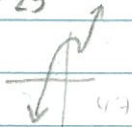


Calculus 30 - Unit 1 - Factoring

1.2 Page 15 1-11, 12a-c, g, 13, 14

1. a) $25 \in \mathbb{Q}$ T f) $\frac{8}{5} \in \mathbb{Q}$ F
 b) $25 \in \mathbb{R}$ T g) $\frac{8}{5} \in \mathbb{R}$ T
 c) $\sqrt{3} \in \mathbb{Q}$ F h) $\sqrt{5} \in \mathbb{R}$ T
 d) $\sqrt{3} \in \mathbb{Q}$ T i) $2 - \sqrt{3} \in \mathbb{Q}$ T
 e) $\sqrt{3} \in \mathbb{R}$ T j) $1 + \sqrt{5} \in \mathbb{R}$ F

2. a) T b) F c) T d) F e) T f) F g) F
 h) T i) T j) $D = b^2 - 4ac$ T k) T l) F
 $= 1 - 4(3)(-2) = 25$
 m) T n) only crosses x-axis once T o) T p) T
 q) F r) T



- 3 a) $49x^2 - 36 = (7x+6)(7x-6)$ b) $25x^2 - 6 = \text{prime}$ c) $25x^4 - 81 = (5x^2+9)(5x^2-9)$

d) $x^8 - 16 = (x^4+4)(x^4-4)$
 $= (x^4+4)(x^2+2)(x^2-2)$

- 4 a) $49x^2 + 36 = \text{prime}$ b) $25x^2 - 6 = (5x+\sqrt{6})(5x-\sqrt{6})$ c) $25x^4 - 81 = (5x^2+9)(5x^2-9)$
 $= (5x^2+9)(\sqrt{5}x+3)(\sqrt{5}x-3)$

Note:

1) $x = 2 + \sqrt{2}$

$(x - (2 + \sqrt{2}))$

$(x - 2 - \sqrt{2})$

d) $x^8 - 16$

$= (x^4+4)(x^4-4)$

$= (x^4+4)(x^2+2)(x^2-2)$

$= (x^4+4)(x^2+2)(x+\sqrt{2})(x-\sqrt{2})$

2) $x = -3 + \sqrt{2}$

$x = -3 + \sqrt{2}$

$(x + 3 - \sqrt{2})$

5. a) $x - 8$

b) $x + 4$

c) $x + \frac{3}{5}$

d) $x - \frac{9}{5}$

e) $x + 3 - \sqrt{2}$

f) $x + 1 + \sqrt{7}$

g) $x - \frac{1 - \sqrt{11}}{3}$

h) $x + \frac{4 + \sqrt{2}}{2}$

1.2 - Continued

6. Plug into calculator... $P(-1) = 0$ $P(1) = 12$
 $\therefore x+1$ is a factor

7. a) $x^2 - x - 56$
 $= (x-8)(x+7)$

b) $x^2 - 5x + 6$
 $= (x-2)(x-3)$

c) $6x^2 - x - 35$
 $= 6x^2 - 15x + 14x - 35$
 $= 3x(2x-5) + 7(2x-5)$
 $= (3x+7)(2x-5)$

d) $12x^2 - 23x + 10$
 $= 12x^2 - 15x - 8x + 10$
 $= 3x(4x-5) - 2(4x-5)$
 $= (4x-5)(3x-2)$

8. a) $6x^2 + 29x + 35$

$a=6$ $x = \frac{-29 \pm \sqrt{(29)^2 - 4(6)(35)}}{2(6)}$
 $b=29$
 $c=35$ $x = \frac{-29 \pm \sqrt{12}}$

$x = \frac{-29 \pm 12}{12}$

$x = -\frac{7}{3}$ or $-\frac{5}{2}$ $(3x+7)(2x+5)$

b) $16x^2 - 82x + 45$

$a=16$ $x = \frac{82 \pm \sqrt{82^2 - 4(16)(45)}}{2(16)}$
 $b=-82$
 $c=45$ $x = \frac{82 \pm \sqrt{3844}}{32}$

$x = \frac{82 \pm 62}{32}$

$x = \frac{9}{2}$ or $\frac{5}{8}$ $(2x-9)(8x-5)$

$x = -\frac{7}{3}$

$3x = -7$

$3x+7=0$

c) $5x^2 + 11x + 7$

$a=5$ $x = \frac{-11 \pm \sqrt{11^2 - 4(5)(7)}}{2(5)}$
 $b=11$
 $c=7$ $x = \frac{-11 \pm \sqrt{-19}}{10}$

prime

d) $60x^2 + 103x - 72$

$a=60$ $x = \frac{-103 \pm \sqrt{103^2 - 4(60)(-72)}}{2(60)}$
 $b=103$
 $c=-72$ $x = \frac{-103 \pm \sqrt{27889}}{120}$

$x = \frac{-103 \pm 167}{120}$

$x = \frac{8}{15}$ or $-\frac{9}{4}$ $(15x-8)(4x+9)$

e) $x^2 + 4x + 2$

$a=1$ $x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$
 $b=4$
 $c=2$ $x = \frac{-4 \pm \sqrt{8}}{2}$

$x = -2 \pm \sqrt{2}$ $(x+2-\sqrt{2})(x+2+\sqrt{2})$

f) $x^2 - 2x - 2$

$a=1$ $x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-2)}}{2(1)}$
 $b=-2$
 $c=-2$ $x = \frac{2 \pm \sqrt{12}}{2}$

$x = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$
 $(x-1+\sqrt{3})(x-1-\sqrt{3})$

1.2 - continued

8 g) $9x^2 - 6x - 1$
 $a=9$ $x = \frac{6 \pm \sqrt{(-6)^2 - 4(9)(-1)}}{2(9)}$
 $b=-6$ $x = \frac{6 \pm \sqrt{72}}{18}$
 $c=-1$ $x = \frac{1 \pm \sqrt{2}}{3}$
 $(x - \frac{1+\sqrt{2}}{3})(x - \frac{1-\sqrt{2}}{3})$

h) $3x^2 - 7x + 1$
 $a=3$ $x = \frac{7 \pm \sqrt{(-7)^2 - 4(3)(1)}}{2(3)}$
 $b=-7$ $x = \frac{7 \pm \sqrt{37}}{6}$
 $c=1$
 $(x - \frac{7+\sqrt{37}}{6})(x - \frac{7-\sqrt{37}}{6})$

9 a) $x^3 + 1$
 $= (x+1)(x^2 - x + 1)$

b) $x^3 - 1000$
 $= (x-10)(x^2 + 10x + 100)$

c) $64x^3 - 125$
 $= (4x-5)(16x^2 + 20x + 25)$

d) $8x^3 + 27$
 $= (2x+3)(4x^2 - 6x + 9)$

e) $x^5 - 1$
 $= (x-1)(x^4 + x^3 + x^2 + x + 1)$

f) $x^5 - 100,000$
 $= (x-10)(x^4 + 10x^3 + 100x^2 + 1000x + 10000)$

g) $x^5 - y^5$
 $= (x-y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

h) $x^7 - y^7$
 $= (x-y)(x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6)$

10 a) $x^3 - 2$
 $= (x - \sqrt[3]{2})(x^2 + \sqrt[3]{2}x + \sqrt[3]{4})$

b) $x^3 + 7$
 $= (x + \sqrt[3]{7})(x^2 - \sqrt[3]{7}x + \sqrt[3]{49})$

c) $3x^3 + 1$
 $= (\sqrt[3]{3}x + 1)(\sqrt[3]{9}x^2 - \sqrt[3]{3}x + 1)$

d) $4x^3 - 7$
 $= (\sqrt[3]{4}x - \sqrt[3]{7})(\sqrt[3]{16}x^2 + \sqrt[3]{28}x + \sqrt[3]{49})$

11 $x^6 - 64$

a) $(x^3 + 8)(x^3 - 8) = (x+2)(x^2 - 2x + 4)(x-2)(x^2 + 2x + 4)$ ← most factored

b) $(x^2 - 4)(x^4 + 4x^2 + 16) = (x+2)(x-2)(x^4 + 4x^2 + 16)$

c) $(x-2)(x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32)$

1.2 - Continued

12 a) $x^3 + 4x^2 + x - 6$ ($x-1$) is a factor

$$\begin{array}{r|rrrr} 1 & 1 & 4 & 1 & -6 \\ & \downarrow & & & \\ & 1 & 5 & 6 & \\ \hline & 1 & 5 & 6 & 10 \end{array}$$

$$(x-1)(x^2 + 5x + 6)$$

$$(x-1)(x+2)(x+3)$$

b) $x^3 - 7x - 6$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & \downarrow & & & \\ & 1 & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 10 \end{array}$$

$$(x-3)(x^2 + 3x + 2)$$

$$(x-3)(x+2)(x+1)$$

c) $3x^3 + 8x^2 - 33x + 10$

$$\begin{array}{r|rrrr} 2 & 3 & 8 & -33 & 10 \\ & \downarrow & & & \\ & 3 & 6 & 28 & -10 \\ \hline & 3 & 14 & -5 & 10 \end{array}$$

$$(x-2)(3x^2 + 14x - 5)$$

$$(x-2)(3x-1)(x+5)$$

$$\begin{array}{l} 3x^2 + 14x - 5 \\ 3x^2 + 15x - 1x - 5 \\ \hline 3x(x+5) - 1(x-1) \\ (3x-1)(x+5) \end{array}$$

g) $x^4 + x^3 - 6x^2 - 4x + 8$

$$\begin{array}{r|rrrrr} 1 & 1 & 1 & -6 & -4 & 8 \\ & \downarrow & & & & \\ & 1 & 2 & -4 & -8 & \\ \hline & 1 & 2 & -4 & -8 & 10 \end{array}$$

$$(x-1)(x^3 + 2x^2 - 4x - 8)$$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -4 & -8 \\ & \downarrow & & & \\ & 1 & 4 & 4 & 10 \end{array}$$

$$(x-1)(x-2)(x^2 + 4x + 4)$$

$$(x-1)(x-2)(x+2)(x+2)$$

13. a) $x^3 + 6x^2 - 4x - 24$
 $= x^2(x+6) - 4(x+6)$
 $= (x+6)(x^2 - 4)$
 $= (x+6)(x+2)(x-2)$

b) $4x^3 - 4x^2 - 3x + 3$
 $= 4x^2(x-1) - 3(x-1)$
 $= (4x^2 - 3)(x-1)$
 $= (2x + \sqrt{3})(2x - \sqrt{3})(x-1)$

c) $x^4 + 3x^3 + 8x + 24$
 $= x^3(x+3) + 8(x+3)$
 $= (x^3 + 8)(x+3)$
 $= (x+2)(x^2 - 2x + 4)(x+3)$

d) $8x^4 + 16x^3 - 3x - 6$
 $= 8x^3(x+2) - 3(x+2)$
 $= (8x^3 - 3)(x+2)$
 $= (x+2)(2x - \sqrt{3})(4x^2 + 2x\sqrt{3} + 3)$

$$\begin{aligned}
 -5 &= 6x^2 + 17x + 12 \\
 &= 6x^2 + 9x + 9x + 12 \\
 &= 2x(3x+4) + 3(3x+4) \\
 &= (3x+4)(2x+3)
 \end{aligned}$$

1.2 - Continued

14

a) $\frac{1}{2}x^2 - \frac{1}{8}$

$$= \frac{1}{8}(4x^2 - 1)$$

$$= \frac{1}{8}(2x-1)(2x+1)$$

d) $\frac{1}{6}x^2y^2 - xy^3 + \frac{3}{2}y^4$

$$= \frac{1}{6}y^2(x^2 - 6xy + 9y^2)$$

$$= \frac{1}{6}y^2(x-3y)(x-3y)$$

g) $3x^{-4} + 6x^{-2}$

$$= 3x^{-4}(1 + 2x^2)$$

j) $b^{-2} - 25$

$$= b^{-2}(1 - 25b^2)$$

$$= b^{-2}(1+5b)(1-5b)$$

m) $x^{3/2} + x^{1/2}$

$$= x^{1/2}(x^3 + 1)$$

$$= x^{1/2}(x+1)(x^2-x+1)$$

p) $a^{5/2}b^{-2/3} - a^{-1/2}b^{2/3}$

$$= a^{-1/2}b^{-2/3}(a^3 - b^3)$$

$$= a^{-1/2}b^{-2/3}(a-b)(a^2+ab+b^2)$$

s) $\frac{1}{18}x^{7/2} - \frac{9}{2}x^{-1/2}$

$$= \frac{1}{18}x^{-1/2}(x^4 - 81)$$

$$= \frac{1}{18}x^{-1/2}(x^2+9)(x^2-9)$$

$$= \frac{1}{18}x^{-1/2}(x^2+9)(x+3)(x-3)$$

b) $\frac{3}{2}x^2 + \frac{17}{4}x + 3$

$$= \frac{1}{4}(6x^2 + 17x + 12)$$

$$= \frac{1}{4}(3x+4)(2x+3)$$

e) $\frac{1}{6}x^2 + \frac{4}{3}x$

$$= \frac{1}{6}x(x+8)$$

h) $x^{-3} - x^{-4} - 12x^{-5}$

$$= x^{-5}(x^2 - x - 12)$$

$$= x^{-5}(x-4)(x+3)$$

k) $1 + 4x^{-1} + 4x^{-2}$

$$= x^{-2}(x^2 + 4x + 4)$$

$$= x^{-2}(x+2)(x+2)$$

n) $t^{2/3} + t^{1/3} - 2t^{-1/3}$

$$= t^{-1/3}(t^2 + t - 2)$$

$$= t^{-1/3}(t+2)(t-1)$$

q) $\frac{1}{8}s^{3/2} + \frac{3}{4}s^{1/2} - 2s^{-1/2}$

$$= \frac{1}{8}s^{-1/2}(s^2 + 6s - 16)$$

$$= \frac{1}{8}s^{-1/2}(s+8)(s-2)$$

t) $t^{1/2} + 6t^{-1/2} - t^{-3/2} - 6t^{-5/2}$

$$= t^{-5/2}(t^3 + 6t^2 - t - 6)$$

c) $\frac{1}{9}x^3 + 3$

$$= \frac{1}{9}(x^3 + 27)$$

$$= \frac{1}{9}(x+3)(x^2-3x+9)$$

f) $x^2 + 8x^{-1}$

$$= x^{-1}(x^3 + 8)$$

$$= x^{-1}(x+2)(x^2-2x+4)$$

i) $16x^{-2}y^{-3} - 24x^{-5}y^{-1}$

$$= 8x^{-5}y^{-3}(2x^3 - 3y^2)$$

l) $\frac{1}{10}s^{-2} - \frac{1}{5}r^{-1}s^{-1} - \frac{4}{5}r^{-2}$

$$= \frac{1}{10}s^{-2}r^{-2}(r^2 - 2rs - 8s^2)$$

$$= \frac{1}{10}s^{-2}r^{-2}(r-4s)(r+2s)$$

o) $\frac{1}{3}v^2 - 3v^{-1/2}$

$$= \frac{1}{3}v^{-1/2}(v^2 - 9)$$

$$= \frac{1}{3}v^{-1/2}(v+3)(v-3)$$

r) $\frac{1}{3}a^{3/2}b^{-1/2} - \frac{3}{4}a^{-1/2}b^{3/2}$

$$= \frac{1}{12}a^{-1/2}b^{-1/2}(4a^2 - 9b^2)$$

$$= \frac{1}{12}a^{-1/2}b^{-1/2}(2a+3b)(2a-3b)$$

1	6	-1	-6
↓	1	7	6

1 7 6 10

$$t^{-5/2}(t-1)(t^2+7t+6)$$

$$t^{-5/2}(t-1)(t+1)(t+6)$$