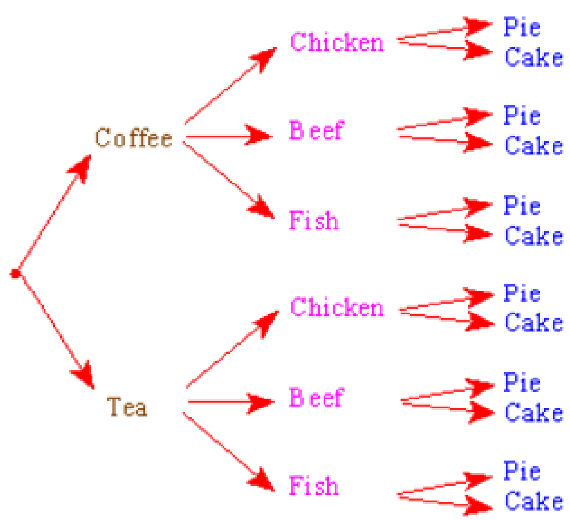
Chapter 11 Permutations, Combinations, and the Binomial Theorem

11.1 Permutations

How safe is your password? It has been suggested that a four-character letters-only password can be hacked in under 10 s. However, an eight-character password with at least one number could take up to 7 years to crack. Why is there such a big difference?

Example 1: You are ordering dinner at a restaurant. How many ways can you order a meal if you have two choices for a drink (coffee or tea), three main courses to choose from (chicken, beef, or fish) and two desserts (pie or cake) ?

Draw a picture — this is known as a TREE DIAGRAM



Follow each path; we end up with 12 different meals that you can order.

12

$\underline{2} \cdot \underline{3} \cdot \underline{2}$

Drink ME D

Another method of determining the number of possible meals is to us the **fundamental counting principle.**

fundamental counting principle

- if one task can be performed in *a* ways and a second task can be performed in *b* ways, then the two tasks can be performed in $a \times b$ ways

Example 2: Nine horses are racing in a derby. How many different ways can these nine horses finish the race?

$9! = 362880$

$\underline{9}\ \underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

$9!$

Example 3

6 people are lining up for a picture. How many different arrangements are possible? if all people are in picture.

6 5 4 3 2 1

$6! = 720$

The arrangement of objects or people in a line is called a linear **permutation**. In a permutation, the order of the objects is important. When the objects are distinguishable from one another, a new order of objects creates a new permutation.

The notation ${}_nP_r$ is used to represent the number of permutations, or arrangements in a definite order, of r items taken from a set of n distinct items. A formula for ${}_nP_r$ is ${}_nP_r = \frac{n!}{(n-r)!}$, $n \in \mathrm{N}$. $r \leq n$

We could also solve our horse race question or picture question using the permutation formula.

$\underline{21}\ \underline{20}\ \underline{19} = 7980$

$${}_{21}P_3 = \frac{21!}{18!}$$

$$= \frac{21 \cdot 20 \cdot 19 \cdot 18 \cdot \text{---} \cdot 1}{18 \cdot 17 \cdot \text{---} \cdot 1}$$

$${}_{21}P_3 =$$

Example 4

18 students are available to be selected as SRC president, vice president and secretary. How many different arrangements are possible?

Example 5:

a) Evaluate ${}_8P_3$

b) Show that 100!+99! = 101(99!)

c) Solve for *n* if ${}_nP_2 = 210$, where *n* is a natural number.

a) ${}_8P_3 = 336$

b) $100! + 99! = 101(99!)$

$99!(100+1)$

$99!(101)$

c) $_nP_2 = 210$

$n(n-1) = 210$

$n^2 - n = 210$

$n^2 - n - 210 = 0$

$(n-15)(n+14) = 0$

$n = 15$ or ~~$n = -14$~~

Your Turn

a) Evaluate ${}_7P_2$ using factorial notation.

b) Show that $5! - 3! = 19(3!)$.

c) Solve for n if ${}_nP_2 = 56$.

Assignment: Handout Front Side Circled Questions

Permutations With Repeating Objects

Say we were asked to find the number of different ways all the letters in the word **REGINA** could be arranged.

$_6P_6 = 6! = 720$

Now say we were asked to find the number of different ways all the letters in the word **SASKATOON** could be arranged.

$$\frac{9!}{2! \cdot 2! \cdot 2!} = \frac{9!}{8} = 45360$$

S O A

Permutations With Repetitions Principle

A set of n objects with a of one kind that are identical, b of a second kind that are identical, and c of a third kind that are identical, and so on, can be arranged in $\frac{n!}{a!b!c!\ldots}$ different ways.

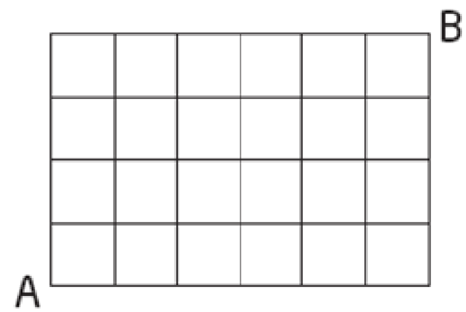
Example 6: In how many ways can all the letters of the word **MINIMUM** be arranged ?

$$\frac{7!}{3!\ 2!} = \frac{7!}{12} =$$

(M) (I)

$$\frac{9!}{(4!\ 3!)}$$

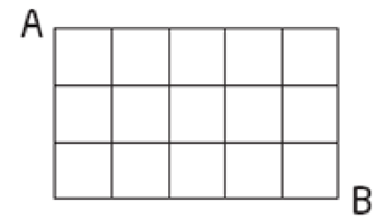
Example 7: How many paths can you follow from A to B in a four by six rectangular grid if you move only up one or to the right?



Your Turn

a) How many different 5-digit numbers can you make by arranging all of the digits of 17 171?

b) In how many different ways can you walk from A to B in a three by five rectangular grid if you must move only down or to the right?



Permutations With Constraints

Example 8

Five people (A, B, C, D, and E) are seated on a bench. In how many ways can they be arranged if

a) E is seated in the middle?

b) A and B must be seated together?

c) A and B cannot be together?

a) $\underline{4}\ \underline{3}\ \underline{1}\ \underline{2}\ \underline{1} = 24$

b) $(\underline{4}\ \underline{3}\ \underline{2}\ \underline{1})2 = 48$

c) total possibilities

− # sitting together

${}_5P_5 - 48$

$120 - 48 = 72$

Your Turn

How many ways can one French poster, two mathematics posters, and three science posters be arranged in a row on a wall if

a) the two mathematics posters must be together on an end?

b) the three science posters must be together?

c) the three science posters cannot all be together?

Solution a) 2 cases

________ ________ ________ ________ ________ ________

________ ________ ________ ________ ________ ________

Solution b) 4 cases

Arrangements Requiring Cases

Example 9

How many 4-digit odd numbers can you make using the digits 1 to 7 if the numbers must be less than 6000? No digits are repeated.

Assignment
Handout Back Page Circled Questions
Page 524
#'s 2,3,4,5,6,7,8,11a,b,13,16