Differential Equations

2.1 Modelling Situations With Differential Equations

Introduction to Differential Equations

What is a Differential Equation?

Differential equations are equations that involve x, y, *and the derivatives of* y.

$$y'' + 2y' = 3y \qquad f''(x) + 2f'(x) = 3f(x) \qquad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 3y$$

We will be interpreting verbal statements that represent differential equations.

Writing A Differential Equation

Variation

Let's recall:

Direct Variation:

"y varies directly as x" or "y is directly proportional to x"

$$y = kx \text{ or } f(x) = kx$$

k is the constant of variation or constant of proportionality

Inverse Variation:

" y varies inversely as x" or "y is inversely proportional to x"

$$y = \frac{k}{x} \text{ or } f(x) = \frac{k}{x}$$

Ex. 1

Katherine runs around a linear path. Her position is given by the function P(t), where t is measured in seconds since she started running and P is measured in miles. During the first second of her run, Katherine's acceleration is proportional to the cube root of the time since she started running. Write a differential equation that describes this relationship, where k is a positive constant of proportionality.

$P(t) = \text{position}$

$\text{velocity} = \dfrac{dP}{dt}$

$\text{acceleration} = \dfrac{d^2P}{dt^2}$

$$\frac{d^2P}{dt^2} = k\sqrt[3]{t}$$

B

Ex. 2

derivative k B

Bacteria in a certain culture increase at a rate proportional to the number present. If the number of bacteria doubles in three hours, in how many hours will the number of bacteria triple?

Write a differential equation that describes this relationship, where k is a positive constant of proportionality

$$\frac{dB}{dt} = kB$$

A rumor spreads among a population of N people at a rate proportional to the product of the number of people who have heard the rumor and the number of people who have not heard the rumor. If p denotes the number of people who have heard the rumor, which of the following differential equations could be used to model this situation with respect to time t, where k is a positive constant?

derv.

k

multi.

(A) $\frac{dp}{dt} = kp$

(B) $\frac{dp}{dt} = kp(N-p)$

(C) $\frac{dp}{dt} = kp(p-N)$

(D) $\frac{dp}{dt} = kt(N-t)$

(E) $\frac{dp}{dt} = kt(t-N)$

$$\frac{dp}{dt} = kp(N-p)$$

Your Turn

The rate of change of A with respect to t is proportional to r

$$\frac{dA}{dt} = kr$$

The rate of change of G with respect to t is proportional to the square of G.

$$\frac{dG}{dt} = kG^2$$

The rate of growth of a population of zombies, z, over time is proportional to the population of zombies.

$$\frac{dz}{dt} = kz$$

The rate of change of J with respect to t is inversely proportional to the square of J.

$$\frac{dJ}{dt} = \frac{k}{J^2}$$

In a community of F farmers, the number x of farmers who own a certain combine changes with respect to time t at a rate that is jointly proportional to the number of farmers who own the combine and to the number of farmers who do not own the combine.

$$\frac{dx}{dt} = kx(F-x)$$

Suppose a virus is spreading in a network of computers. There are *M* computers in the network. The rate of change in the number of infected computers is jointly proportional to the number of computers already infected, *j*, and the number of computers not yet infected.

$$\frac{dj}{dt} = kj(m-j)$$

The rate of change of a level of response R with respect to the level of a stimulus s is a joint proportionality between the level of the response and the inverse of the level of the stimulus. This DE is known as the Brentano-Stevens Law.

$$\frac{dR}{ds} = \frac{kR}{s}$$

Barometric pressure p (measured in millibars) changes with respect to altitude a (measured in feet above sea level) at a rate that is directly proportional to the altitude.

$$\frac{dP}{da} = ka$$

Assignment Handout