

1.9 Function Operations

Arithmetic Operations

Function Addition	$(f + g)(x) = f(x) + g(x)$ with domain $P \cap Q$.
Function Subtraction	$(f - g)(x) = f(x) - g(x)$ with domain $P \cap Q$.
Function Multiplication	$(fg)(x) = f(x) \cdot g(x)$ with domain $P \cap Q$.
Function Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with domain $P \cap Q$ excluding values of x for which $g(x) = 0$.

Your Turn #1

If $f(x) = \frac{x+2}{3}$ and

$g(x) = \frac{3}{x-2}$, find each of

the following:

(a) $(f+g)(x)$

(b) $(f-g)(x)$

(c) $(fg)(x)$

(d) $\left(\frac{f}{g}\right)(x)$

(e) the domain of $(fg)(x)$.

$$\frac{(x-2)}{(x-2)} \left(\frac{(x+2)}{3} + \frac{3}{x-2} \left(\frac{3}{3} \right) \right)$$

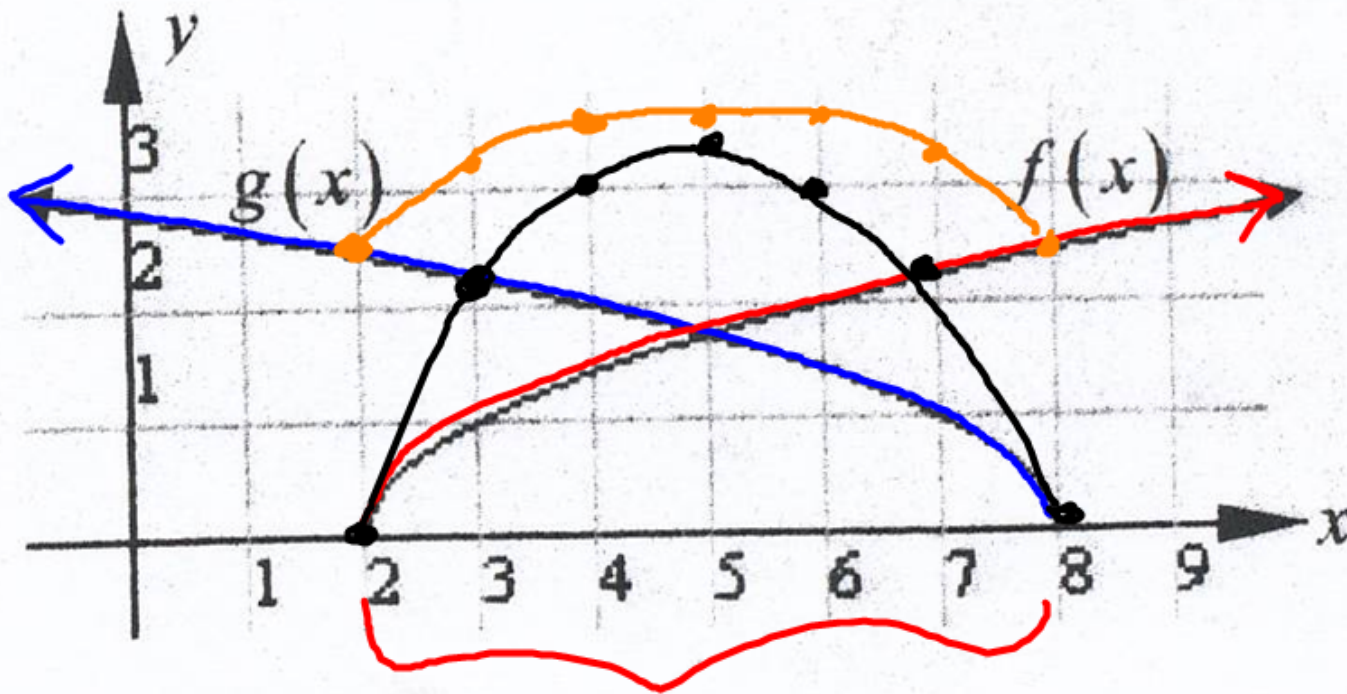
$$= \frac{x^2 - 4}{3(x-2)} + \frac{9}{3(x-2)}$$

$$= \frac{x^2 + 5}{3(x-2)}$$

(c) $\left(\frac{x+2}{3} \right) \left(\frac{3}{x-2} \right) = \left(\frac{x+2}{x-2} \right)$

$$x \neq 2$$

Ex.1 Find $(f+g)(x)$ or $f(x)+g(x)$



$$(f+g)(x)$$

Function Composition

Another way in which functions can be combined is known as **function composition**. This is when we develop a new function by creating a function within a function.

$$f(g(x)) \text{ or } g(f(x))$$
$$(f \circ g)(x) \text{ or } (g \circ f)(x)$$

$$c) f(f(-2))$$

$$f(-2) = (-2)^2 - 1 = 3$$

$$f(3) = (3)^2 - 1 = 8$$

Your Turn #3

If $f(x) = x^2 - 1$ and

$g(x) = \frac{1}{x-3}$, find.

(a) $(f \circ g)(1)$

(b) $(g \circ f)(0)$

(c) $(f \circ f)(-2)$

(d) $(g \circ g)(5)$

(e) $(f \circ g)(x)$

(f) $(g \circ f)(x)$

(g) $(f \circ f)(x)$

(h) $(g \circ g)(x)$

b)

$$a) f(g(1))$$

$$g(1) = \frac{1}{1-3}$$

$$= -\frac{1}{2}$$

$$f(-\frac{1}{2})$$

$$= (-\frac{1}{2})^2 - 1 = -\frac{3}{4}$$

$$b) g(f(0))$$

$$f(0) = -1$$

$$g(-1) = \frac{1}{-1-3} = -\frac{1}{4}$$

$$e) f(g(x))$$
$$f\left(\frac{4}{x-3}\right) = \left(\frac{4}{x-3}\right)^2 - 1$$

$$= \frac{16}{(x-3)^2} - 1$$

$$f) g(f(x))$$

$$g(x^2-1) = \frac{4}{x^2-1-3} = \frac{4}{x^2-4}$$

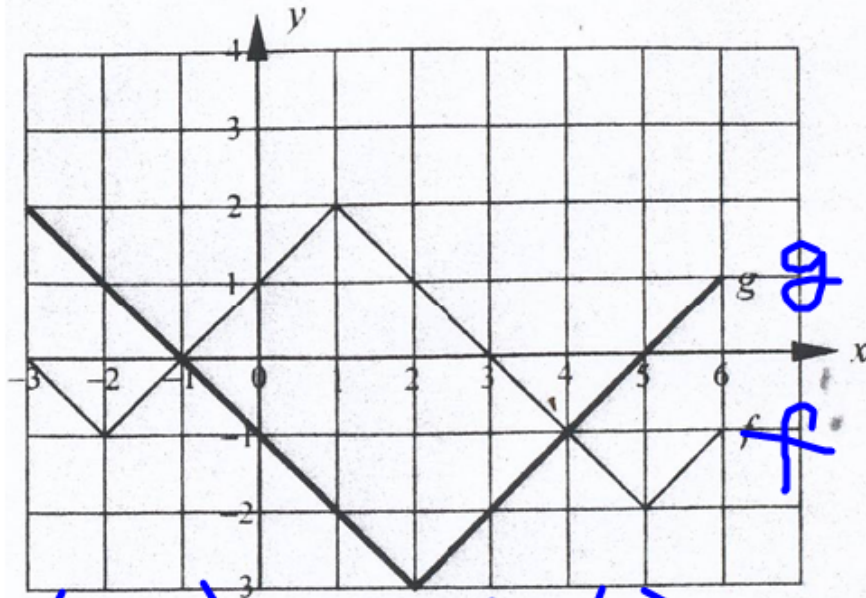
Ex.2 Find the following:

(a) $(f+g)(2)$ (b) $(f-g)(5)$ (c) $(fg)(1)$ (d) $\left(\frac{f}{g}\right)(-2)$

(e) $(f \circ g)(6)$ (f) $(g \circ f)(-1)$ (g) $(f \circ f)(-2)$ (h) $(g \circ g)(1)$

a) $f(2) + g(2)$
 $1 + (-3)$
 $= -2$

c) $f(1) \cdot g(1)$
 $2 \cdot (-2)$
 $= -4$



e) $f(g(6)) = f(1) = 2$

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x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.
- (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
- (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.
- (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.
- (d) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

Domains of Composite Functions

$$f(g(x)) \text{ or } g(f(x))$$
$$(f \circ g)(x) \text{ or } (g \circ f)(x)$$

Ex Given that: $f(x) = x^2$ $D : [4, \infty)$
 $g(x) = \sqrt{x-4}$

find the domain of

a) $f(g(x))$

$$g(x) = \sqrt{x-4}$$

$$x-4 \geq 0$$

$$x \geq 4$$

$$D: [4, \infty)$$

b) $g(f(x))$

$$\begin{aligned} f(\sqrt{x-4}) \\ &= (\sqrt{x-4})^2 \\ &= x-4 \\ D: x \in \mathbb{R} \end{aligned}$$

$$g(f(x))$$

$$\cancel{f(x)} \quad f(x) = x^2$$

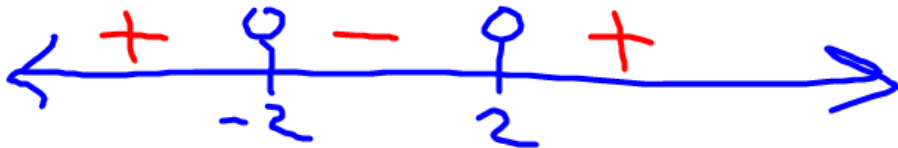
$$D: x \in \mathbb{R}$$

$$g(x^2) = \sqrt{x^2 - 4}$$

$$x^2 - 4 \geq 0$$

$$(-\infty, -2] \cup [2, \infty)$$

$$(x-2)(x+2) \geq 0$$



Function Decomposition

The ability to **compose** and **decompose** functions is essential in order to fully understand that calculus that is to follow!

$$h(x) = f(g(x))$$

Your Turn #5

Find functions $f(x)$ and

$g(x)$ so that

$$h(x) = (f \circ g)(x).$$

(a) $h(x) = (2x^2 + 3x)^4$

(b) $h(x) = \sqrt[5]{\sin 3x}$

(c) $h(x) = \frac{4}{2x+7}$

(d) $h(x) = 2^{\tan 3x}$

(e) $h(x) = \log_7(x^3 - 8)$

(f) $h(x) = \sin(3 \cos x^3)$

a) $h(x) = (2x^2 + 3x)^4$

$$g(x) = 2x^2 + 3x$$

$$f(x) = x^4$$

b) $h(x) = \sqrt[5]{\sin 3x}$

$$g(x) = \sin 3x$$

$$f(x) = \sqrt[5]{x}$$

Assignment
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Written Exercises

1. Ask your teacher for four copies of the grid below. On these grids draw the graphs of each of the following:

(a) $(f + g)(x)$ (b) $(f - g)(x)$ (c) $(fg)(x)$ (d) $\left(\frac{f}{g}\right)(x)$

