1.8 Domain and Range

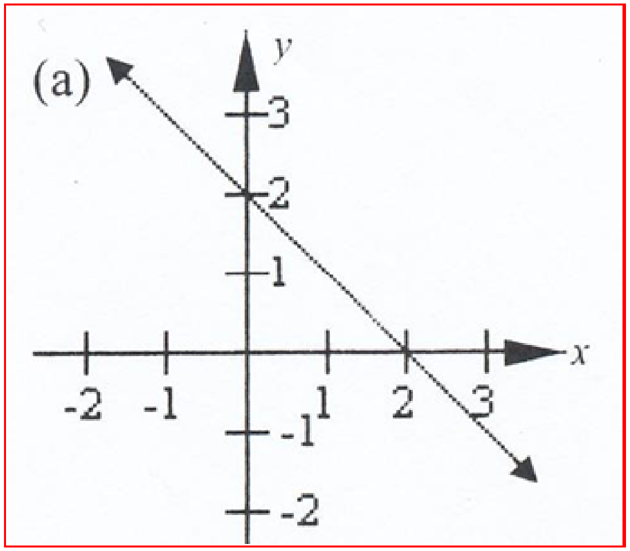
x values

y values

Domain of a function is the set of x values for which a value of y can be determined.

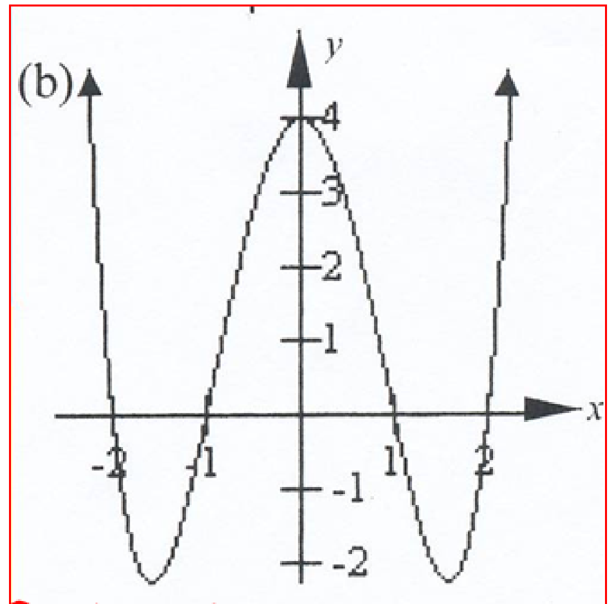
Range of a function consists of all the y values of a function.

Determining Domain and Range from the Graph of a Function



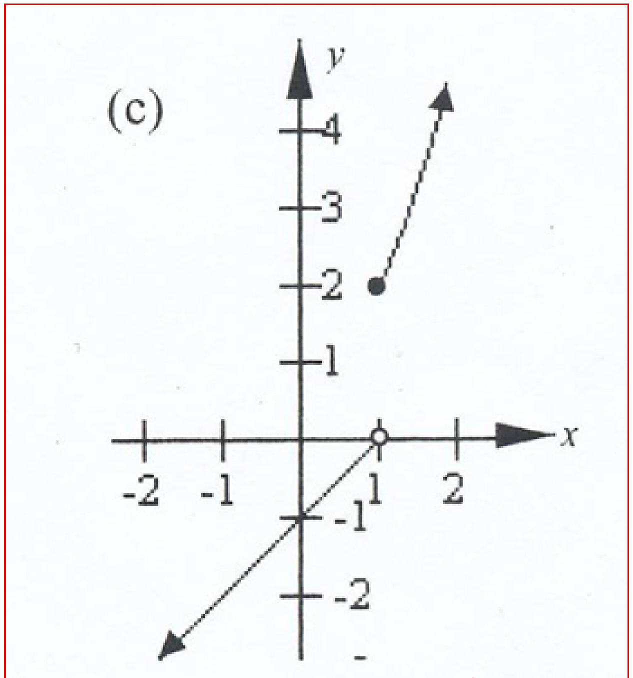
D $(-\infty, \infty)$

R $(-\infty, \infty)$



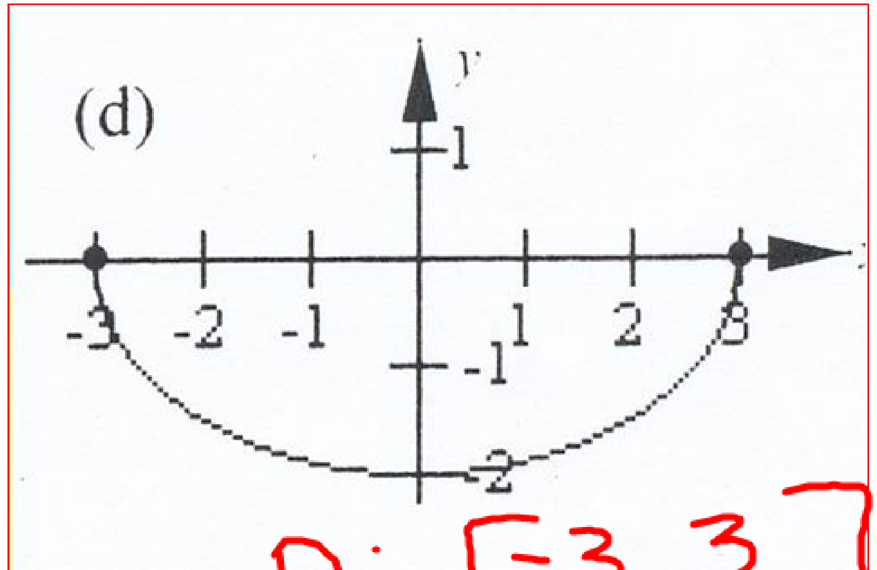
D: $(-\infty, \infty)$

R $[-2, \infty)$



D: $(-\infty, \infty)$

R: $(-\infty, 0) \cup [2, \infty)$



D: $[-3, 3]$

R: $[-2, 0]$

Determining the Domain By Examining The Function

To find the domain of a function without seeing the graph we must be careful to exclude any value of x that will result in:

Division by 0.

Taking the even root of a negative number.

Finding the logarithm of a non-positve number.

A trig function being undefined

Ex.1 Determine the domain of the following functions:

a) $f(x) = x^6 - 15x^3$

D: $(-\infty, \infty)$

b) $f(x)=\dfrac{10}{x^3-9x}=\dfrac{10}{x(x-3)(x+3)}$

D: $x \neq 0, 3, -3$

$(-\infty,-3)\cup(-3,0)\cup(0,3)\cup(3,\infty)$

c) $f(x)=\sqrt{x^2+x-30}$

$$x^2+x-30 \geq 0$$

$$(x+6)(x-5) \geq 0$$

$$\begin{array}{ccccc} + & 0 & - & 0 & + \\ -7 & -6 & 0 & 5 & 6 \end{array}$$

$$(-\infty,-6] \cup [5,\infty)$$

d) $h(x) = \log_5(2x - 7)$

$2x - 7 > 0$

$2x > 7$

$x > 7/2$

$(7/2, \infty)$

e) $g(x)=\sqrt[3]{\frac{2}{x}-4}$

D: $x \neq 0$

Domain and Range

Another domain and range video

Determining the Range By Examining The Function

This is harder to do. Later in our course we will develop strategies that will assist us. For now, we need to use a good deal of common sense and past knowledge.

Ex.2 Determine the range of the following functions:

a) $f(x) = x^3 - 9x$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

b) $f(x) = (x^2 + 5)^2 = x^4 + 10x^2 + 25$

D: $(-\infty, \infty)$

R $[25, \infty)$

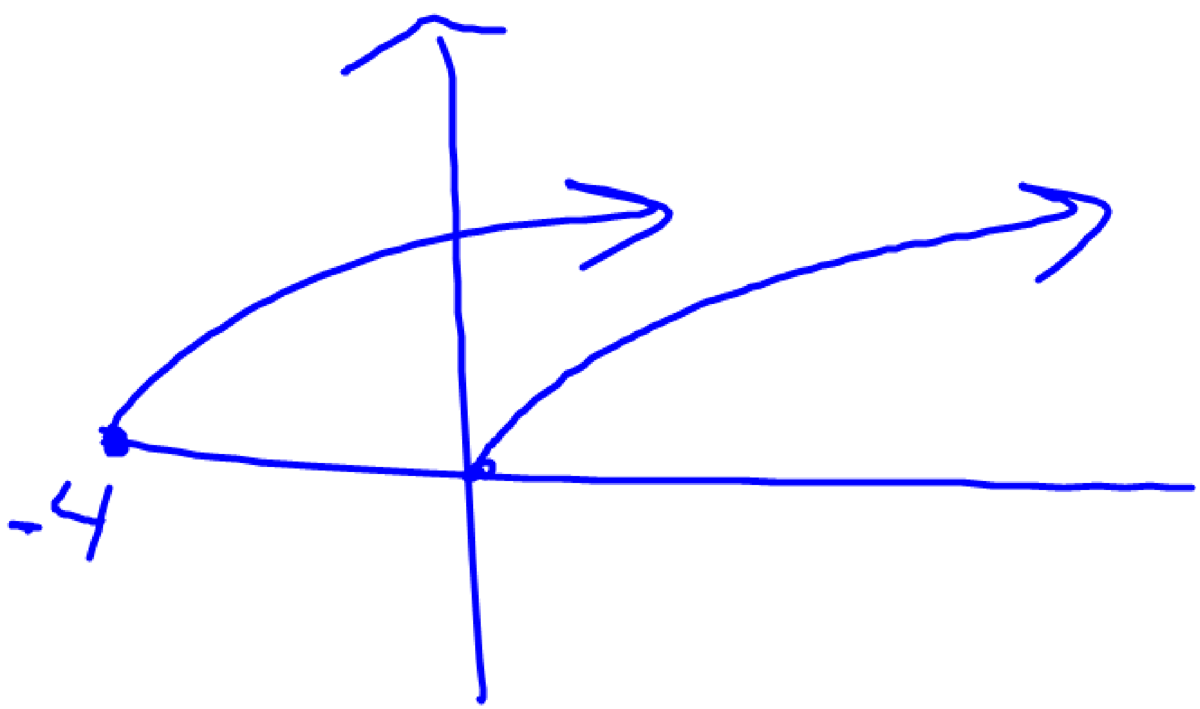
x	y
0	25
1	36
-1	36

c) $f(x)=\sqrt{4+x}$

$= \sqrt{x+4}$

D: $[-4, \infty)$

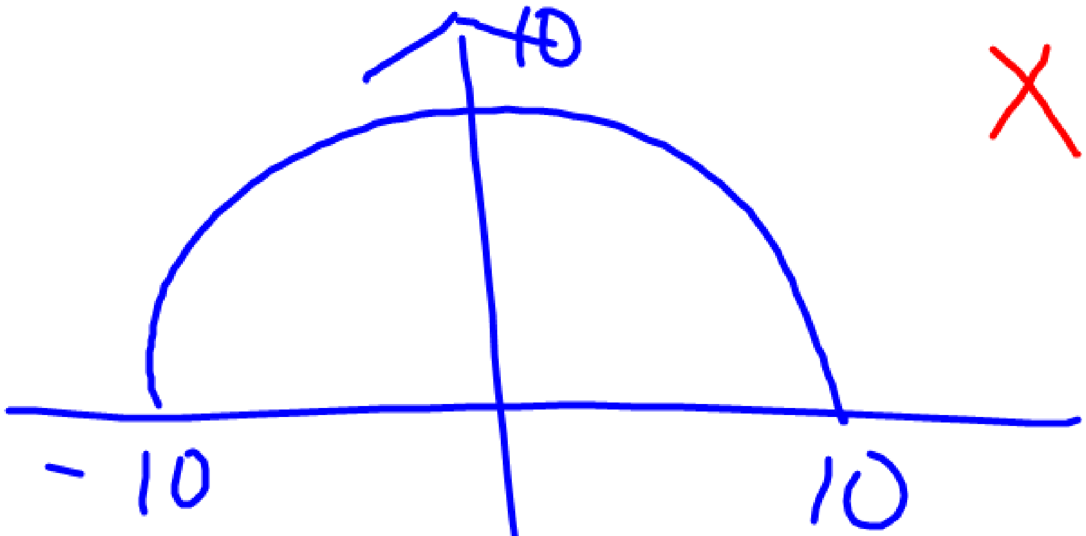
R: $[0, \infty)$



d) $f(x)=\sqrt{100-x^2}$

$\sqrt{r^2-x^2}$

$x^2+y^2=r^2$

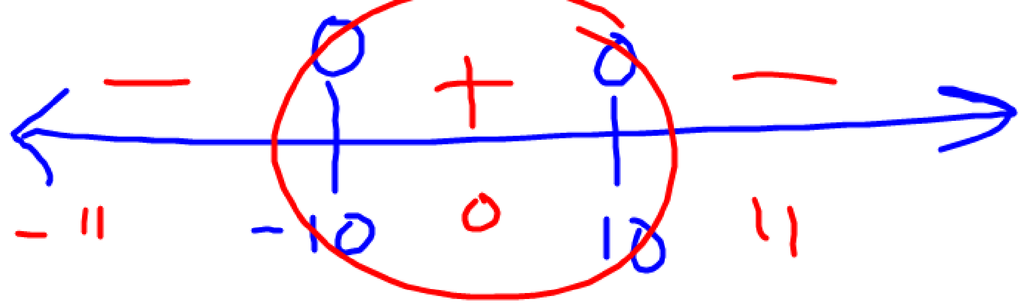


D = [-10, 10]

R: [0, 10]

$$100 - x^2 \geq 0$$

$$(10-x)(10+x) \geq 0$$



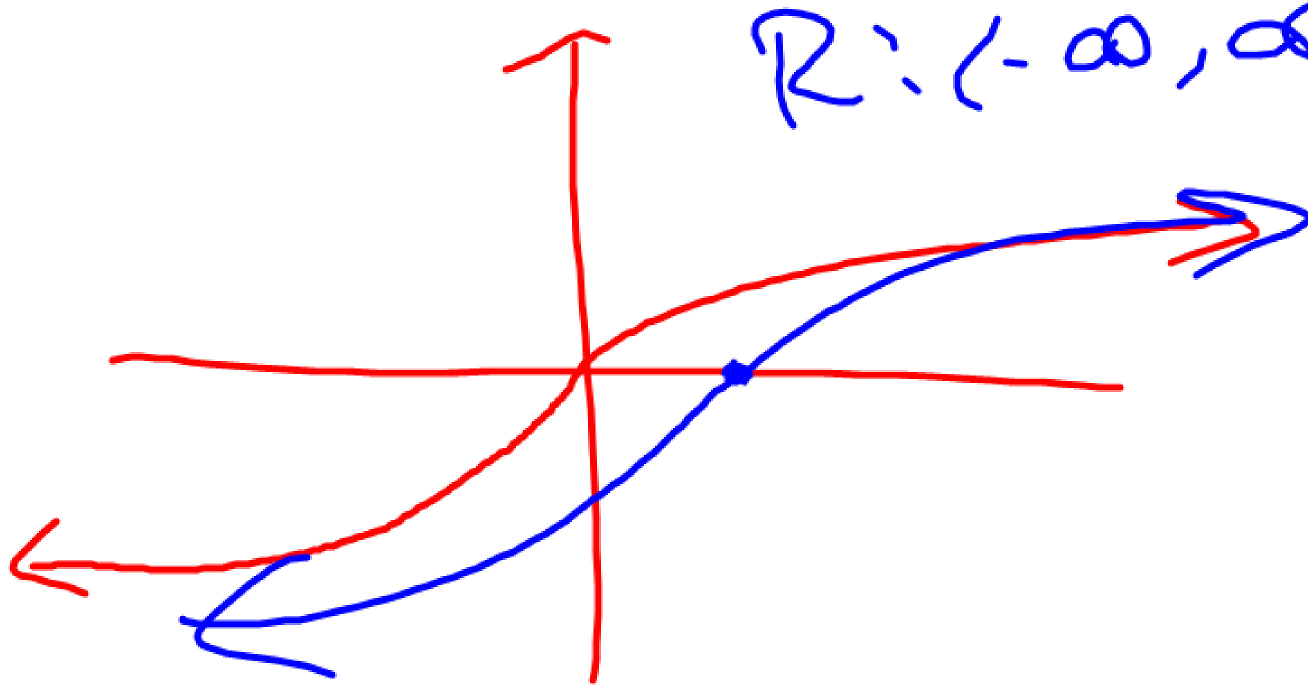
x	y
−10	0
0	10
10	0

$$\sqrt{100 - x^2}$$

e) $f(x)=\sqrt[3]{x-1}$

D: $(-\infty, \infty)$

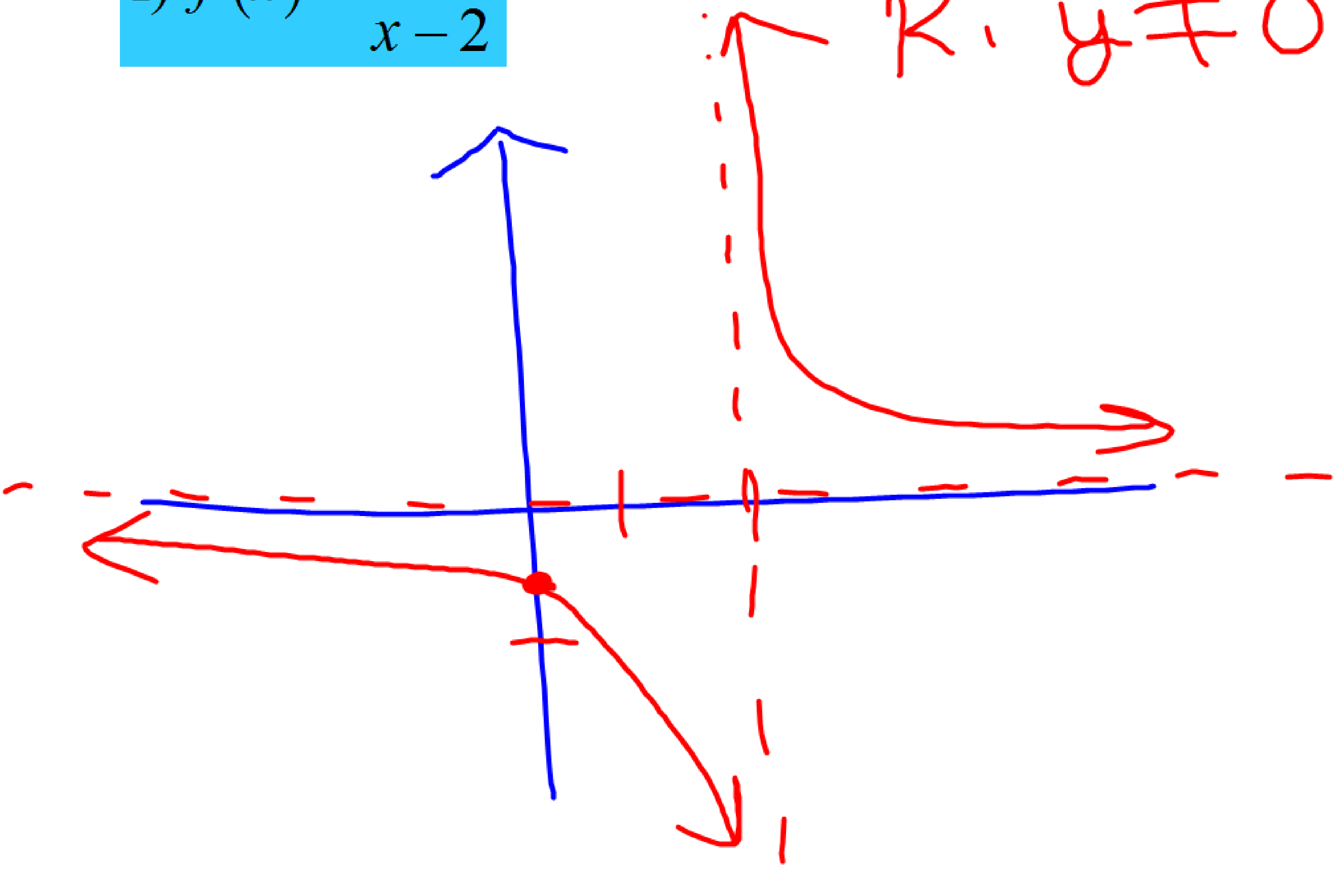
R: $(-\infty, \infty)$



f) $f(x)=\dfrac{3}{x-2}$

D: $x \neq 2$

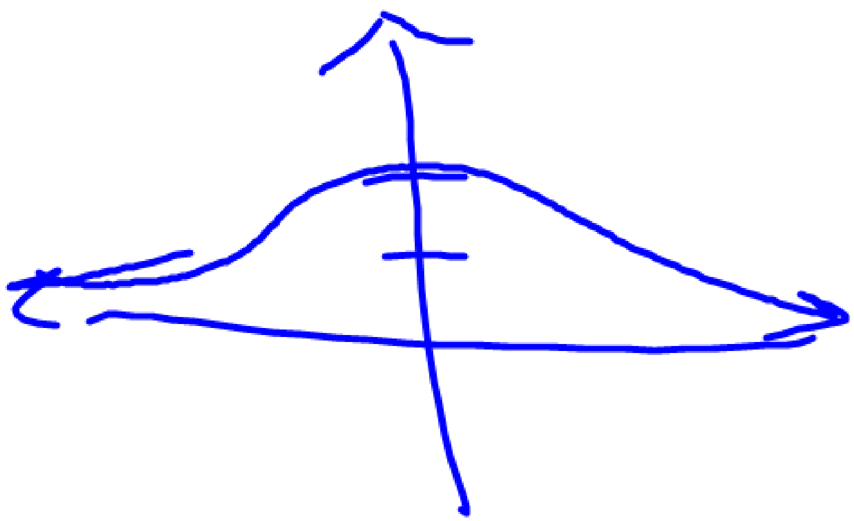
R: $y \neq 0$



g) $f(x) = \dfrac{4}{x^2 + 2}$

D: $(-\infty, \infty)$

R: $(0, 2]$



x	y
0	2
± 1	$4/3$
± 2	$2/3$
± 3	$4/11$
$\vdots$	

h) $f(x) = x^2 + 10x$

$D: (-\infty, \infty)$

$y = a(x-p)^2 + q$

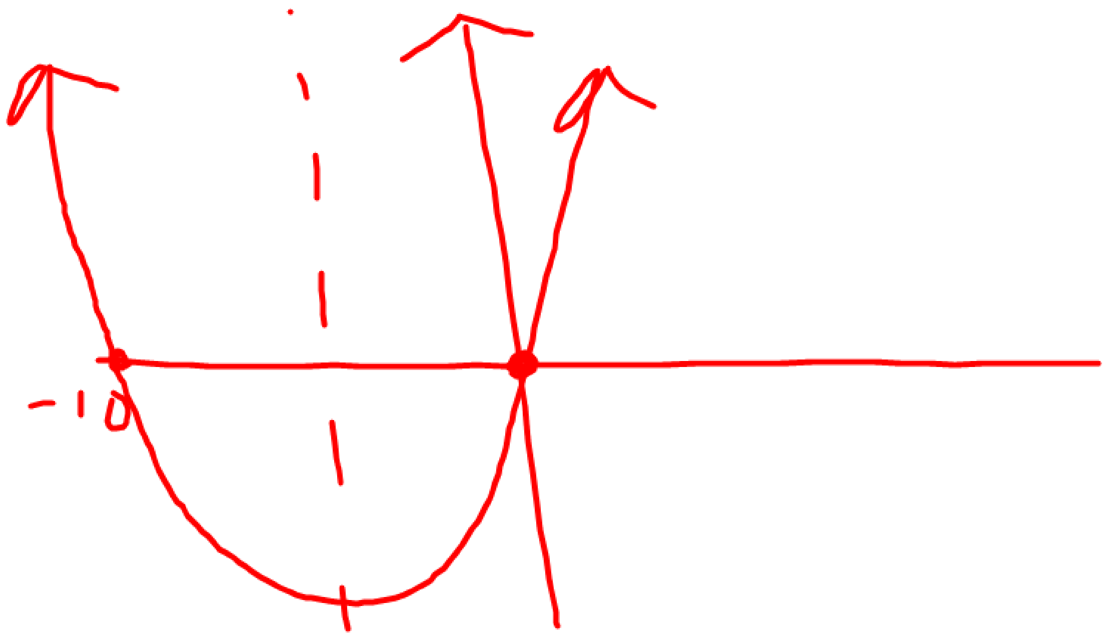
$f(x) = x^2 + 10x + \underline{25} - \underline{25}$

$f(x) = (x+5)^2 - 25$

$R: [-25, \infty)$

-5

$f(x) = x(x+10) = -25$



TO FIND THE RANGE, CONSIDER THESE QUESTIONS

- Is the function a translation of a function whose graph I already know?
- Is it a polynomial function? If so examine the degree and the leading coefficient.
- What happens to y as x becomes very large positive (negative)?
- What happens to y near any vertical asymptotes? Use a sign analysis.
- Is there a horizontal asymptote line?
- What is the y-intercept?
- If you are dealing with a root function, is there are largest (smallest) value for the expression beneath the radical sign?

Assignment

Page 99

#'s1-21,27,31,33,35,37,~~39~~,42

$$\boxed{x^4+6x^2+8}$$

$$y=(x^2+4)(x^2+2)$$

D: $(-\infty,\infty)$

R: $[8,\infty)$

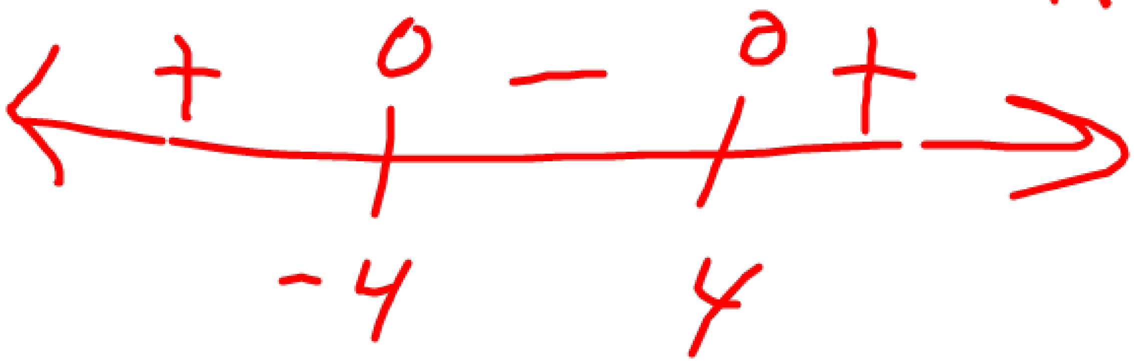
$$y = \sqrt{x^2 - 16}$$

$$D: (-\infty, -4] \cup [4, \infty)$$

$$x^2 - 16 \geq 0$$

$$R: [0, \infty)$$

$$(x-4)(x+4) \geq 0$$



$y = \log_3(x - 11)$

$x - 11 > 0$

$x > 11$