph the functions $y = g(x)$ and $y = \sqrt{g(x)}$.
- b) Identify the domain and range of each function and any invariant points.

$$y = 3x + 6$$

x	y
-2	0
0	6
2	12
-1	3
1	9

D: $x \in \mathbb{R}$
R: $y \in \mathbb{R}$



$$y = \sqrt{3x + 6}$$

D: $x \geq -2$
R: $y \geq 0$

Invariant Points

$(-2, 0)$ $(-5/3, 1)$

Relative Locations of $y = f(x)$ and $y = \sqrt{f(x)}$

The domain of $y = \sqrt{f(x)}$ consists only of the values in the domain of $f(x)$ for which $f(x) \geq 0$.

The range of $y = \sqrt{f(x)}$ consists of the square roots of the values in the range of $y = f(x)$ for which $\sqrt{f(x)}$ is defined.

The graph of $y = \sqrt{f(x)}$ exists only where $f(x) \geq 0$. You can predict the location of $y = \sqrt{f(x)}$ relative to $y = f(x)$ using the values of $f(x)$.

$$3x + 6 = 1$$

$$3x = -5$$

$$x = -\frac{5}{3}$$

Value of $f(x)$	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Relative Location of Graph of $y = \sqrt{f(x)}$	The graph of $y = \sqrt{f(x)}$ is undefined.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ intersect on the x-axis.	The graph of $y = \sqrt{f(x)}$ is above the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ intersects the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ is below the graph of $y = f(x)$.

Example

$$y = -0.5x^2 + 2$$

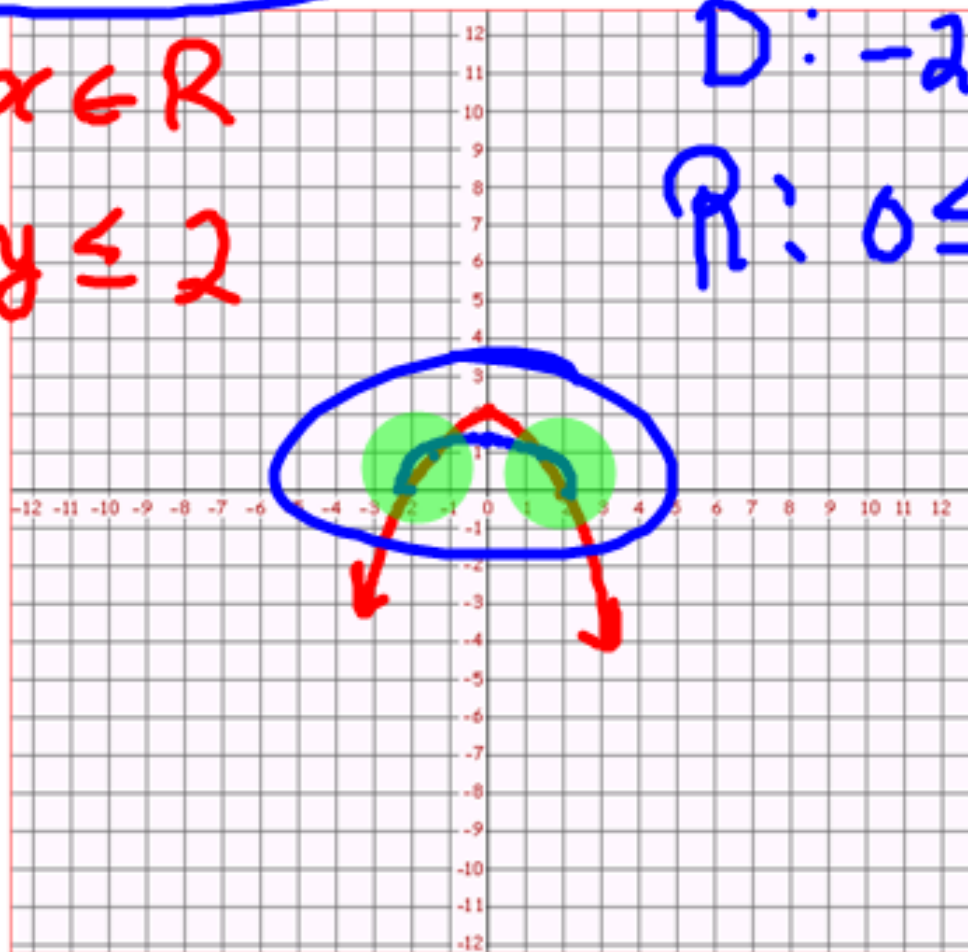
Identify and compare the domains and ranges of the functions in each pair.

a) $y = 2 - 0.5x^2$ and $y = \sqrt{2 - 0.5x^2}$

x	y
0	2
2	0
-2	0

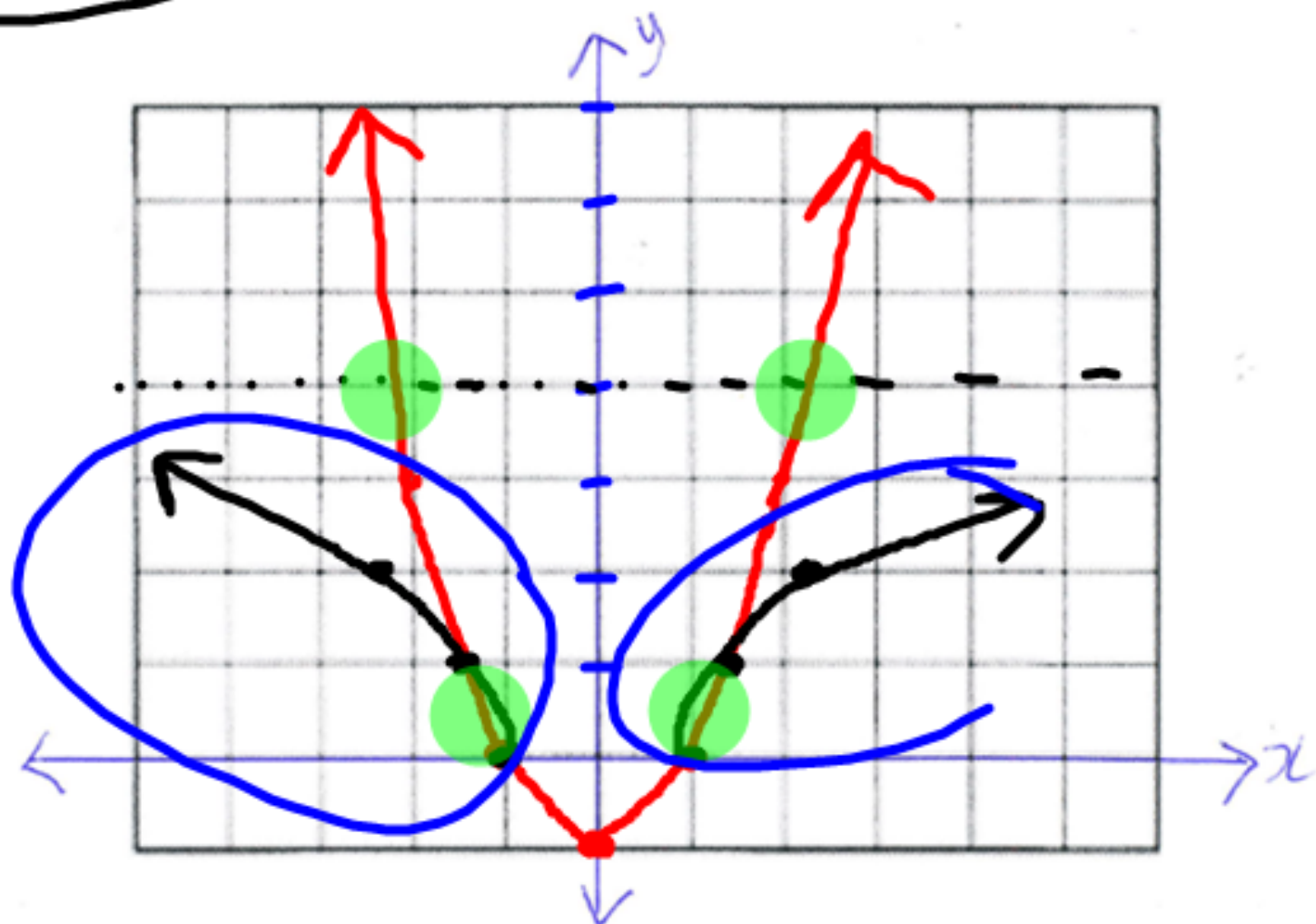
$D: x \in \mathbb{R}$
 $R: y \leq 2$

$D: -2 \leq x \leq 2$
 $R: 0 \leq y \leq 2$



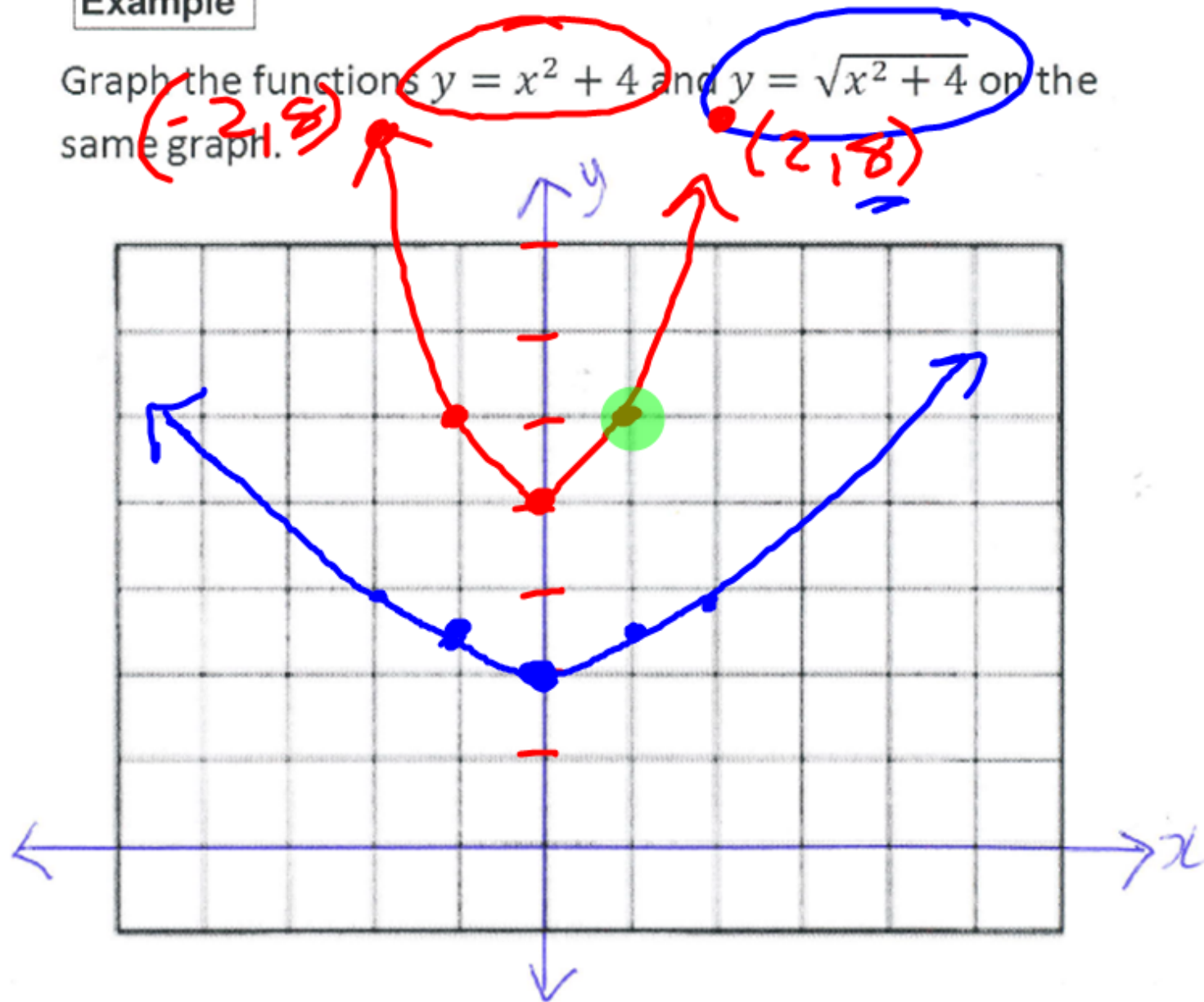
Your Turn

Identify and compare the domains and ranges of the functions $y = x^2 - 1$ and $y = \sqrt{x^2 - 1}$. Verify your answers.



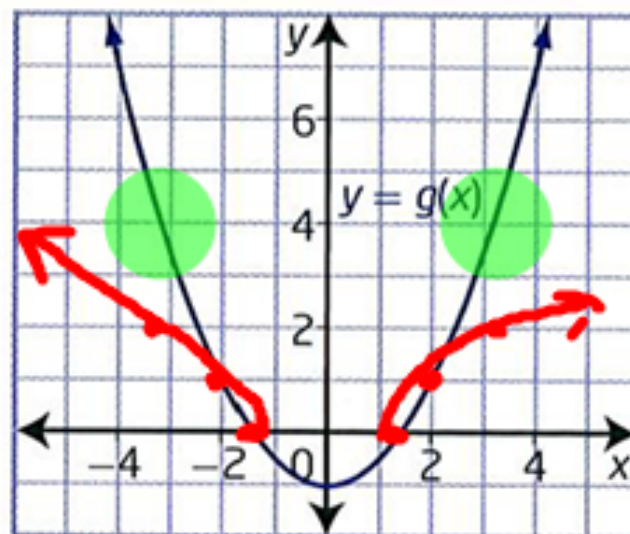
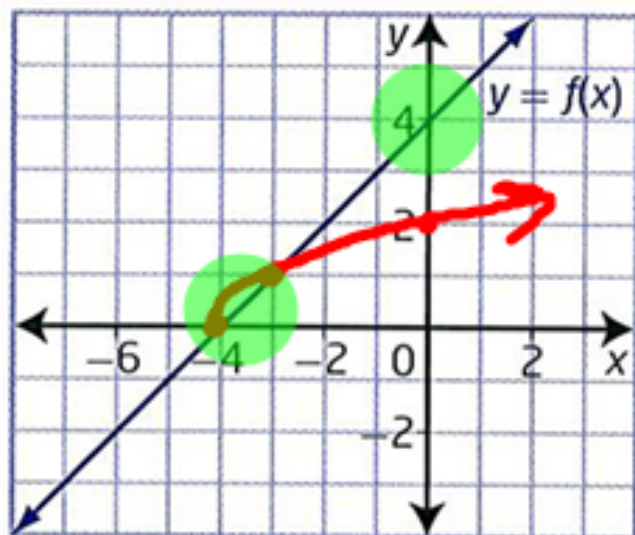
Example

Graph the functions $y = x^2 + 4$ and $y = \sqrt{x^2 + 4}$ on the same graph.



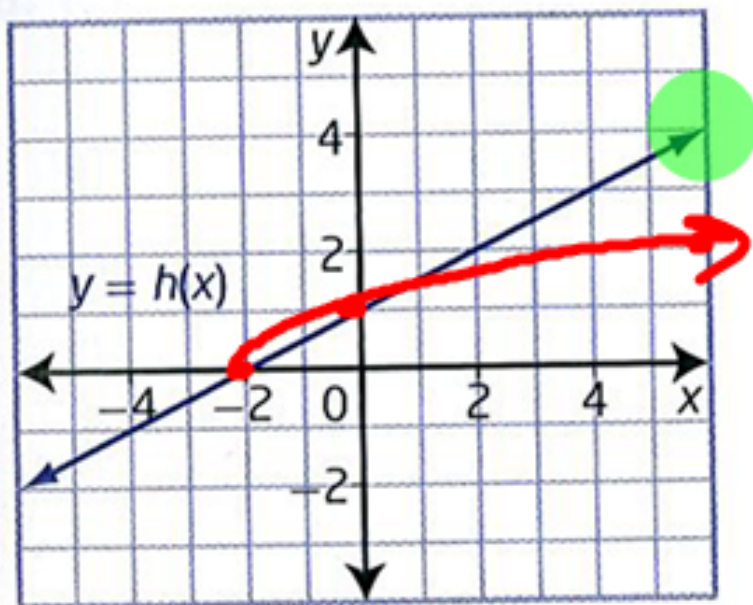
Example

Using the graphs of $y = f(x)$ and $y = g(x)$, sketch the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.



Your Turn

Using the graph of $y = h(x)$, sketch the graph of $y = \sqrt{h(x)}$.



Find the Domain and the Range for each of the following:

$$y = -\sqrt{2(x-4)} + 1$$

$$y = \sqrt{-3x+18} - 2$$

$$D: x \geq 4$$

$$R: y \leq 1$$

$$D: x \geq 0$$

$$R: y \geq 0$$

$$y = \sqrt{x}$$

$$y = \sqrt{-3(x-6)} - 2$$

$$D: x \leq 6$$

$$R: y \geq -2$$

Key Ideas

- You can use values of $f(x)$ to predict values of $\sqrt{f(x)}$ and to sketch the graph of $y = \sqrt{f(x)}$.
- The key values to consider are $f(x) = 0$ and $f(x) = 1$.
- The domain of $y = \sqrt{f(x)}$ consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$.
- The range of $y = \sqrt{f(x)}$ consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.
- The y -coordinates of the points on the graph of $y = \sqrt{f(x)}$ are the square roots of the y -coordinates of the corresponding points on the original function $y = f(x)$.

What do you know about the graph of $y = \sqrt{f(x)}$ at $f(x) = 0$ and $f(x) = 1$? How do the graphs of $y = f(x)$ and $y = \sqrt{f(x)}$ compare on either side of these locations?

Assignment Page 86

#'s 2, 3,4,5.a,c, 6. a,c, 8, 11, 16a,b,