Solving Inequalities

Calculus 30 1.4 P. 33 1-21

1. $2x-6 > \frac{7}{2}x-4$

$4x-12 > 7x-8$

$-3x > 4$

$x < \frac{-4}{3}$

$\{x: x \in (-\infty, -\frac{4}{3})\}$

2. $-2(1-3x) \leq 7+9x$

$-2+6x \leq 7+9x$

$-3x \leq 9$

$x \geq -3$

$\{x: x \in [-3, \infty)\}$

3. $-7 < 2x+9 \leq 1$

$-7 < 2x+9$	$2x+9 \leq 1$
$-2x < 16$	$2x \leq -8$
$x > -8$	$x \leq -4$

$\{x: x \in (-8, -4]\}$

4. $10 \geq 4-3x \geq 1$

$10 \geq 4-3x$	$4-3x \geq 1$
$3x \geq -6$	$-3x \geq -3$
$x \geq -2$	$x \leq 1$

$\{x: x \in [-2, 1]\}$

5. $\frac{1}{21} < \frac{3}{x} < \frac{1}{6}$

reciprocal property →

$21 > \frac{x}{3} > 6$

$21 > \frac{x}{3}$ $\quad \frac{x}{3} > 6$

$63 > x$ $\quad x > 18$

$x < 63$

$\{x: x \in (18, 63)\}$

6. $\frac{-1}{2} \geq \frac{6}{x-2} \geq -3$

reciprocal property →

$-2 \leq \frac{x-2}{6} \leq -\frac{1}{3}$

$-2 \leq \frac{x-2}{6}$ $\quad \frac{x-2}{6} \leq -\frac{1}{3}$

$-12 \leq x-2$ $\quad 3(x-2) \leq -6$

$-10 \leq x$ $\quad 3x-6 \leq -6$

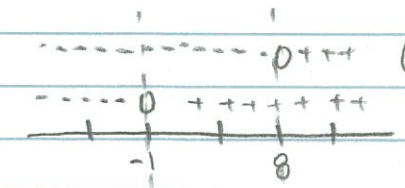
$x \geq -10$ $\quad 3x \leq 0$

$x \leq 0$

$\{x: x \in [-10, 0]\}$

7. $x^2-7x-8 \leq 0$

$(x-8)(x+1) \leq 0$

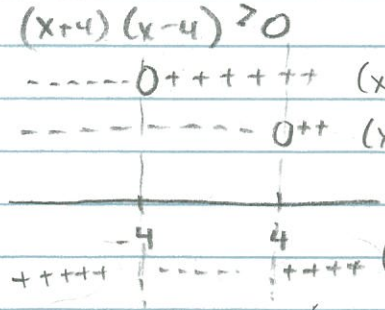


$\{x: x \in [-1, 8]\}$

8. $x^2 \geq 16$

$x^2-16 \geq 0$

$(x+4)(x-4) \geq 0$



$\{x: x \in (-\infty, -4) \cup (4, \infty)\}$

1.4 - Continued

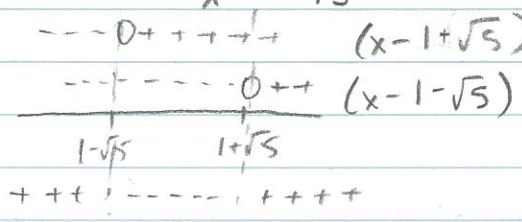
9. $x^2 - 2x - 4 \geq 0$

$a = 1$ $\quad x = \dfrac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$b = -2$

$c = -4$ $\quad x = \dfrac{2 \pm \sqrt{4+16}}{2}$

$x = 1 \pm \sqrt{5}$



$\{x : x \in (-\infty, 1-\sqrt{5}] \cup [1+\sqrt{5}, \infty)\}$

10. $-x^2 + 4x \geq 7$

$0 \geq x^2 - 4x + 7$

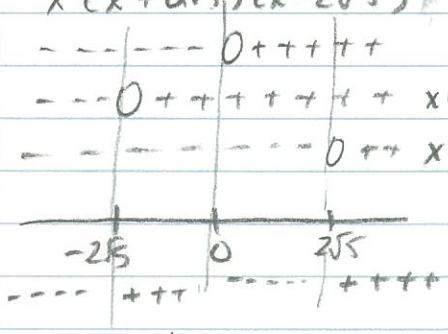
$d = b^2 - 4ac = 16 - 4(1)(7)$

$\emptyset = -12$ ← complex

11. $x^3 - 20x < 0$

$x(x^2 - 20) < 0$

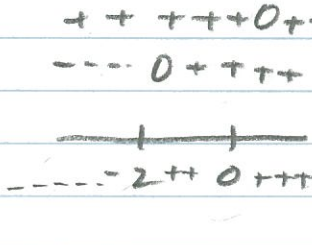
$x(x + 2\sqrt{5})(x - 2\sqrt{5}) < 0$



$\{x : x \in (-\infty, -2\sqrt{5}) \cup (0, 2\sqrt{5})\}$

12. $x^3 + 2x^2 > 0$

$x^2(x+2) > 0$

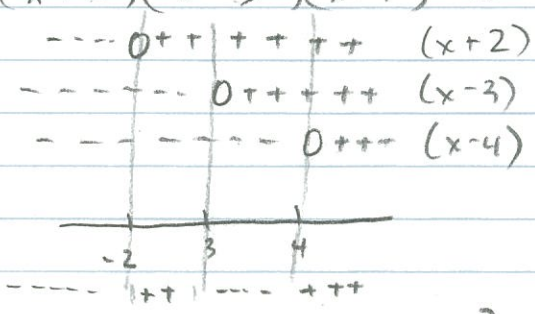


$(-2, 0) \cup (0, \infty)$

$\{x : x \in (-2, \infty), x \neq 0\}$

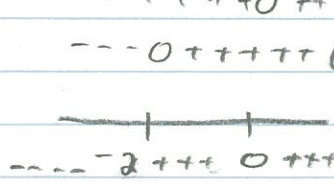
13. $x^3 - 5x^2 - 2x + 24 \geq 0$

$(x+2)(x-3)(x-4) \geq 0$



$\{x : x \in [-2, 3] \cup [4, \infty)\}$

14. $\dfrac{x^2}{x+2} < 0$

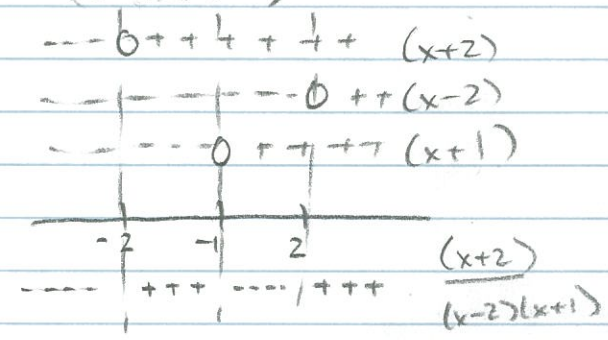


$\{x : x \in (-\infty, -2)\}$ because undefined at -2

1.4 continued

15. $\frac{x+2}{x^2-x-2} \geq 0$

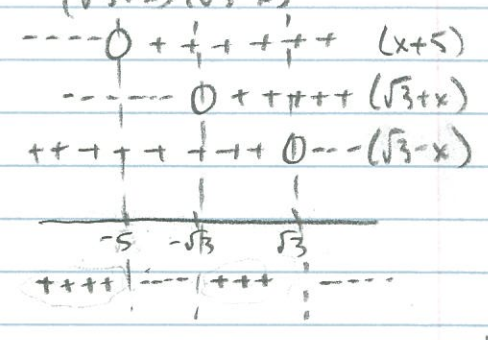
$\frac{x+2}{(x-2)(x+1)} \geq 0$



$\{x : x \in [-2, -1) \cup (2, \infty)\}$ because undefined at 2, -1

16. $\frac{x+5}{3-x^2} \geq 0$

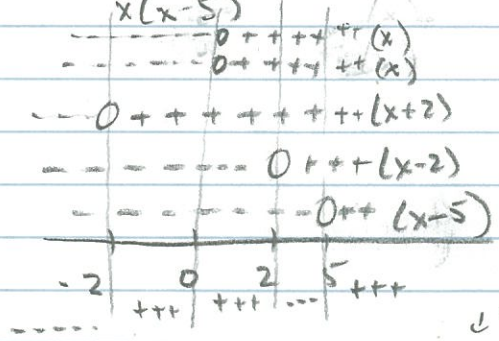
$\frac{x+5}{(\sqrt{3}+x)(\sqrt{3}-x)} \geq 0$



$\{x : x \in (-\infty, -5] \cup (-\sqrt{3}, \sqrt{3})\}$ ↑ curved bracket; undefined at $\sqrt{3}, -\sqrt{3}$

17. $\frac{x^3-4x}{x^2-5x} < 0$

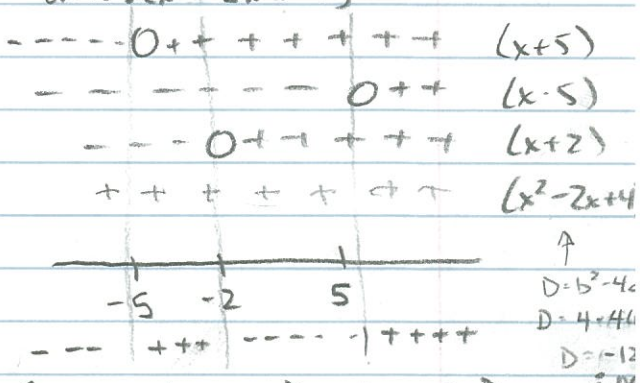
$\frac{x(x+2)(x-2)}{x(x-5)} < 0$



$\{x : x \in (-\infty, -2) \cup (2, 5)\}$ ↓ undefined at 0, 5

18. $\frac{x^2-25}{x^3+8} < 0$

$\frac{(x+5)(x-5)}{(x+2)(x^2-2x+4)} < 0$



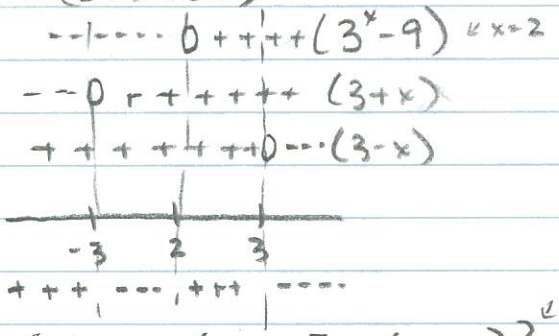
↑ $D = b^2 - 4c$, $D = 4 - 4(1)$, $D = -12$, ∴ no real roots

$\{x : x \in (-\infty, -5) \cup (-2, 5)\}$

1.4 Continued

19 $\frac{3^x - 9}{9 - x^2} \leq 0$

$\frac{3^x - 9}{(3+x)(3-x)} \leq 0$



$\{x: x \in (-3, 2] \cup (3, \infty)\}$ ↙ undefined at 3, -3

20 $\frac{\log x - \log 2}{2 \log x} > 0$

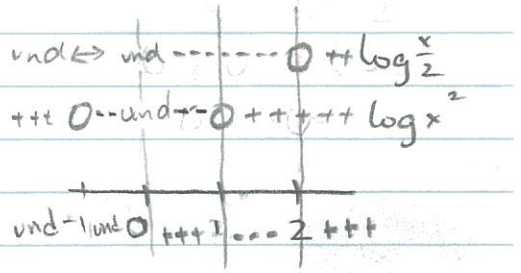
$\frac{\log \frac{x}{2}}{\log x^2} > 0$

Find roots:

$\log_{10} \frac{x}{2} = 0$ $\log_{10} x^2 = 0$

$\frac{x}{2} = 10^0$ $x^2 = 10^0$

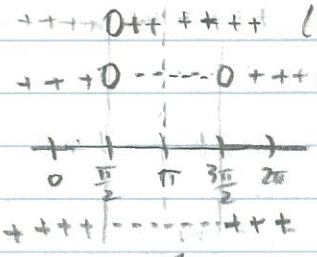
$x = 2$ $x = \pm 1$



$\{x: x \in (0,1) \cup (2, \infty)\}$

answer: $\{x: x \in (0,1) \cup (2, \infty)\}$

21. $\frac{1 - \sin x}{\cos x} > 0$ $x \in [0, 2\pi]$



$\{x: x \in [0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi]\}$ ↙ undefined at $\frac{\pi}{2}, \frac{3\pi}{2}$ indeterminate at $\frac{\pi}{2}$

$0 = -\sin\theta + 1$

$\cos\theta = 0$

$-\sin x + 1$