

(17-35 odd, 42-55 odd, 59,60,61)

Absolute Values
1.S P 46 1-61

Calculus 30

1. $|47| = 47$

2. $|-33| = 33$

3. $|\sqrt{35}| = \sqrt{35}$

4. $|\sqrt{11}| = \sqrt{11}$

5. $|-8-19|$
 $= |-27|$
 $= 27$

6. $|-4-(-17)|$
 $= |-4+17|$
 $= |13|$
 $= 13$

7. $|\frac{2}{3} - \frac{3}{4}|$
 $= |\frac{8}{12} - \frac{9}{12}|$
 $= |\frac{-1}{12}|$
 $= \frac{1}{12}$

8. $|3\frac{2}{5} - 4\frac{1}{10}|$
 $= |\frac{17}{5} - \frac{41}{10}|$
 $= |\frac{34}{10} - \frac{41}{10}|$
 $= |\frac{-7}{10}|$
 $= \frac{7}{10}$

9. $|\sqrt{7}-7|$ ← represents neg. # because $\sqrt{7}-7 = -ve \#$

$= -(\sqrt{7}-7)$

$= -\sqrt{7}+7$

$= 7-\sqrt{7}$

10. $|\sqrt{14}-\sqrt{13}|$ ← positive #
 $= \sqrt{14}-\sqrt{13}$

11. $|\sqrt[3]{9}-\sqrt{10}|$ ← neg. #
 $= -(\sqrt[3]{9}-\sqrt{10})$
 $= \sqrt{10}-\sqrt[3]{9}$

12. $|2\pi-10|$ ← neg #
 $= -(2\pi-10)$
 $= 10-2\pi$

13. $|3e-1|$ ← pos #
 $= 3e-1$

14. $|2e-3\pi|$ ← neg #
 $= -(2e-3\pi)$
 $= 3\pi-2e$

15. $|\pi^2-e^2|$ ← pos # $= |-(\pi^2+e^2)|$
 $= \pi^2+e^2$

16. $|\pi-e-1|$ ← neg #
 $= -(\pi-e-1)$
 $= (1+e-\pi)$

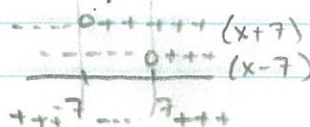
17. $|x-10| = \begin{cases} x-10, & x \in [10, \infty) \\ -(x-10), & x \in (-\infty, 10) \end{cases}$

18. $|x+4| = \begin{cases} x+4, & x \in [-4, \infty) \\ -(x+4), & x \in (-\infty, -4) \end{cases}$

19. $|3x+15| = \begin{cases} 3x+15, & x \in [-5, \infty) \\ -(3x+15), & x \in (-\infty, -5) \end{cases}$

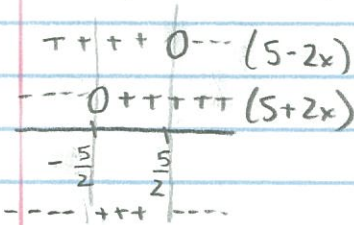
20. $|16-2x| = \begin{cases} 16-2x, & x \in (-\infty, 8] \\ -(16-2x), & x \in (8, \infty) \end{cases}$

21. $|x^2-49| = \begin{cases} x^2-49, & x \in (-\infty, -7] \cup [7, \infty) \\ -(x^2-49), & x \in (-7, 7) \end{cases}$

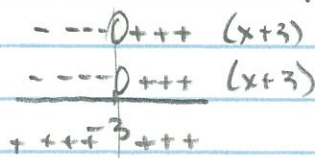


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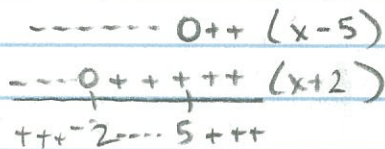
$$22. |25 - 4x^2| = \begin{cases} 25 - 4x^2, & x \in \left[-\frac{5}{2}, \frac{5}{2}\right] \\ -(25 - 4x^2), & x \in (-\infty, -\frac{5}{2}] \cup (\frac{5}{2}, \infty) \end{cases}$$



$$23. |x^2 + 6x + 9| = \begin{cases} x^2 + 6x + 9, & x \in (-\infty, \infty) \\ -(x^2 + 6x + 9) \text{ never} \end{cases}$$



$$24. |x^2 - 3x - 10| = \begin{cases} x^2 - 3x - 10, & x \in (-\infty, -2] \cup [5, \infty) \\ -(x^2 - 3x - 10), & x \in (-2, 5) \end{cases}$$

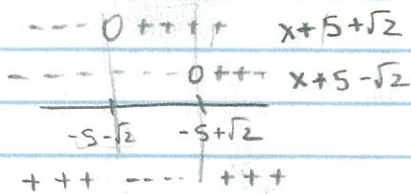


$$25. |x^2 + 10x + 23| = \begin{cases} x^2 + 10x + 23, & x \in (-\infty, -5 - \sqrt{2}] \cup [-5 + \sqrt{2}, \infty) \\ -(x^2 + 10x + 23), & x \in (-5 - \sqrt{2}, -5 + \sqrt{2}) \end{cases}$$

$$x = \frac{-10 \pm \sqrt{100 - 4(1)(23)}}{2}$$

$$x = \frac{-10 \pm \sqrt{3}}{2}$$

$$x = -5 \pm \sqrt{2}$$



$$26. |x^2 - 7x + 13| = \begin{cases} x^2 - 7x + 13, & x \in (-\infty, \infty) \\ -(x^2 - 7x + 13) \text{ never} \end{cases}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 - 52}}{2}$$

$$x = \frac{7 \pm \sqrt{-3}}{2}$$

Discriminant \rightarrow negative
 \therefore always positive

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27. $| -x^2 + 4x - 5 | = \begin{cases} -x^2 + 4x - 5, \text{ never} \\ -(-x^2 + 4x - 5), x \in (-\infty, \infty) \end{cases}$
 $D = b^2 - 4ac$
 $= (4)^2 - 4(-1)(-5)$
 $= 16 - 20 = -4 \leftarrow \text{always negative}$

28. $| 27 - x^3 | = \begin{cases} 27 - x^3, x \in (-\infty, 3] \\ -(27 - x^3), x \in (3, \infty) \end{cases}$
 $++++0----(3-x)$
 $++++++(9+3x+x^2)$
 $---3---$
 $D = b^2 - 4ac$
 $= (3)^2 - 4(9)(-1)$
 $= 9 - 36 = -27$
undefined at 0

29. $| \frac{x-2}{x} | = \begin{cases} \frac{x-2}{x}, x \in (-\infty, 0) \cup [2, \infty) \\ -(\frac{x-2}{x}), x \in (0, 2) \end{cases}$
 $-----0++x-2$
 $---0++++x$
 $0 \quad 2$
 $+++---+++$

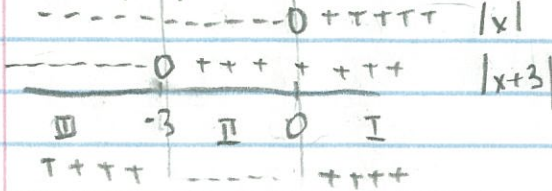
30. $| \frac{x^2-9}{x+1} | = \begin{cases} \frac{x^2-9}{x+1}, x \in [-3, 1) \cup [3, \infty) \\ -(\frac{x^2-9}{x+1}), x \in (-\infty, -3) \cup (-1, 3) \end{cases}$
 $-----0++++(x+3)$
 $-----0++(x-3)$
 $-----0++++(x+1)$
 $-3 \quad -1 \quad 3$
 $+++---+++$
undefined at -1

* 31. $| \frac{2x-5}{2x-5} | = \begin{cases} 1, x \in (\frac{5}{2}, \infty) \\ -1, x \in (-\infty, \frac{5}{2}) \end{cases}$
 $-----++++(2x-5)$
 $\frac{5}{2}$
undefined at 5/2

32. $| \frac{x^3+27}{x-3} | = \begin{cases} (\frac{x^3+27}{x-3}), x \in (-\infty, -3] \cup (3, \infty) \\ -(\frac{x^3+27}{x-3}), x \in (-3, 3) \end{cases}$
 $-----0++++(x+3)$
 $-----0+++ (x-3)$
 $++++++(x^2-3x+9)$
 $-3 \quad 3$
 $+++---+++$
undefined at 3

WS

33. $|x| + |x+3|$



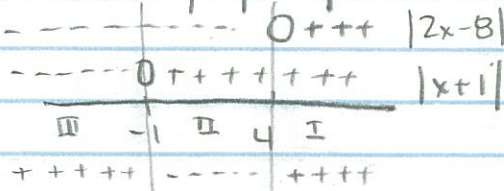
$$|x| + |x+3| = \begin{cases} 2x+3, & x \in [0, \infty) \\ 3, & x \in (-3, 0) \\ -2x-3, & x \in (-\infty, -3] \end{cases}$$

I $\rightarrow (x) + (x+3) = 2x+3$

II $\rightarrow -(x) + (x+3) = 3$

III $\rightarrow -(x) + -(x+3) = -2x-3$

34. $|2x-8| - |x+1|$



$$|2x-8| - |x+1| = \begin{cases} x-9, & x \in [4, \infty) \\ -3x+7, & x \in (-1, 4) \\ -x+9, & x \in (-\infty, -1] \end{cases}$$

I $\rightarrow (2x-8) - (x+1) = x-9$

II $\rightarrow -(2x-8) - (x+1) = -3x+7$

III $\rightarrow -(2x-8) - -(x+1) = -x+9$

35. $|x^2| - |2x|$

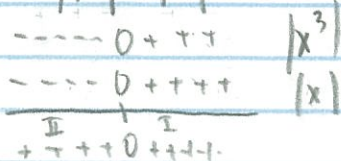


$$|x^2| - |2x| = \begin{cases} x^2-2x, & x \in [0, \infty) \\ x^2+2x, & x \in (-\infty, 0) \end{cases}$$

I $\rightarrow (x^2) - (2x) = x^2-2x$

II $\rightarrow (x^2) - -(2x) = x^2+2x$

36. $|x^3| + |x|$



$$|x^3| + |x| = \begin{cases} x^3+x, & x \in [0, \infty) \\ -x^3-x, & x \in (-\infty, 0) \end{cases}$$

I $\rightarrow (x^3) + (x) = x^3+x$

II $\rightarrow -(x^3) + -(x) = -x^3-x$

1.5

$|bc| = |b| |c|$

37. $|20 - 5x| = 5|x - 4|$
 $|-5(x - 4)| = 5|x - 4|$
 $|-5||x - 4| = 5|x - 4|$
 $5|x - 4| = 5|x - 4|$

38. $|x^3 - 4x^2| = x^2|x - 4|$
 $|x^2(x - 4)| = x^2|x - 4|$
 $|x^2||x - 4| = x^2|x - 4|$
 $x^2|x - 4| = x^2|x - 4|$

39. $\sqrt{16x^2 + 8x + 1} = |4x + 1|$
 $\sqrt{(4x + 1)^2} = |4x + 1|$
 $-(4x + 1) = |4x + 1|$
 $(\sqrt{b^2} = |b|)$

40. $\left| \frac{3 - x}{-2} \right| = \frac{|x - 3|}{2}$
 $\left| \frac{-1(x - 3)}{-2} \right| = \frac{|x - 3|}{2}$
 $\frac{|x - 3|}{2} = \frac{|x - 3|}{2}$

$\left(\frac{|b|}{|c|} = \frac{|b|}{|c|} \right)$

41. $|b - a| = |a - b|$
 $|-1(a - b)| = |a - b|$
 $|a - b| = |a - b|$

42. $|-b^3| = |b^3|$
 $|-1| |b^3| = |b^3|$
 $|b^3| = |b^3|$

43. $|4 - 5x| = 24$
 $4 - 5x = 24$ $4 - 5x = -24$
 $-5x = 20$ $-5x = -28$
 $x = -4$ $x = \frac{28}{5}$

$\left\{ -4, \frac{28}{5} \right\}$

44. $|x^2 + 11x| = 10$
 $x^2 + 11x = 10$ $x^2 + 11x = -10$
 $x^2 + 11x - 10 = 0$ $x^2 + 11x + 10 = 0$
 $x = \frac{-11 \pm \sqrt{11^2 - 4(1)(-10)}}{2}$ $(x + 1)(x + 10) = 0$
 $x = \frac{-11 \pm \sqrt{161}}{2}$ $x = -1$ $x = -10$

$\left\{ -1, -10, \frac{-11 \pm \sqrt{161}}{2} \right\}$

45. $|3x - x^2| = -4$
 \emptyset can never equal negative number

46. $|3x - 7| = 2x + 2$
 $3x - 7 = 2x + 2$ $3x - 7 = -(2x + 2)$
 $x = 9$ $3x - 7 = -2x - 2$
 $5x = 5$
 $x = 1$
 $\{1, 9\}$

47. $|2 - x| + |x + 5| = 7$
 $++ + + + + 0 \dots \dots \dots (2 - x)$
 $\dots \dots \dots 0 + + + + + + + |x + 5|$
 $\text{II} \quad -5 \quad \text{II} \quad 2 \quad \text{I}$

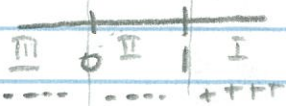
I $\rightarrow -(2 - x) + (x + 5) = 7$ II $\rightarrow (2 - x) + (x + 5) = 7$
 $-2 + x + x + 5 = 7$ $2 - x + x + 5 = 7$
 $2x = 4$ all # work
 $x = 2$ III $\rightarrow (2 - x) - (x + 5) = 7$
 $2 - x - x - 5 = 7$
 $-2x = 10$
 $x = -5$
 $[-5, 2]$ because right at $x = -5$

1.5

48. $|x^2 - 3|x-1|| = 1$

+++0+++ $|x^2|$

---0++ $|x-1|$



I $\rightarrow (x^2) - 3(x-1) = 1$

$x^2 - 3x + 3 = 1$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2 \quad x = 1$

II $\rightarrow (x^2) - -3(x-1) = 1$

III $x^2 + 3x - 3 = 1$

$x^2 + 3x - 4 = 1$

$(x+4)(x-1) = 0$

$x = -4 \quad x = 1$

^{part of region 3}

$\{-4, 1, 2\}$

49. $|1-2x| \leq 11$

$-11 \leq 1-2x \leq 11$

$-11 \leq 1-2x \quad 1-2x \leq 11$

$2x \leq 12$

$x \leq 6$

$-2x \leq 10$

$x \geq -5$

$\{x: x \in [-5, 6]\}$

50. $|5x+1| \geq 6$

$5x+1 \geq 6 \quad \text{or} \quad 5x+1 \leq -6$

$5x \geq 5$

$x \geq 1$

$5x \leq -7$

$x \leq -7/5$

$\{x: (-\infty, -7/5] \cup [1, \infty)\}$

51. $|5-x^2| \geq 4$

$5-x^2 > 4$

$1-x^2 > 0$

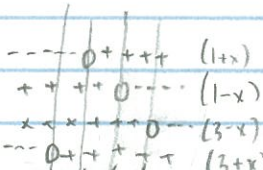
$(1+x)(1-x) > 0$

$\{x: x \in (-\infty, -3) \cup (-1, 1) \cup (3, \infty)\}$

$5-x^2 < -4$

$9-x^2 < 0$

$(3+x)(3-x) < 0$



52. $|x^2+x-12| > -1$

is true for all values of x.

$\{x: x \in (-\infty, \infty)\}$

53. $|x^2-5x| < 6$

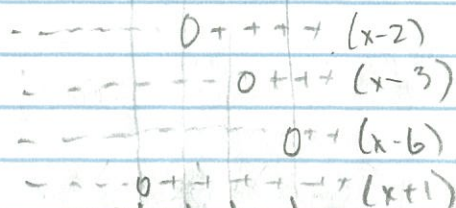
$-6 < x^2-5x < 6$

$-6 < x^2-5x \quad x^2-5x < 6$

$0 < x^2-5x+6 \quad x^2-5x-6 < 0$

$x^2-5x+6 > 0 \quad (x-6)(x+1) < 0$

$(x-2)(x-3) > 0$



$\{x: x \in (-1, 2) \cup (3, 6)\}$

1.5
54. $\left| \frac{1}{x} \right| \geq 5$

$$\frac{1}{x} \geq 5 \quad \frac{1}{x} \leq -5$$

$$1 \geq 5x \quad 1 \leq -5x$$

$$\frac{1}{5} \geq x \quad -\frac{1}{5} \leq x$$

← undefined at 0

$$\{x: x \in \left[-\frac{1}{5}, 0\right) \cup \left(0, \frac{1}{5}\right]\}$$

55. $|2x-5| > x-1$

$$2x-5 > x-1$$

$$x > 4$$

$$2x-5 < -(x-1)$$

$$2x-5 < -x+1$$

$$3x < 6$$

$$x < 2$$

$$\{x: (-\infty, 2) \cup (4, \infty)\}$$

56. $\left| \frac{x}{x+3} \right| > \frac{1}{2}$

tough one →

if $x+3$ is positive, $x > -3$ if $x+3$ is positive, $x > -3$

$$\frac{x}{x+3} > \frac{1}{2} \quad \frac{x}{x+3} < -\frac{1}{2}$$

$$x+3 > 2x \quad -x-3 < 2x$$

$$-x > -3$$

$$-3x < 3$$

$$x < 3 \quad \leftarrow \text{not possible}$$

$$x > -1 \quad \leftarrow \text{not possible}$$

if $x+3$ is negative

$$x < -3$$

if $x+3$ is negative

$$x < -3$$

$$\frac{x}{x+3} < \frac{1}{2}$$

$$\frac{x}{x+3} > -\frac{1}{2}$$

note sign switching

$$x+3 < 2x$$

$$-x-3 > 2x$$

$$-x < -3$$

$$-3x > 3$$

$$x > 3$$

$$x < -1$$

$$(-\infty, -3) \cup (3, \infty)$$

$$(-3, -1)$$

$$\{x: x \in (-\infty, -3) \cup (-3, -1) \cup (3, \infty)\}$$

57. $\left| \frac{x}{2-x} \right| \leq 3$

if $2-x$ is positive $x < 2$

$$\frac{x}{2-x} \leq 3$$

$$x \leq 3(2-x)$$

$$x \leq 6-3x$$

$$4x \leq 6$$

$$x \leq \frac{3}{2}$$

if $2-x$ is neg. $x > 2$

$$\frac{x}{2-x} \geq 3$$

$$x \geq 6-3x$$

$$4x \geq 6$$

$$x \geq \frac{3}{2}$$

if $2-x$ is pos. $x < 2$

$$-3 \leq \frac{x}{2-x}$$

$$-6+3x \leq x$$

$$-6 \leq -2x$$

$$3 \geq x$$

if $2-x$ is neg $x > 2$

$$-3 \geq \frac{x}{2-x}$$

$$-3(2-x) \geq x$$

$$-6+3x \geq x$$

$$-6 \geq -2x$$

$$3 \leq x$$

$$\{x: x \in (-\infty, \frac{3}{2}] \cup [3, \infty)\}$$

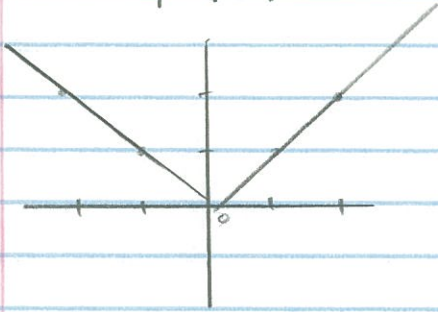
$$(3, \infty)$$

Hilroy

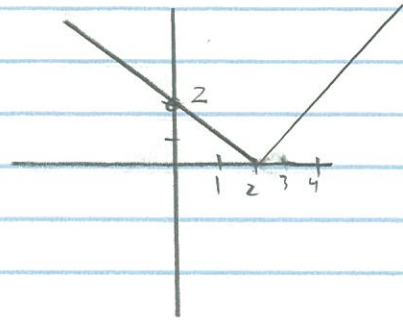
7
check

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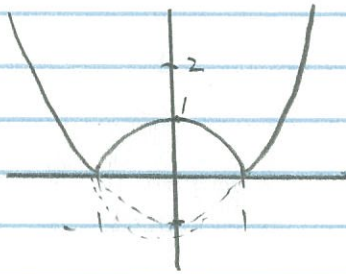
58. $y = |x|$



59. $y = |2-x|$



60. $y = |x^2 - 1|$



$$y = x^2 - 2x - 3$$
$$y = x^2 - 2x + 1 - 1 - 3$$
$$y = (x-1)^2 - 4$$

$V(1, -4)$

61. $y = |x^2 - 2x - 3|$

