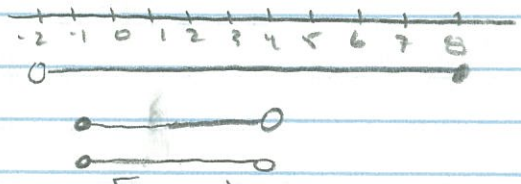
Calculus 30- 1.6- Chapter Review

1. a) $\sqrt{9} = 3 \rightarrow$ rational
   b) $\sqrt[3]{9} \rightarrow$ irrational
   c) 14, 2322322232223... $\rightarrow$ irrational
   d) $\frac{-11}{5} \rightarrow$ rational
   e) $5\pi \rightarrow$ irrational
   F) $e^3 \rightarrow$ irrational
   g) $-8.\overline{254} \rightarrow$ rational
   h) $\sqrt{-16} \rightarrow$ neither

2. Real numbers $\rightarrow$ all but h

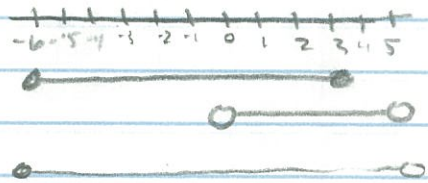
3. a) $x \leq -2 \rightarrow (-\infty, -2]$
   b) $-8 < x \leq 3 \rightarrow (-8, 3]$
   c) $x \geq \pi \rightarrow [\pi, \infty)$
   d) $10 < x < 11 \rightarrow (10, 11)$
   e) $\frac{2}{3} \leq x \leq \frac{3}{4} \rightarrow \left[\frac{2}{3}, \frac{3}{4}\right]$
   F) $-e \leq x < e \rightarrow [-e, e)$

4. a) $(-2, 8] \cap [-1, 4)$



$[-1, 4)$

b) $[-6, 3] \cup (0, 5)$



$[-6, 5)$

5. a) $d^2 - 8d - 9$
   $= (d-9)(d+1)$

   b) $18v^2 + 45v - 50$
   $18v^2 + 60v - 15v - 50$
   $= 6v(3v+10) - 5(3v+10)$
   $= (6v-5)(3v+10)$

   c) $3x^2 - 15$
   $= 3(x^2 - 5)$
   $= 3(x+\sqrt{5})(x-\sqrt{5})$

   d) $j^3 + 125$ he has a 10
   $= (j+5)(j^2 - 5j + 25)$

   e) $8b^3 - 7$
   $= (2b - \sqrt[3]{7})(4b^2 + 2\sqrt[3]{7}b + \sqrt[3]{49})$

   f) $250s^3 - 10s$
   $= 10s(25s^2 - 1)$
   $= 10s(5s+1)(5s-1)$

   g) $x^2 - 8x + 9$
   $a = 1$
   $b = -8$
   $c = 9$
   $x = \frac{8 \pm \sqrt{64 - 4(1)(9)}}{2}$
   $x = \frac{8 \pm \sqrt{28}}{2}$
   $x = 4 \pm \sqrt{7}$
   $(x - 4 + \sqrt{7})(x - 4 - \sqrt{7})$

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h) $w^2 - 6w + 10$

$a=1$, $b=-6$, $c=10$

$x = \frac{6 \pm \sqrt{36 - 4(1)(10)}}{2}$

$x = \frac{6 \pm \sqrt{-4}}{2}$ prime

i) $x^{\frac{7}{3}} - 4x^{\frac{1}{3}}$

$= x^{\frac{1}{3}}(x^2 - 4)$

$= x^{\frac{1}{3}}(x+2)(x-2)$

j) $t^{-1} + 4t^{-2} + 4t^{-3}$

$t^{-3}(t^2 + 4t + 4)$

$= t^{-3}(t+2)(t+2)$

k) $2am^{\frac{5}{3}} - 6am^{\frac{2}{3}} + 4am^{-\frac{1}{3}}$

$= 2am^{-\frac{1}{3}}(m^2 - 3m + 2)$

$= 2am^{-\frac{1}{3}}(m-2)(m-1)$

l) $x^{\frac{3}{2}}y^{-\frac{3}{2}} + x^{\frac{1}{2}}y^{\frac{3}{2}}$

$= x^{\frac{1}{2}}y^{-\frac{3}{2}}(x^3 + y^3)$

$= x^{\frac{1}{2}}y^{-\frac{3}{2}}(x+y)(x^2 - xy + y^2)$

m) $6 - \frac{1}{6}y^2$

$= -\frac{1}{6}(y^2 - 36)$

$= -\frac{1}{6}(y+6)(y-6)$

n) $\frac{1}{5}t^2 + \frac{1}{30}t - \frac{1}{2}$

$= \frac{1}{30}(6t^2 + t - 15)$

$6t^2 + t - 15$

$6t^2 + 10t - 9t - 15$

$2t(3t+5) - 3(3t+5)$

$(2t-3)(3t+5)$

$= \frac{1}{30}(2t-3)(3t+5)$

o) $p^4 - 4$

$= (p^2+2)(p^2-2)$

$= (p^2+2)(p+\sqrt{2})(p-\sqrt{2})$

p) $h^7 - 1$

$= (h-1)(h^6 + h^5 + h^4 + h^3 + h^2 + h + 1)$

q) $x^3 - 3x^2 - 10x + 24$ ↙ calculate

$= (x-2)(x-4)(x+3)$

r) $x^4 + x^3 - 10x^2 - 4x + 24$

$= (x+3)(x+2)(x-2)^2$ ← calc

s) $2x^3 + 6x^2 - 18x - 54$

$= 2(x+3)^2(x-3)$ ← calc

6 a) $f(x) = \frac{x^3 - 9x}{x^2 - 7x + 12} = \frac{x(x^2-9)}{(x-3)(x-4)} = \frac{x(x+3)(x-3)}{(x-3)(x-4)}$ $N = \{0, -3, 3\}$ $D = \{3, 4\}$

a) zero → $\{0, -3\}$

b) undefined → $\{4\}$

c) indeterminate → $\{3\}$

b) $f(x) = \frac{x^3 - x^2 - 6x}{x^3 + x^2 - 4x - 4} = \frac{x(x+2)(x-3)}{(x+2)(x+1)(x-2)}$ $N = \{0, -2, 3\}$ $D = \{-2, -1, 2\}$

a) zero → $\{0, 3\}$

b) undefined → $\{-1, 2\}$

c) indeterminate → $\{-2\}$

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6 c) $F(x) = \frac{x(5^x-25)}{x^2-3x+2} = \frac{x(5^x-25)}{(x-2)(x-1)}$ $N=\{0,2\}$ $D=\{2,1\}$

a) zero → $\{0\}$

b) undefined → $\{1\}$

c) indeterminate → $\{2\}$

7 $F(x) = \frac{x^2-x-6}{x^2+5x+6} = \frac{(x-3)(x+2)}{(x+3)(x+2)}$ $N=\{3,-2\}$ $D=\{-3,-2\}$

a) x-intercept (zero) → $\{3\}$

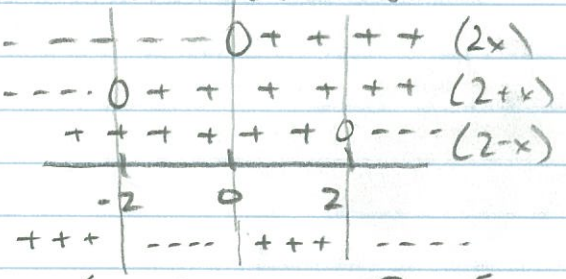
b) hole (indeterminate) → $\{-2\}$

c) V. asymptote (undefined) → $\{-3\}$

8. a) $8x-2x^3 \geq 0$

$2x(4-x^2) \geq 0$

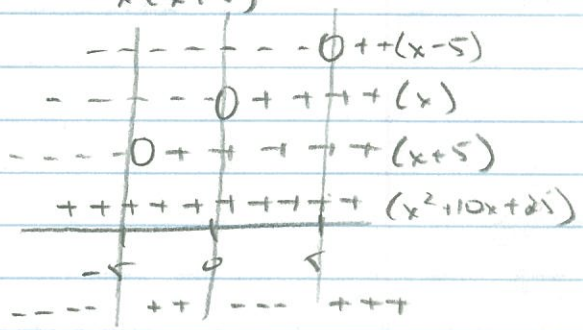
$2x(2+x)(2-x) \geq 0$



$\{x : x \in (-\infty,-2] \cup [0,2]\}$

b) $\frac{x^3-125}{x^2+5x} < 0$

$\frac{(x-5)(x^2+10x+25)}{x(x+5)} < 0$



$\{x : x \in (-\infty,-5) \cup (0,5)\}$

c) $\frac{1}{4} < \frac{3}{x+3} < \frac{1}{3}$

$4 > \frac{x+3}{3} > 3$

$4 > \frac{x+3}{3}$ $\quad \frac{x+3}{3} > 3$

$12 > x+3$ $\quad x+3 > 9$

$9 > x$ $\quad x > 6$

$x < 9$

$\{x : x \in (6,9)\}$

d) $\frac{2^x-4}{\log_2 x} > 0$

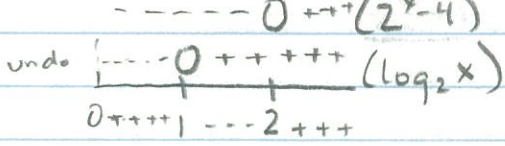
$\log_2 x = 0$

$2^0 = x$

$1 = x$ ✓

test: $\log_2(0.5) =$

$\log_2(3) = 1.6$



$\{x : x \in (0,1) \cup (2,\infty)\}$

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8 e) $|x-3| > 3$

$x-3 > 3$ $\quad$ $x-3 < -3$

$x > 6$ $\quad$ $x < 0$

$\{x : x \in (-\infty, 0) \cup (6, \infty)\}$

f) $\left|\frac{1}{2}x+4\right| \leq 10$

$-10 \leq \frac{1}{2}x+4 \leq 10$

$-10 \leq \frac{1}{2}x+4$ $\quad$ $\frac{1}{2}x+4 \leq 10$

$-20 \leq x+8$ $\quad$ $x+8 \leq 20$

$-28 \leq x$ $\quad$ $x \leq 12$

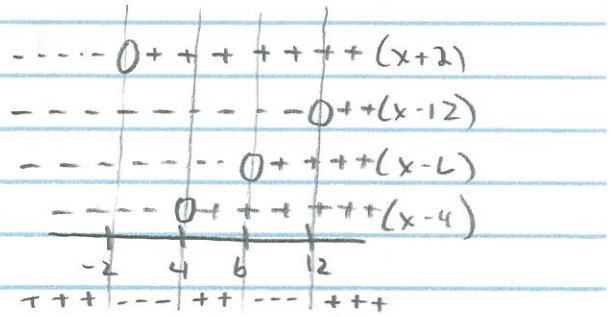
$\{x : x \in [-28, 12]\}$

g) $|x^2-10x| \geq 24$

$x^2-10x \geq 24$ $\quad$ $x^2-10 \leq -24$

$x^2-10x-24 \geq 0$ $\quad$ $x^2-10x+24 \leq 0$

$(x-12)(x+2) \geq 0$ $\quad$ $(x-6)(x-4) \leq 0$



$\{x : x \in (-\infty, -2] \cup [4, 6] \cup [12, \infty)\}$

h) $|2-3x| = 11$

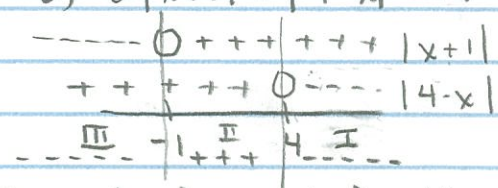
$2-3x = 11$ $\quad$ $2-3x = -11$

$-3x = 9$ $\quad$ $-3x = -13$

$x = -3$ $\quad$ $x = \frac{13}{3}$

$\{-3, \frac{13}{3}\}$

i) $2|x+1| + |4-x| = 14$



I $\rightarrow$ $2(x+1) + -(4-x) = 14$

$2x+2-4+x = 14$

$3x = 16$

$x = \frac{16}{3}$ ✓ lies in Region I

II $\rightarrow$ $2(x+1) + (4-x) = 14$

$2x+2+4-x = 14$

$x = 8$ ← does not lie in Region 2

III $\rightarrow$ $-2(x+1) + (4-x) = 14$

$-2x-2+4-x = 14$

$-3x = 12$

$x = -4$ ✓ lies in Region 3

$\{-4, \frac{16}{3}\}$

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8 j) $|x^2-3| \leq 2x$

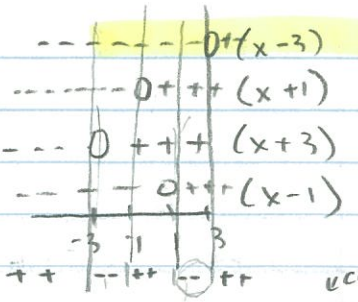
$-2x \leq x^2-3 \leq 2x$

$-2x \leq x^2-3$ $\quad x^2-3 \leq 2x$

$0 \leq x^2+2x-3$ $\quad x^2-2x-3 \leq 0$

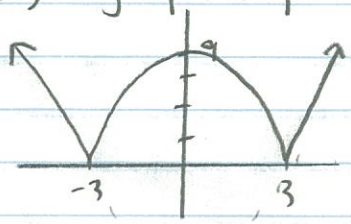
$0 \leq (x+3)(x-1)$ $\quad (x-3)(x+1) \leq 0$

$x^2-3 \geq -2x$ $\quad x^2-3 \leq 2x$

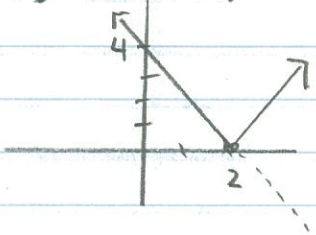


$\{x : x \in [1,3]\}$ ↙ can't be between $(-3,-1)$ because $2x$ would give -ve answer.

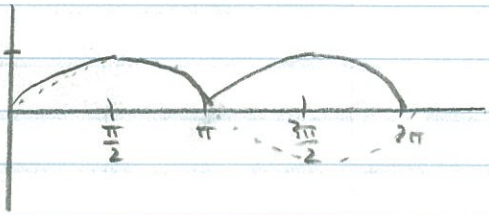
9. a) $y = |9-x^2|$



b) $y = |2x-4|$



c) $y = |\sin x|$ for $x \in [0, 2\pi)$



10. a) $|5-7x| = \begin{cases} 5-7x & x \in (-\infty, \frac{5}{7}] \\ -(5-7x) & x \in (\frac{5}{7}, \infty) \end{cases}$

b) $|x^2+3x+1| = \begin{cases} (x^2+3x+1) & x \in (-\infty, \frac{-3-\sqrt{5}}{2}] \cup [\frac{-3+\sqrt{5}}{2}, \infty) \\ -(x^2+3x+1) & x \in (\frac{-3-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}) \end{cases}$

$a=1$ $\quad x = \frac{-3 \pm \sqrt{9-4(1)(1)}}{2}$

$b=3$

$c=1$ $\quad x = \frac{-3 \pm \sqrt{5}}{2}$

$x + \frac{3+\sqrt{5}}{2}$

$x + \frac{3-\sqrt{5}}{2}$

-2.62 -0.38

c) $\left|\frac{x+6}{6-x}\right| = \begin{cases} \left(\frac{x+6}{6-x}\right) & x \in [-6, 6) \\ -\left(\frac{x+6}{6-x}\right) & x \in (-\infty, -6) \cup (6, \infty) \end{cases}$ ↙ undefined at 6

$x+6$

$6-x$

-6 6

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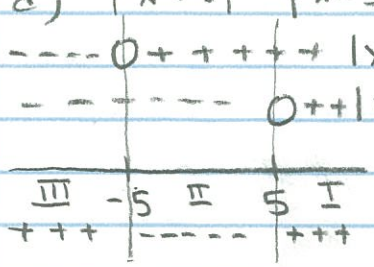
10 d) $|x+5| - |x-5| = \begin{cases} 10, & x \in [5, \infty) \\ 2x, & x \in (-5, 5) \\ -10, & x \in (-\infty, -5] \end{cases}$



I → $(x+5) - (x-5)$
$= x+5-x+5$
$= 10$

II → $(x+5) - -(x-5)$
$= x+5+x-5$
$= 2x$

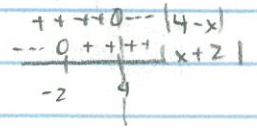
III → $-(x+5) - (-(x-5))$
$= -x-5+x-5$
$= -10$

11. $|-4-(-3)|$
$= |-4+3|$
$= |-1| = 1$ (C)

12. $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ x, & \text{if } x < 0 \end{cases}$
(B)

13. $\sqrt{x^2-6x+9}$
(D) $= \sqrt{(x-3)^2}$
$= |x-3|$

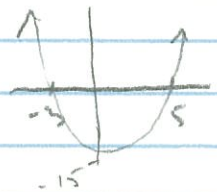
14. $|4-x| - |x+2|$ if $x > 4$
(G) $-(4-x) - (x+2)$
$-4+x-x-2 = (-6)$



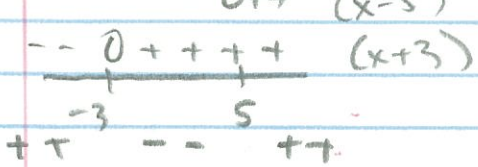
15. $|2-x| \geq 10$
$2-x \geq 10$ $\quad 2-x \leq -10$
$-8 \geq x$ $\quad 12 \leq x$
$x \leq -8$ $\quad x \geq 12$

$\{x : x \in (-\infty, -8) \cup (12, \infty)\}$
(H)

16. $y = x^2 - 2x - 15$
$y = (x-5)(x+3)$
$x = 5 \quad x = -3$



below x axis → $(-3, 5)$
(F)



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17. $|25-x^2| = 25-x^2 \quad x \in [-5,5]$ (F)

$(5+x)$: − − − 0 + + + + +

$(5-x)$: + + + + 0 − − −

	−5		5	
$(5+x)$	−	+	+	+
$(5-x)$	+	+	0	−
	−	+	0	−

18. $|f(x)| = -f(x) \quad x \in (-2,0) \cup (4,\infty)$ (H)

		−2		0		4	
$f(x)$	+	+	−	−	0	+	+
$-f(x)$	−	−	+	+	0	−	−
	−	−	0	+	0	−	−

19. height between 160 - 200cm.

$h-180 \leq 20$ (J)

Proof: $-20 \leq h-180 \leq 20$

$-20 \leq h-180$ → $160 \leq h$ → $h \geq 160$

$h-180 \leq 20$ → $h \leq 200$

$[160, 200]$

20. D. $16x^2-7$ irrational, not rational

$= (4x+\sqrt{7})(4x-\sqrt{7})$