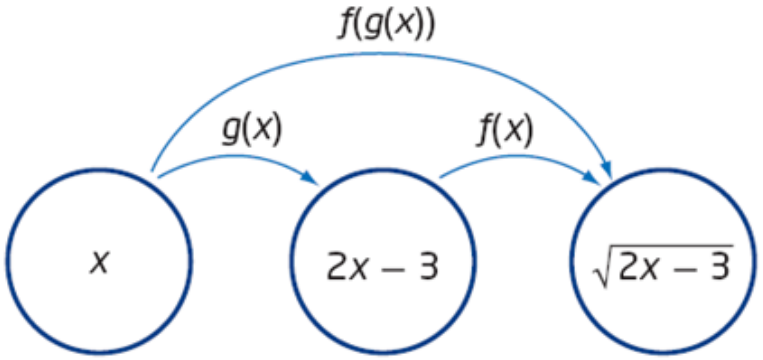
10.3 Composite Functions

Composite functions are functions that are formed from two functions, $f(x)$ and $g(x)$, in which the output or result of one of the functions is used as the input for the other function. For example, if $f(x) = \sqrt{x}$ and $g(x) = 2x - 3$, then the composition of $f(x)$ and $g(x)$ is $f(g(x)) = \sqrt{2x - 3}$, as shown in the mapping diagram.



$f(g(x)) = \sqrt{2x-3}$

$g(x) = 2x - 3$

$f(x) = \sqrt{x}$

I like to use the analogy that a composite functions are like these wooden dolls.

Ex.2 If $f(x)=x^2+2x$ and $g(x)=3-x$, then find the following :

a) $f(g(5))$ $= f(-2) = (-2)^2 + 2(-2) = 0$

$g(5) = 3-5 = -2$

b) $g(f(3))$ $= g(15) = 3-15 = -12$

$f(3) = (3)^2 + 2(3) = 15$

c) $g(g(2+h))$

$g(2+h) = 3-(2+h) = 3-2-h$

$= 1-h$

$g(1-h) = 3-(1-h)$

$= 3-1+h$

$= 2+h$

$d)\ f(g(x))$

$f(3-x)$

$=(3-x)^2+2(3-x)$

$=9-6x+x^2+6-2x$

$=x^2-8x+15$

$e)\ g(f(x))$

$g(x^2+2x)$

$=3-(x^2+2x)$

$=3-x^2-2x$

Your Turn: Let $f(x) = x^2 + 2x$ and $g(x) = x^2$. Determine the equation of each composite function.

a) $f(g(x))$

$f(x^2) = (x^2)^2 + 2(x^2)$

$f(g(x)) = x^4 + 2x^2$

$b)\ g(f(x))$

$g(x^2+2x)$

$=(x^2+2x)^2$

$g(f(x))=x^4+4x^3+4x^2$

c) Find value(s) for x that make f(g(x))=g(f(x)).

$x^4 + 2x^2 = x^4 + 4x^3 + 4x^2$

$0 = 4x^3 + 2x^2$

$0 = 2x^2(2x+1)$

$2x^2 = 0$ or $2x+1=0$

$x = 0$

$x = -1/2$

Assignment Handout
#'s 1 i-p, 3e-h, 6,7