11.2 Combinations

Say we wanted a group of 3 people to represent our class in a school challenge. In how many ways could we choose this group?

Is order important here?

$$24C_3 = 2024$$

This is not a permutation then, but a combination.

$$nC_r = \frac{n!}{(n-r)!\,r!}$$

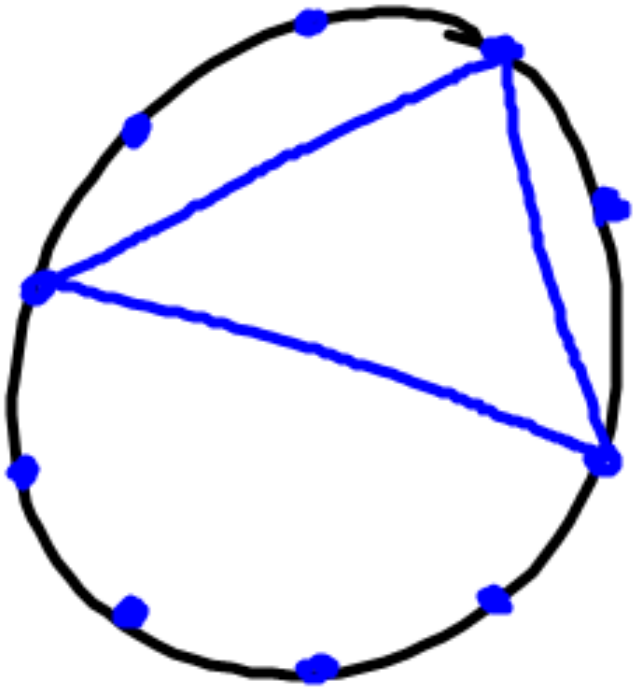
A combination is a selection of a group of objects, taken from a larger group, for which the kinds of objects selected is important, but not the order in which they are selected.

The notation ${}_nC_r$, or $\binom{n}{r}$, represents the number of combinations of n items taken r at a time, where $n \geq r$ and $r \geq 0$.

Why must $n \geq r \geq 0$?

$$
\begin{aligned}
{}_nC_r &= \frac{{}_nP_r}{r!} \\
&= \frac{\dfrac{n!}{(n-r)!}}{r!} \\
&= \frac{n!}{(n-r)!r!}
\end{aligned}
$$

Example 1: Ten points lie on the circumference of a circle. How many triangles can be formed if three of the points are used as the vertices of each triangle ?



$$10C_3 = 120$$

Example 2: From students present today how many 10 person committees can we form having 5 boys and 5 girls.

17 of boys

7 of girls

Combinations from several sets.

$_{17}C_5$ Boys

$_{7}C_5$ GIRLS

6188 X 21 = 129948

Combinations from Several Sets

To find the total combinations from several sets, we simply multiply the individual combinations from each set.

Example: From a standard deck of playing cards, how many 5 card hands are possible that:

a) all red
b) all clubs
c) 2 red and 3 black
d) at least 3 red

a) $26C5 = 65780$

b) $13C5 = 1287$

c) $(26C2) \cdot (26C3)$

$325 \cdot 2600 = 845000$

d) 3R 2B OR 4R 1B OR 5R

$(26C_3)(26C_2) + (26C_4)(26C_1) + (26C_5)$

$(2600)(325) + (1495)(26) + 67780$

$845000 + 38870 + 67780$

$= 951650$

(12)

$$\frac{n!}{(n-r)!\,r!}$$

$$n C_2 = 100 C_{98}$$

$$\frac{n!}{(n-2)!\,2!} = \frac{100!}{2!\,98!}$$

$n = 100$

$$\frac{49 \quad 48 \quad 47 \quad 46 \quad 45 \quad 44}{1.00683 \times 10^{10}}$$

$$49C_6 = 13\,983\,816$$

$$20\,000\,000$$

Your Turn

In how many ways can the debating club coach select a team from six grade 11 students and seven grade 12 students if the team has

a) four members?

b) four members, only one of whom is in grade 11?

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#'s 1, 2, 3, 4, 6a,b, 11, 13, 17, 18
Handout Circled Questions