

Solving Inequalities

Calculus 30 1.4 P. 33 1-21

1. $2x - 6 > \frac{7}{2}x - 4$

$4x - 12 > 7x - 8$

$-3x > 4$

$x < -\frac{4}{3}$

$\{x: x \in (-\infty, -\frac{4}{3})\}$

2. $-2(1-3x) \leq 7+9x$

$-2+6x \leq 7+9x$

$-3x \leq 9$

$x \geq -3$

$\{x: x \in [-3, \infty)\}$

3. $-7 < 2x+9 \leq 1$

$-7 < 2x+9$

$-2x < 16$

$x > -8$

$2x+9 \leq 1$

$2x \leq -8$

$x \leq -4$

$\{x: x \in (-8, -4]\}$

4. $10 \geq 4-3x \geq 1$

$10 \geq 4-3x$

$3x \geq -6$

$x \geq -2$

$4-3x \geq 1$

$-3x \geq -3$

$x \leq 1$

$\{x: x \in [-2, 1]\}$

5. $\frac{1}{21} < \frac{3}{x} < \frac{1}{6}$

reciprocal property \rightarrow

$21 > \frac{x}{3} > 6$

$21 > \frac{x}{3} \quad \frac{x}{3} > 6$

$63 > x \quad x > 18$

$x < 63$

$\{x: x \in (18, 63)\}$

6. $-\frac{1}{2} \geq \frac{6}{x-2} \geq -3$

reciprocal

property \rightarrow

$-2 \leq \frac{x-2}{6} \leq -\frac{1}{3}$

$-2 \leq \frac{x-2}{6}$

$-12 \leq x-2$

$-10 \leq x$

$x \geq -10$

$\frac{x-2}{6} \leq -\frac{1}{3}$

$3(x-2) \leq -6$

$3x-6 \leq -6$

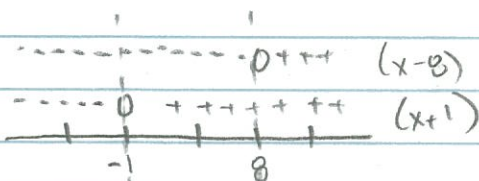
$3x \leq 0$

$x \leq 0$

$\{x: x \in [-10, 0]\}$

7. $x^2 - 7x - 8 \leq 0$

$(x-8)(x+1) \leq 0$



$\{x: x \in [-1, 8]\}$

8. $x^2 > 16$

$x^2 - 16 > 0$

$(x+4)(x-4) > 0$

Number line for $(x+4)(x-4) > 0$. Roots are at $x = -4$ and $x = 4$. The regions $x < -4$ and $x > 4$ are shaded with '+' signs, indicating the inequality is satisfied there.

Number line for $(x+4)(x-4) > 0$. Roots are at $x = -4$ and $x = 4$. The regions $x < -4$ and $x > 4$ are shaded with '+' signs, indicating the inequality is satisfied there.

$\{x: x \in (-\infty, -4) \cup (4, \infty)\}$

1.4 - Continued

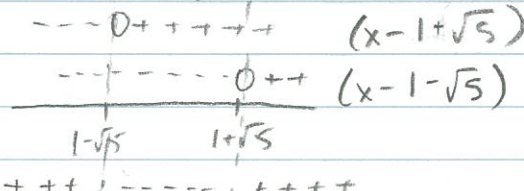
9. $x^2 - 2x - 4 \geq 0$

$a=1$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$b=-2$ $x = \frac{2 \pm \sqrt{4+16}}{2}$

$c=-4$

$x = 1 \pm \sqrt{5}$



$\{x: x \in (-\infty, 1-\sqrt{5}] \cup [1+\sqrt{5}, \infty)\}$

10. $-x^2 + 4x \geq 7$

$0 \geq x^2 - 4x + 7$

$d = b^2 - 4ac = 16 - 4(1)(7)$

$\phi = -12 \leftarrow \text{complex}$

11. $x^3 - 20x < 0$

$x(x^2 - 20) < 0$

$x(x+2\sqrt{5})(x-2\sqrt{5}) < 0$

Number line for problem 11. Roots are at $-2\sqrt{5}$, 0 , and $2\sqrt{5}$. The regions $(-\infty, -2\sqrt{5})$ and $(0, 2\sqrt{5})$ are shaded positive, while the regions $(-2\sqrt{5}, 0)$ and $(2\sqrt{5}, \infty)$ are shaded negative.

$\{x: x \in (-\infty, -2\sqrt{5}) \cup (0, 2\sqrt{5})\}$

12. $x^3 + 2x^2 > 0$

$x^2(x+2) > 0$

Number line for problem 12. Roots are at -2 and 0 . The region $(-2, 0)$ is shaded positive, and the region $(0, \infty)$ is also shaded positive. The region $(-\infty, -2)$ is shaded negative.

$\{x: x \in (-2, \infty), x \neq 0\}$

13. $x^3 - 5x^2 - 2x + 24 \geq 0$

$(x+2)(x-3)(x-4) \geq 0$

Number line for problem 13. Roots are at -2 , 3 , and 4 . The regions $(-2, 3)$ and $(4, \infty)$ are shaded positive, while the regions $(-\infty, -2)$ and $(3, 4)$ are shaded negative.

$\{x: x \in [-2, 3] \cup [4, \infty)\}$

14. $\frac{x^2}{x+2} < 0$

Number line for problem 14. Roots are at -2 and 0 . The region $(-2, 0)$ is shaded negative, and the region $(0, \infty)$ is shaded positive. The region $(-\infty, -2)$ is shaded positive.

$\{x: x \in (-\infty, -2)\}$ because undefined at -2

1.4 continued

15. $\frac{x+2}{x^2-x-2} \geq 0$

$\frac{x+2}{(x-2)(x+1)} \geq 0$

--- 0 + + + + + (x+2)
 --- 0 + + (x-2)
 --- 0 + + + + (x+1)

--- 2 --- 1 --- 2 (x+2)
 --- (x-2)(x+1)

$\{x: x \in [-2, -1) \cup (2, \infty)\}$ because undefined at 2, -1

16. $\frac{x+5}{3-x^2} \geq 0$

$\frac{x+5}{(\sqrt{3}+x)(\sqrt{3}-x)} \geq 0$

--- 0 + + + + + (x+5)
 --- 0 + + + + + (\sqrt{3}+x)
 + + + + + + - - - - - (\sqrt{3}-x)

--- -5 --- -\sqrt{3} --- \sqrt{3}

--- (x+5)

$\{x: x \in (-\infty, -5] \cup (-\sqrt{3}, \sqrt{3})\}$ undefined at $\sqrt{3}, -\sqrt{3}$ curved brackets

17. $\frac{x^3-4x}{x^2-5x} < 0$

$\frac{x(x+2)(x-2)}{x(x-5)} < 0$

--- 0 + + + + + (x)
 --- 0 + + + + + (x)
 --- 0 + + + + + (x+2)
 --- 0 + + + + + (x-2)
 --- 0 + + (x-5)

--- -2 --- 0 --- 2 --- 5 + + +

--- (x+2)(x-2)(x-5)

$\{x: x \in (-\infty, -2) \cup (2, 5)\}$ undefined at 0, 5

18. $\frac{x^2-25}{x^3+8} < 0$

$\frac{(x+5)(x-5)}{(x+2)(x^2-2x+4)} < 0$

--- 0 + + + + + (x+5)
 --- 0 + + (x-5)
 --- 0 + - + + + (x+2)
 + + + + + + + + + (x^2-2x+4)

--- -5 --- -2 --- 5

--- (x+5)(x-5)

$\{x: x \in (-\infty, -5) \cup (-2, 5)\}$ $D = b^2 - 4ac = 4 - 4 \cdot 1 \cdot 4 = -12$ no real root

1.4 Continued

19 $\frac{3^x - 9}{9 - x^2} \leq 0$

$\frac{3^x - 9}{(3+x)(3-x)} \leq 0$

--|---0+++ (3^x - 9) ← x=2

--0++++ (3+x)

++++0--- (3-x)



+++ , --- , +++ , ---

$\{x: x \in (-3, 2] \cup (3, \infty)\}$ ← undefined at 3, -3

answer: $\{x: x \in (0, 1) \cup (2, \infty)\}$

20 $\frac{\log x - \log 2}{2 \log x} > 0$

$\frac{\log \frac{x}{2}}{\log x^2} > 0$

Find roots:

$\log_{10} \frac{x}{2} = 0$

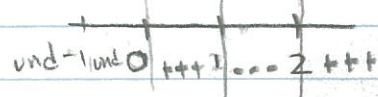
$\frac{x}{2} = 10^0$
x = 2

$\log_{10} x^2 = 0$

$x^2 = 10^0$
x = ±1

und ↔ und ---|---0+++ log $\frac{x}{2}$

+++ 0---und---0++++ log x²

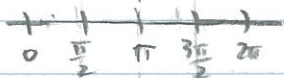


$\{x: x \in (0, 1) \cup (2, \infty)\}$

21. $\frac{1 - \sin x}{\cos x} > 0 \quad x \in [0, 2\pi]$

+++ 0+++ (1 - sin x) ← v //

+++ 0---0+++ cos x

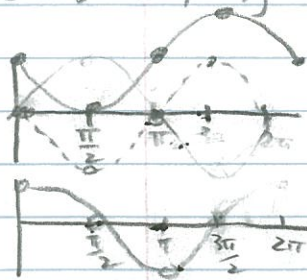


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$\{x: x \in [0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi]\}$ ← undefined at π/2, 3π/2; indeterminate at π

0 = -sin θ + 1

cos θ = 0



-sin x + 1

