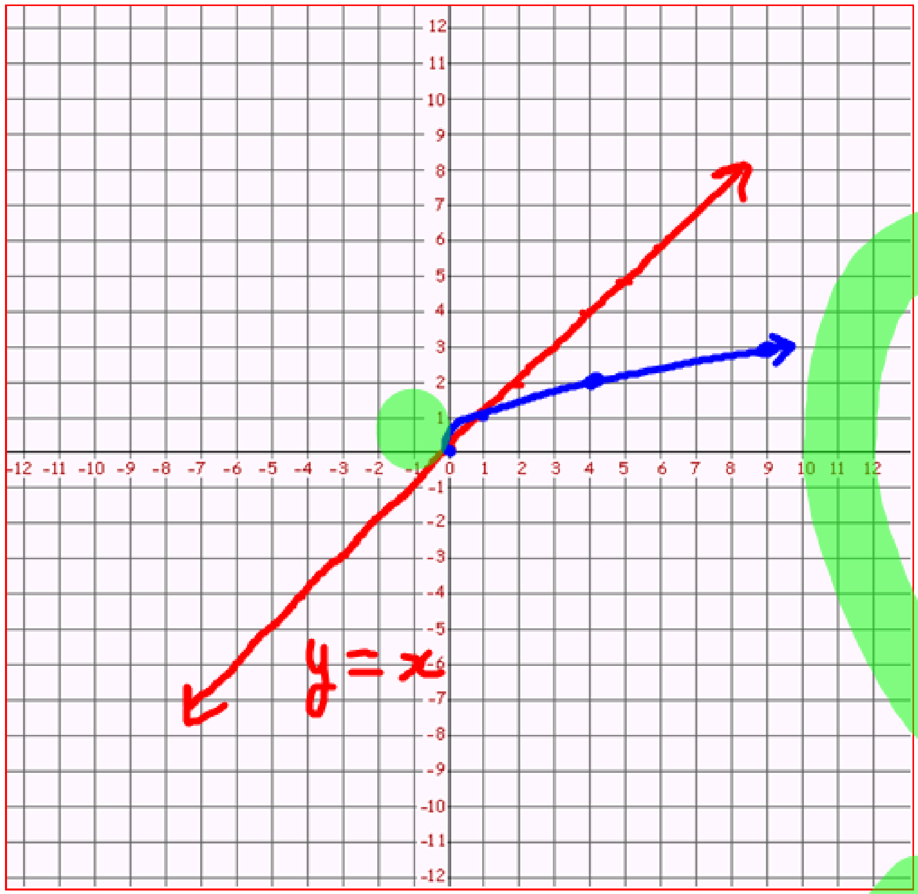
Unit 2 Radical Functions

2.1 Radical Functions and Transformations

Let's examine the function $y = \sqrt{x}$.

$y = x$



$y = \sqrt{x}$

$\sqrt{0} = 0$

$\sqrt{\frac{1}{9}} = \frac{1}{3}$

$\sqrt{\frac{1}{4}} = \frac{1}{2}$

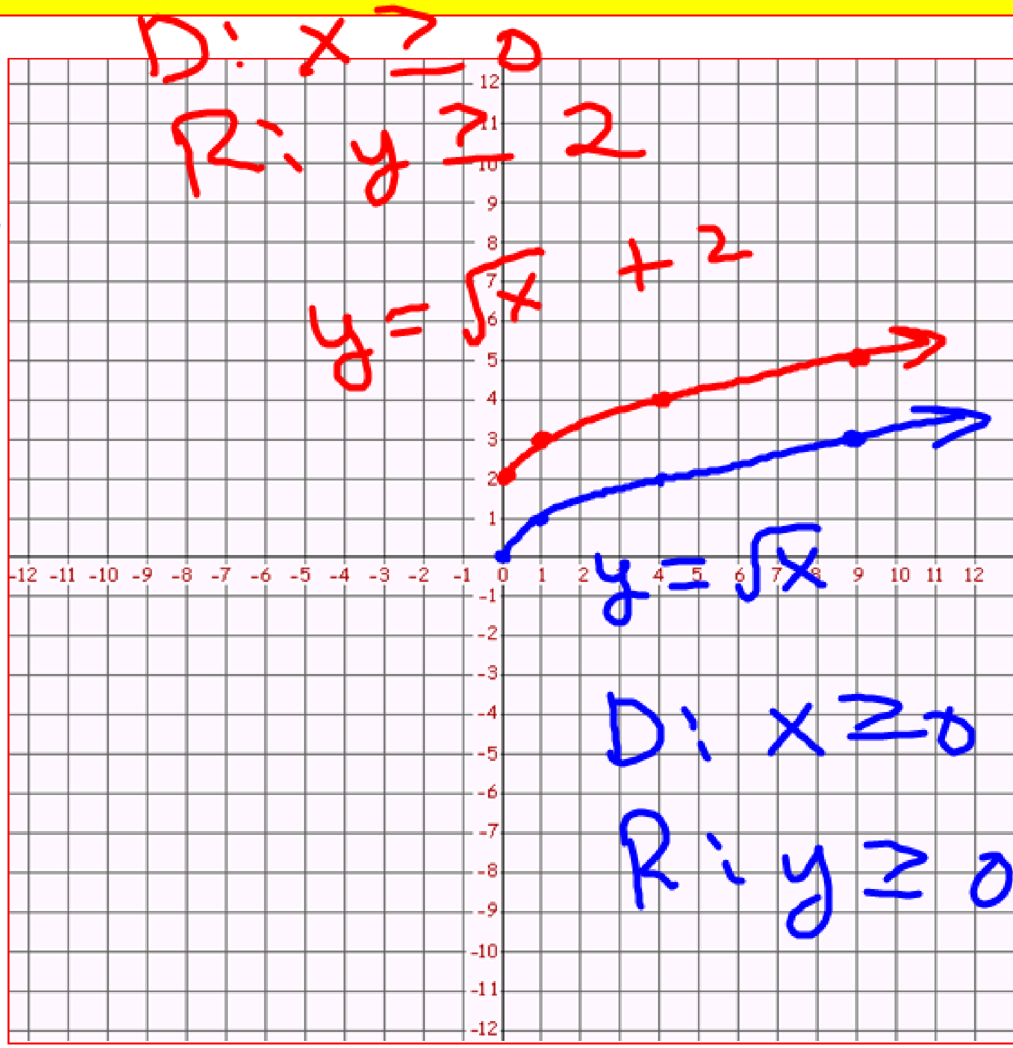
$\sqrt{1} = 1$

$\sqrt{4} = 2$

$\sqrt{9} = 3$

Based on our work from chapter 1, what do you think what do you think the graph of $y = \sqrt{x} + 2$ will look like?

x	y
0	0
1	1
4	2
9	3



$k = 2$

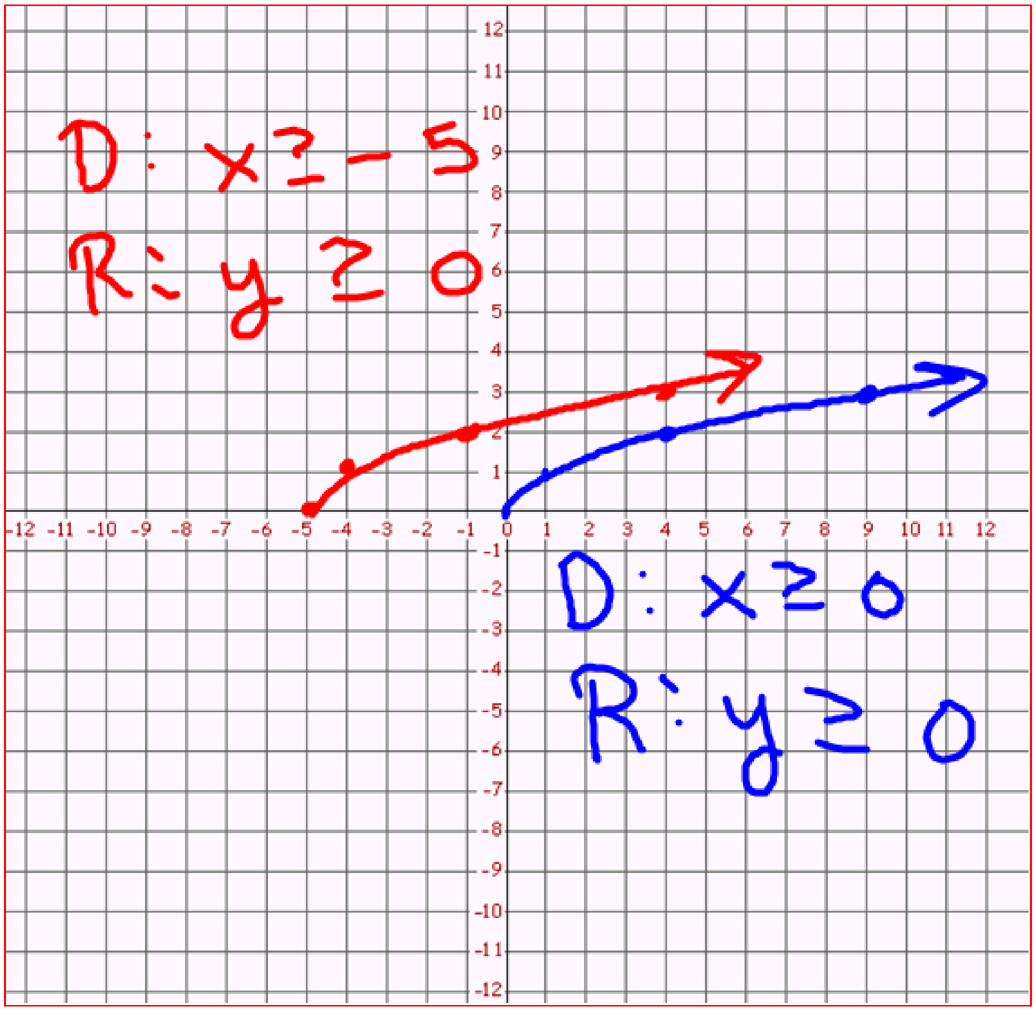
shift graph 2 up.

$h = -5$

Your Turn

Sketch the graph of the function $y = \sqrt{x+5}$ using a table of values. State the domain and range.

x	y
0	0
1	1
4	2
9	3



x	y
−5	0
−4	1
−1	2
4	3

Graphing Radical Functions Using Transformations

You can graph a radical function of the form $y = a\sqrt{b(x - h)} + k$ by transforming the graph of $y = \sqrt{x}$ based on the values of a, b, h, and k. The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

- Parameter a results in a vertical stretch of the graph of $y = \sqrt{x}$ by a factor of $|a|$. If $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x-axis.
- Parameter b results in a horizontal stretch of the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{|b|}$. If $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y-axis.
- Parameter h determines the horizontal translation. If $h > 0$, the graph of $y = \sqrt{x}$ is translated to the right h units. If $h < 0$, the graph is translated to the left $|h|$ units.
- Parameter k determines the vertical translation. If $k > 0$, the graph of $y = \sqrt{x}$ is translated up k units. If $k < 0$, the graph is translated down $|k|$ units.

Graphing Radical Functions Using Transformations

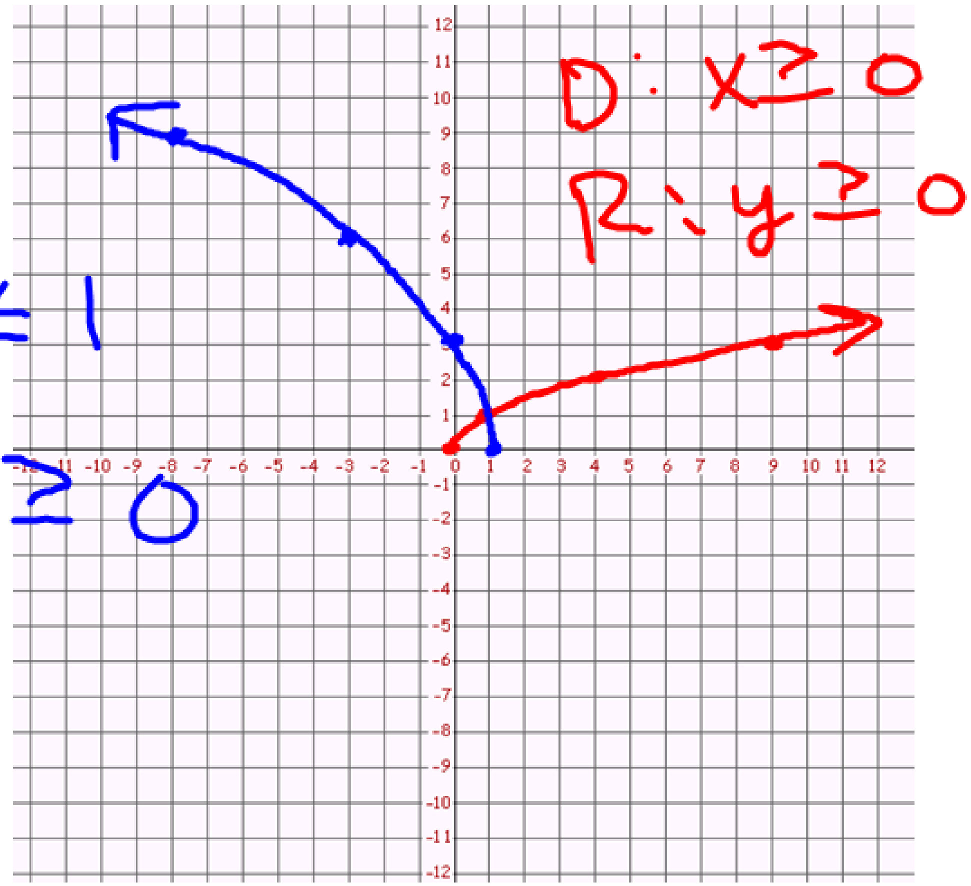
Sketch the graph $y = 3\sqrt{-(x-1)}$ using transformations. Compare the domain and range to those of $y = \sqrt{x}$ and identify any changes.

Mapping notation $(x, y) \rightarrow (\frac{x}{b} + h, ay + k)$

x	y
0	0
1	1
4	2
9	3

D: $x \leq 1$

R: $y \geq 0$



$a = 3$
$b = -1$
$h = 1$

$$(-x+1,\ 3y)$$

x	y
0	0
1	1
4	2
9	3

x	y
1	0
0	3
-3	6
-8	9

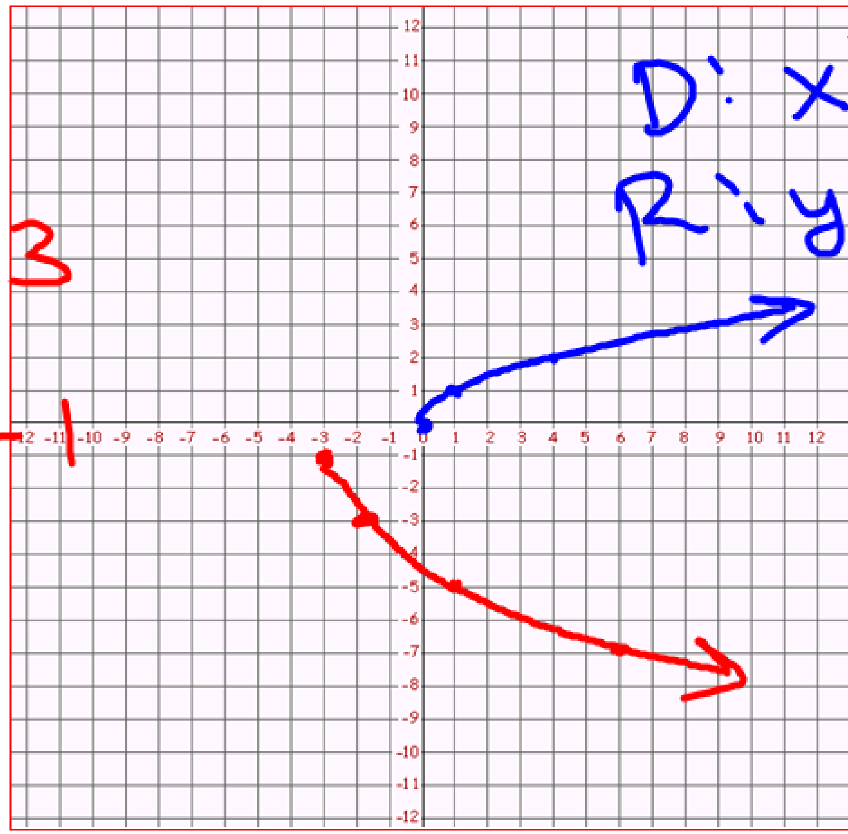
Your Turn

a) Sketch the graph of the function $y = -2\sqrt{x + 3} - 1$ by transforming the graph of $y = \sqrt{x}$.

b) Identify the domain and range of $y = \sqrt{x}$ and describe how they are affected by the transformations.

D: $x \geq -3$

R: $y \leq -1$



$a = -2 \quad h = -3 \quad k = -1$

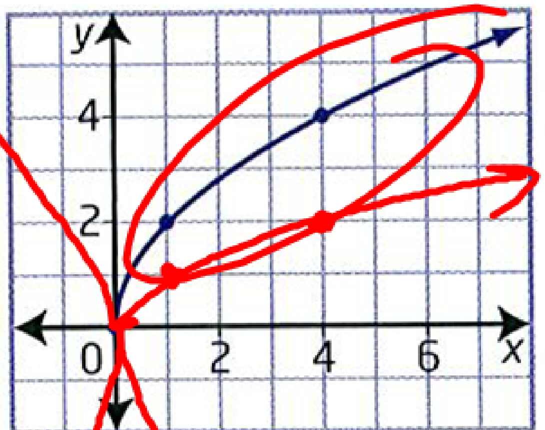
$(x - 3, -2y - 1)$

x	y
0	0
1	1
4	2
9	3

x	y
−3	−1
−2	−3
1	−5
6	−7

Determining a Radical Function From a Graph

Mayleen is designing a symmetrical pattern. She sketches the curve shown and wants to determine its equation and the equation of its reflection in each quadrant. The graph is a transformation of the graph of $y = \sqrt{x}$. What are the equations of the four functions Mayleen needs to work with?



I $y = 2\sqrt{x}$

II. $y = 2\sqrt{-x}$

III $y = -2\sqrt{-x}$

IV $y = -2\sqrt{x}$

$$\left(\frac{1}{2}\right)^2 = \left(\sqrt{b}\right)^2$$

Your Turn

a) Determine two forms of the equation for the function shown. The function is a transformation of the function $y = \sqrt{x}$.

$$\frac{1}{4} = b$$

b) Show algebraically that the two equations are equivalent.

c) What is the equation of the curve reflected in each quadrant?

$$y = \frac{1}{2}\sqrt{x}$$

$$1 = a\sqrt{4}$$

$$1 = a(2)$$

$$y = \sqrt{\frac{1}{4}x}$$

$$1 = \sqrt{b(4)}$$

$$1 = 2\sqrt{b}$$

Your Turn

A company estimates its cost of production using the function $C(n) = 20\sqrt{n} + 1000$, where C represents the cost, in dollars, to produce n items.

a) Describe the transformations represented by this function as compared to $C = \sqrt{n}$.

b) Graph the function using technology. What does the shape of the graph imply about the situation?

c) Interpret the domain and range in this context.

d) Use the graph to determine the expected cost to produce 12 000 items.

a) $C(n) = 20\sqrt{n} + 1000$

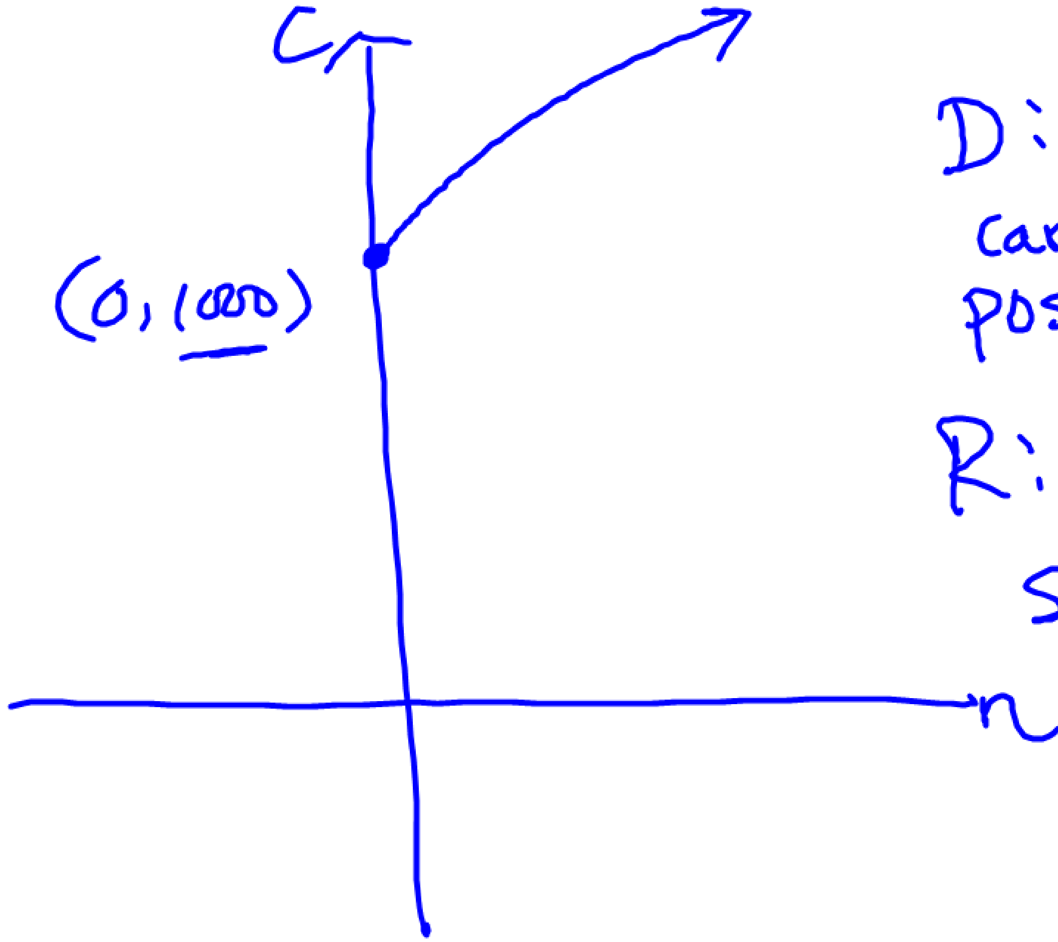
$a = 20$ vert. stretch by factor of 20

$K = 1000$ shifted 1000 units up.

$(x, 20y + 1000)$

x	y
0	0
1	1
4	2
9	3

x	y
0	1000
1	1020
4	1040
9	1060



D: $n \geq 0$

can only make positive # of items

R: $C \geq 1000$

set up cost is $\$1000$ to produce "n" items

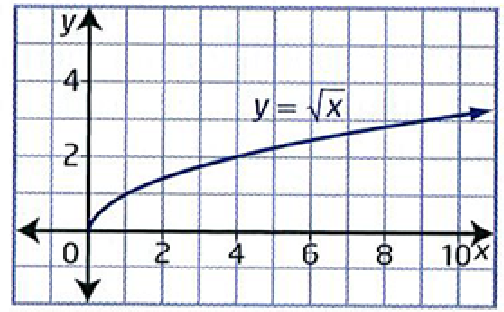
$$C(n) = 20\sqrt{n} + 1000$$

$$C(12000) = 20\sqrt{12000} + 1000$$

$$= \$3190.89$$

Key Ideas

- The base radical function is $y = \sqrt{x}$. Its graph has the following characteristics:
  - a left endpoint at (0, 0)
  - no right endpoint
  - the shape of half of a parabola
  - a domain of $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and a range of $\{y \mid y \geq 0, y \in \mathrm{R}\}$



- You can graph radical functions of the form $y = a\sqrt{b(x - h)} + k$ by transforming the base function $y = \sqrt{x}$.
- You can analyse transformations to identify the domain and range of a radical function of the form $y = a\sqrt{b(x - h)} + k$.

How does each parameter affect the graph of $y = \sqrt{x}$?

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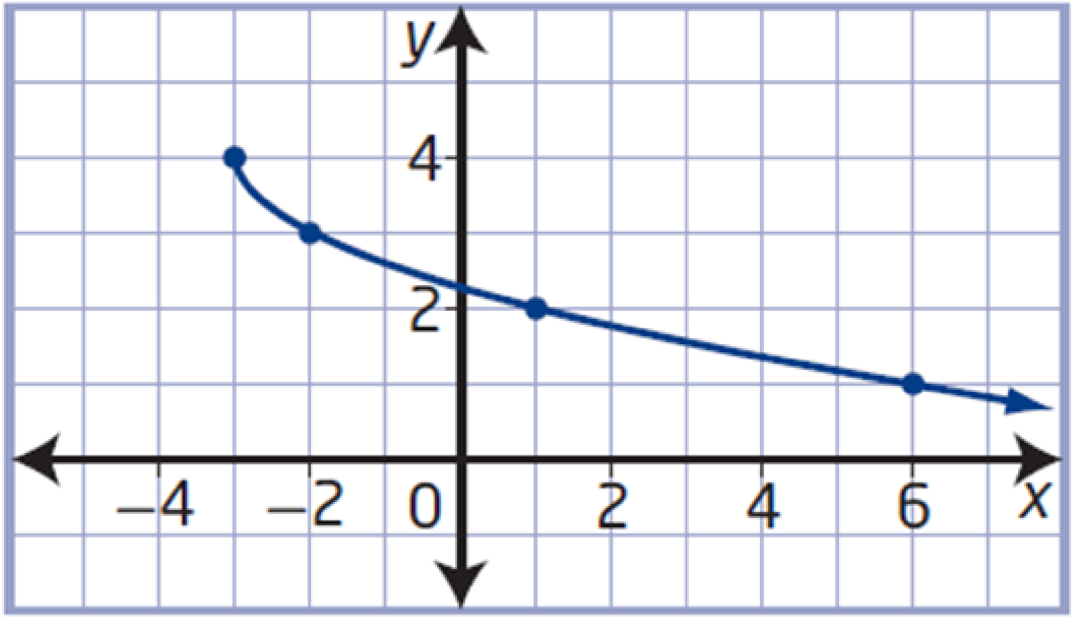
#'s 1a,b,c, 2,3,5a-d,10,12,16

a)



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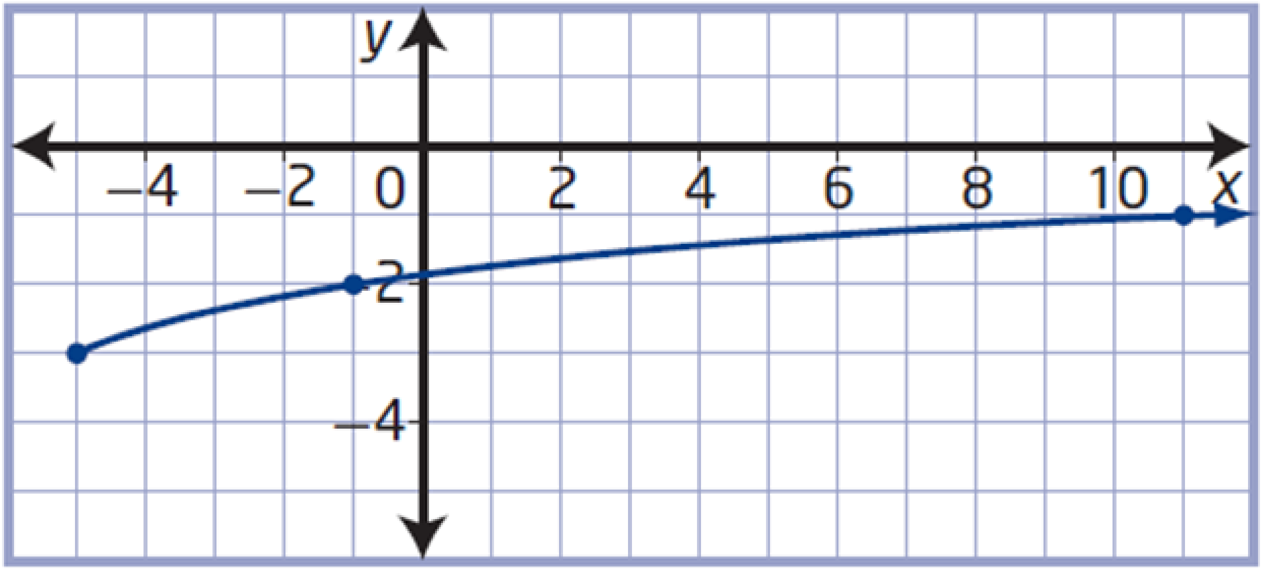
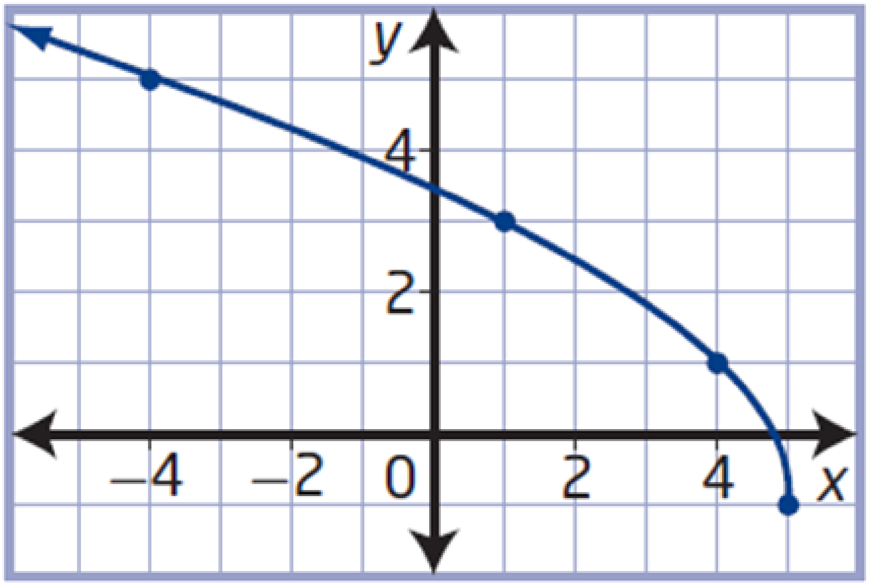


c)



d)