2.3 Piecewise Functions

We were introduced to **piecewise functions** last unit when we studied absolute value functions!

$$|x| = \begin{cases} x, \text{if } x \geq 0 \\ -x, \text{if } x < 0 \end{cases}$$

In a piecewise function the function behaves differently in different parts of its domain!

A truck rental company charges a minimum $75 rental fee or $25 per hour.

$$f(x)=\begin{cases}75, \text{ if } 0<x\le 3\\ 25x, \text{ if } x>3\end{cases}$$

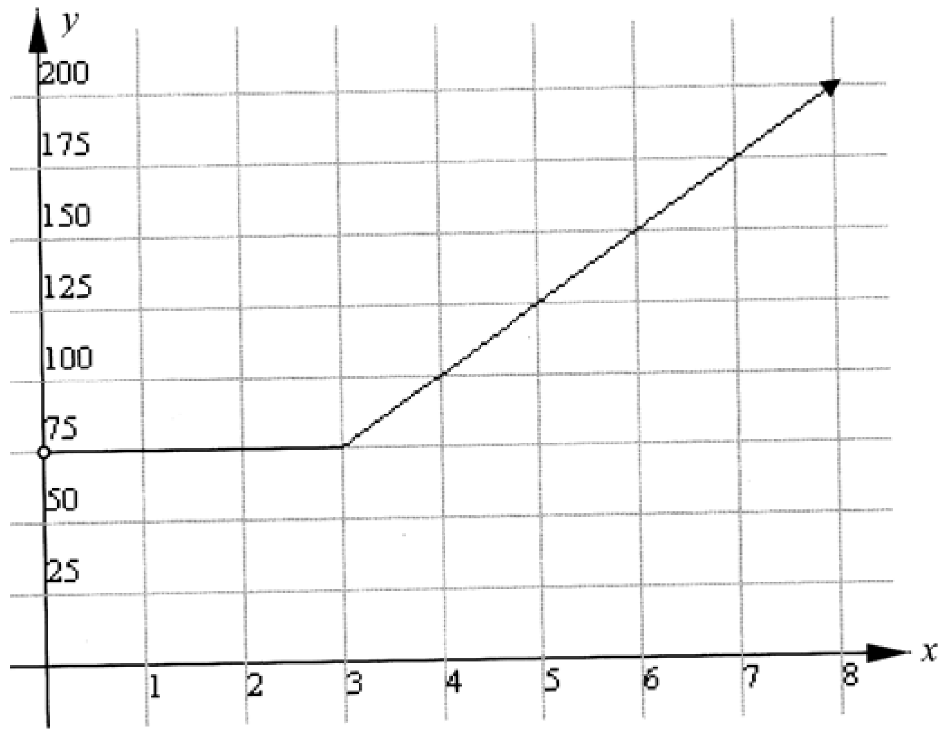
Your Turn #1

By referring to the illustration at left, what is the rental charge for using the truck: $75

(a) 1 hour? $150

(b) 6 hours?

(c) 7.25 hours? $181.25



Graphing Piecewise Functions

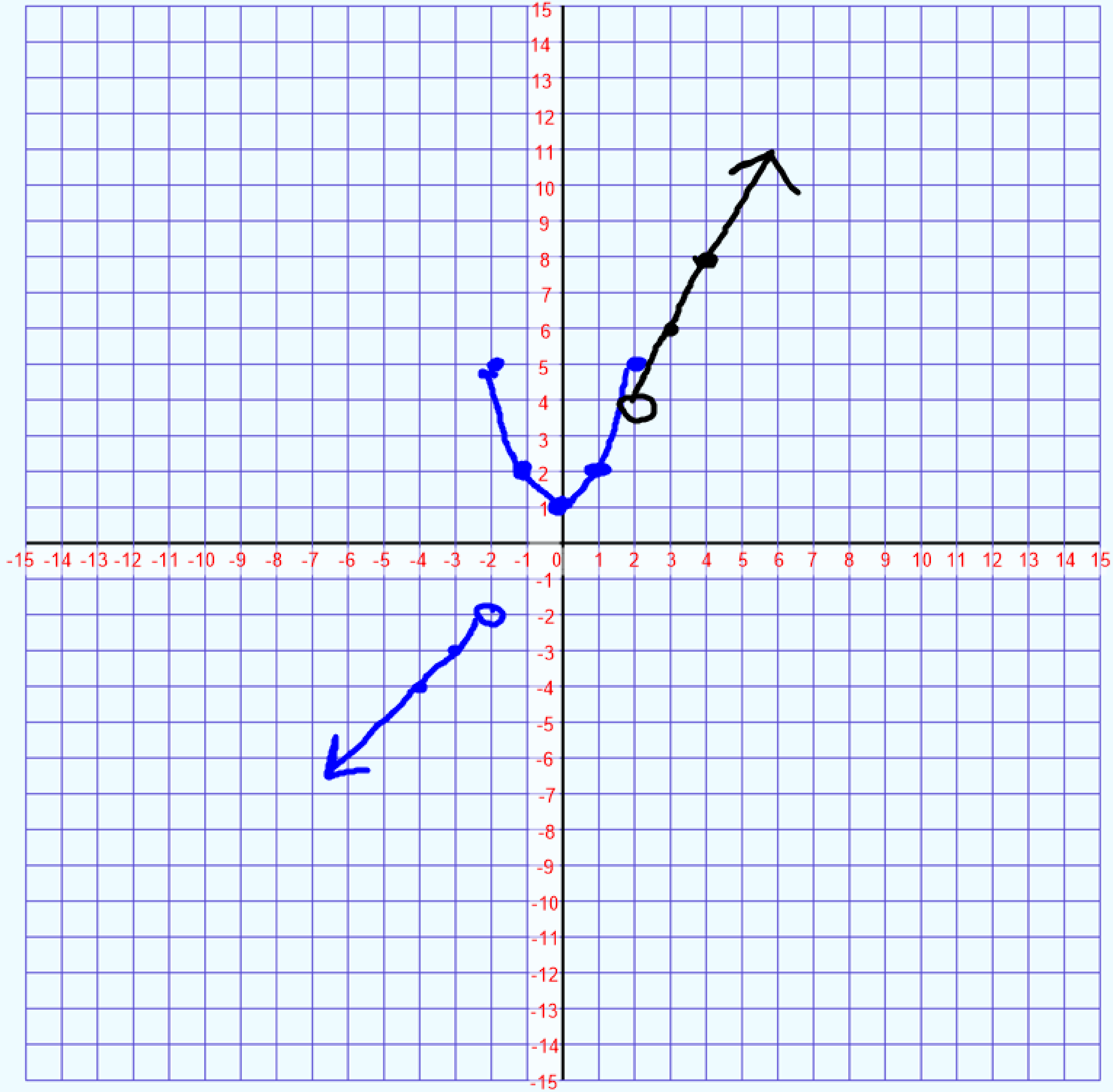
Ex.1 Graph the following:

$$f(x)=\begin{cases} x, \text{ if } x \in (-\infty,-2) \\ x^2+1, \text{ if } x \in [-2,2] \\ 2x, \text{ if } x \in (2,\infty) \end{cases}$$

x	y=x
-2	-2
-3	-3
-4	-4

x	$y=x^2+1$
-2	5
-1	2
0	1
1	2
2	5

x	y=2x
2	4
3	6
4	8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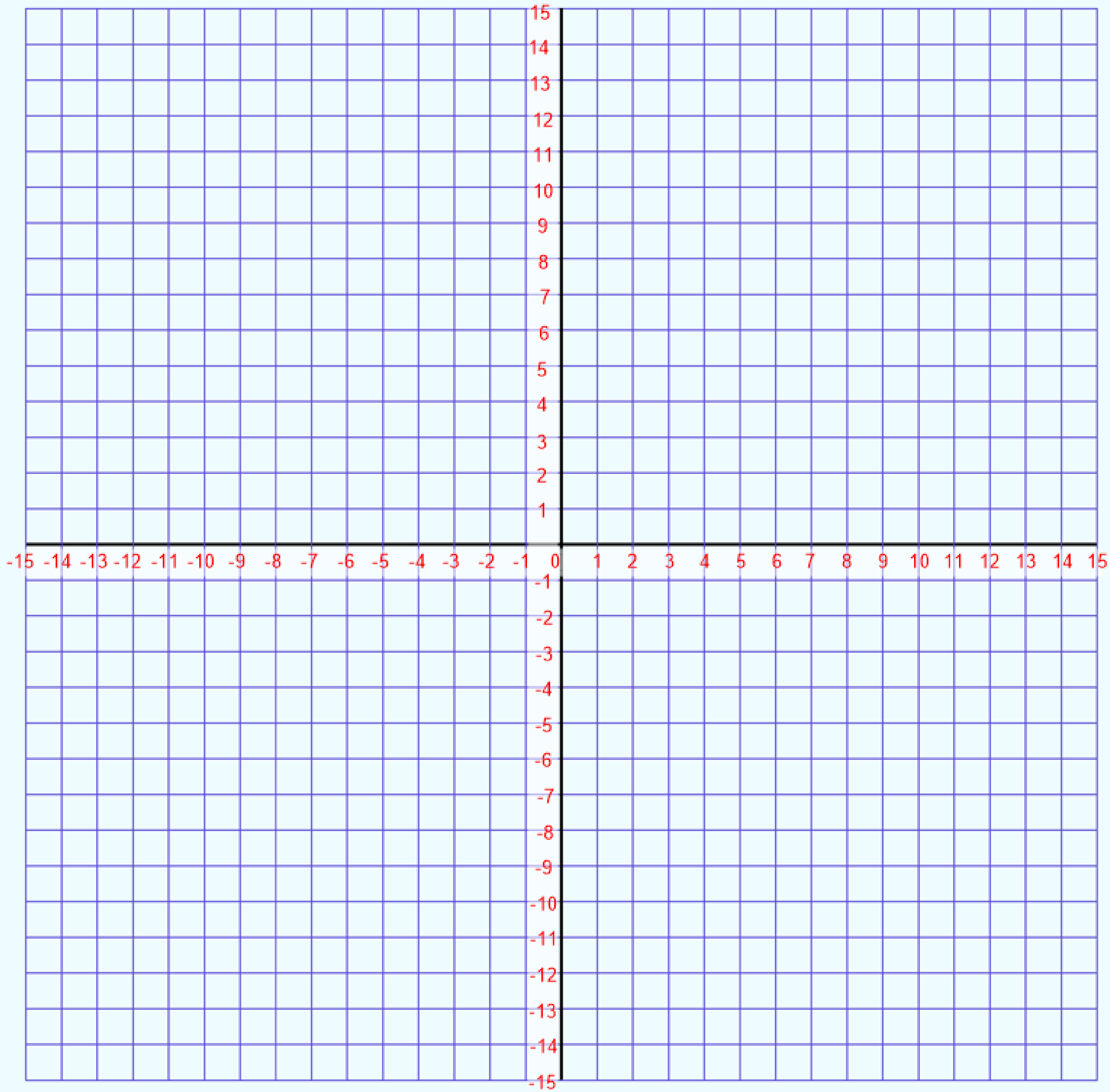
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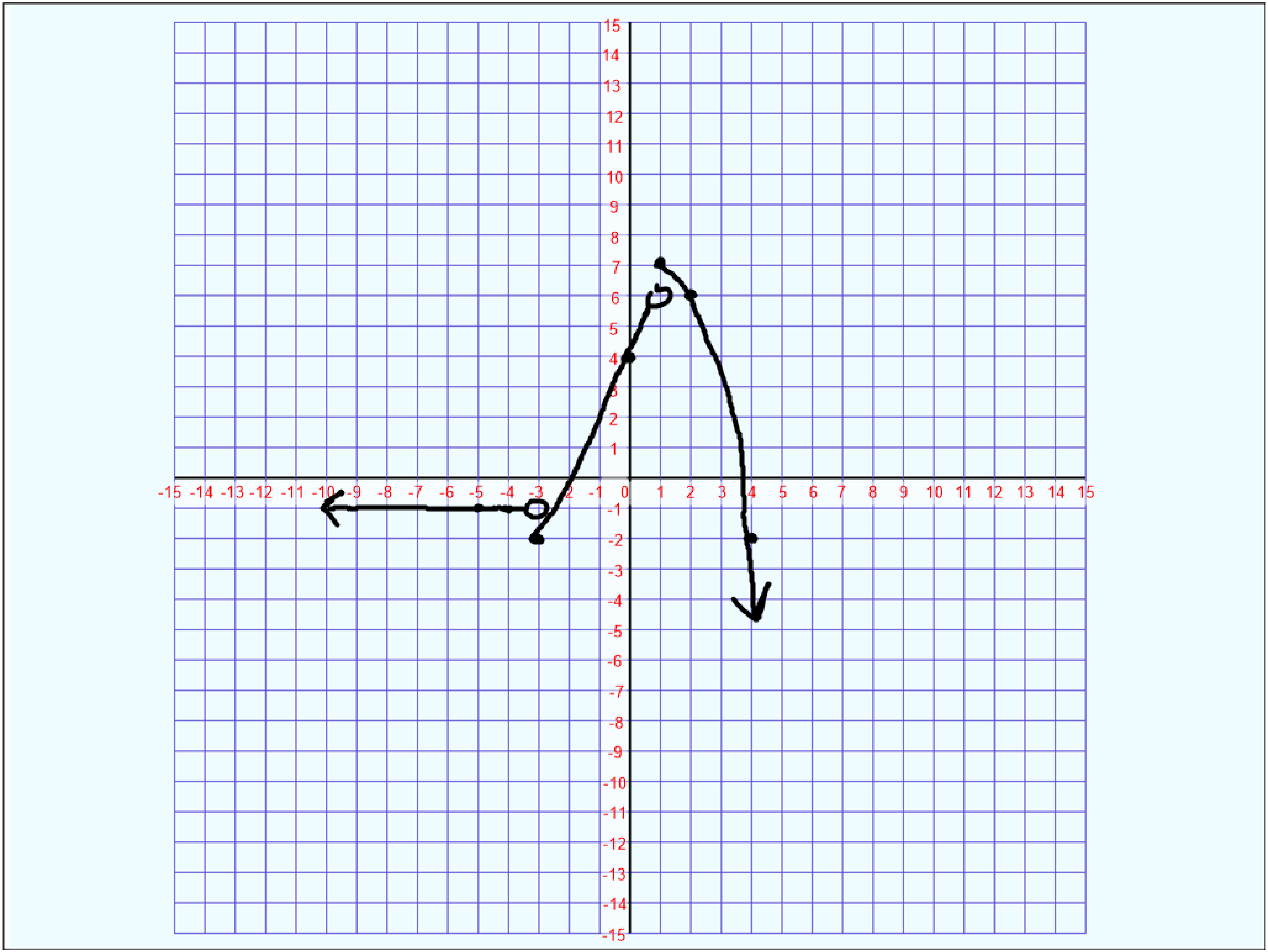
Ex.2 Graph the following:

$$g(x)=\begin{cases} -1, \text{ if } x\in(-\infty,-3) \\ 2x+4, \text{ if } x\in[-3,1) \\ -(x-1)^2+7, \text{ if } x\in[1,\infty) \end{cases}$$

x	y = -1
-3	-1
-4	-1
-5	-1

x	y = 2x + 4
-3	-2
0	4
1	6

x	$y=-(x-1)^2+7$
1	7
2	6
4	-2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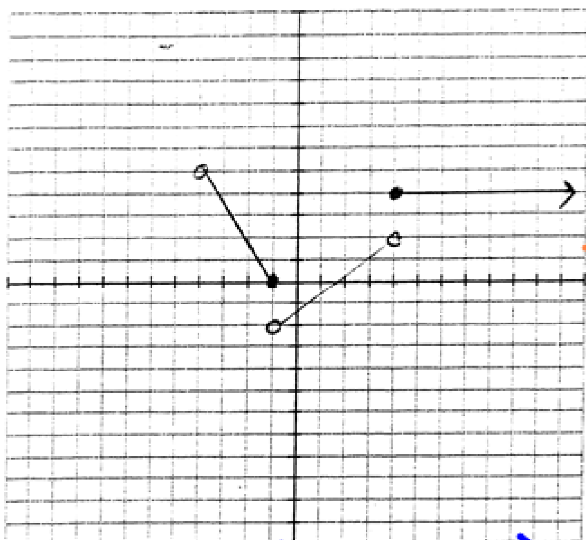
Developing Functions From Graphs of Piecewise Functions

Ex.3 Find the equation of the piecewise function whose graph is given below.



$$f(x)=\begin{cases} -\frac{5}{3}x-\frac{5}{3}, & \text{if } x\in(-4,-1] \\ \frac{4}{5}x-\frac{6}{5}, & \text{if } x\in(-1,4) \\ 4, & \text{if } x\in[4,\infty) \end{cases}$$

$$y-y_1=m(x-x_1)$$

$$(y-0)=-\frac{5}{3}(x+1)$$

$$y=-\frac{5}{3}x-\frac{5}{3}$$

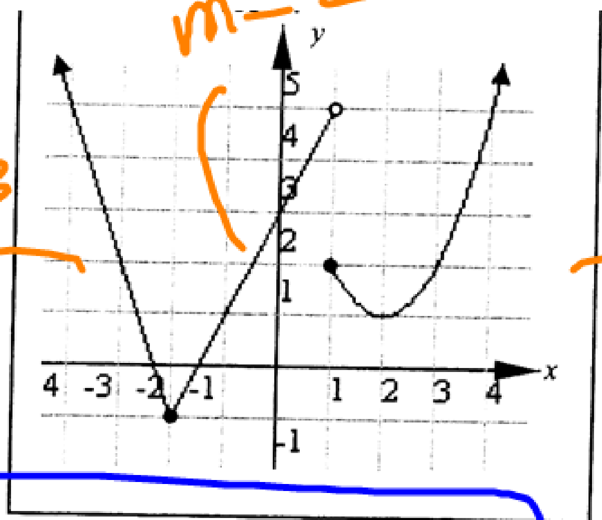
$$y-y_1=m(x-x_1)$$

$$y-2=\frac{4}{5}(x-4)$$

$$y-2=\frac{4}{5}x-\frac{16}{5}$$

$$y=\frac{4}{5}x-\frac{6}{5}$$

Ex.4 Find the equation of the piecewise function whose graph is given below.



$m = 2$

$m = -3$

$$f(x) = \begin{cases} -3x - 7, & \text{if } x \in (-\infty, -2] \\ 2x + 3, & \text{if } x \in (-2, 1) \\ (x-2)^2 + 1, & \text{if } x \in [1, \infty) \end{cases}$$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = -3(x + 2)$$

$$y = -3x - 7$$

$$y + 1 = 2(x + 2)$$

$$y = 2x + 3$$

$$y = a(x-h)^2 + k$$

$$y = a(x-2)^2 + 1$$

$$2 = a(1-2)^2 + 1$$

$$1 = a(-1)^2$$

$$1 = a$$

Assignment

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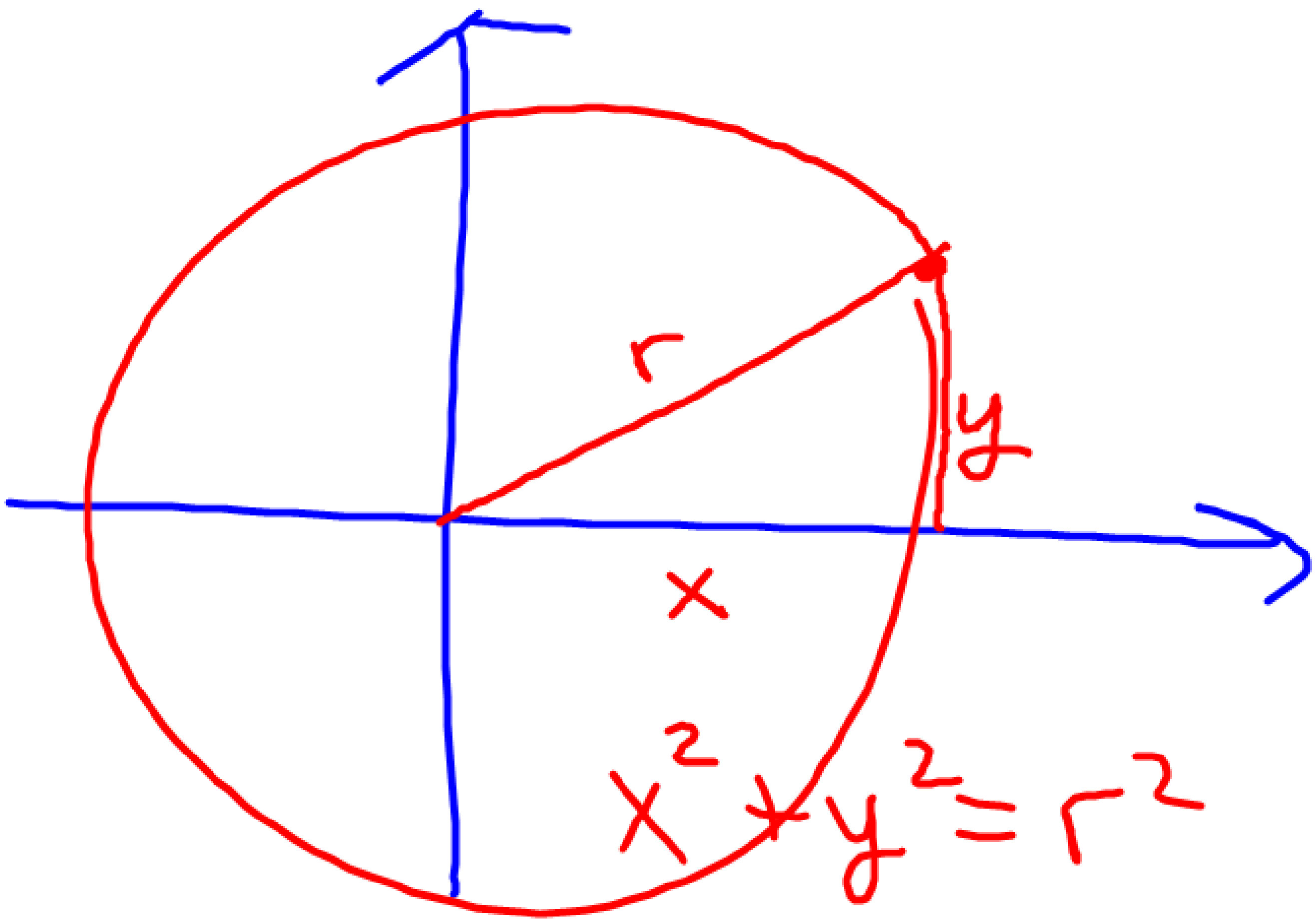
#'s 4,5,6

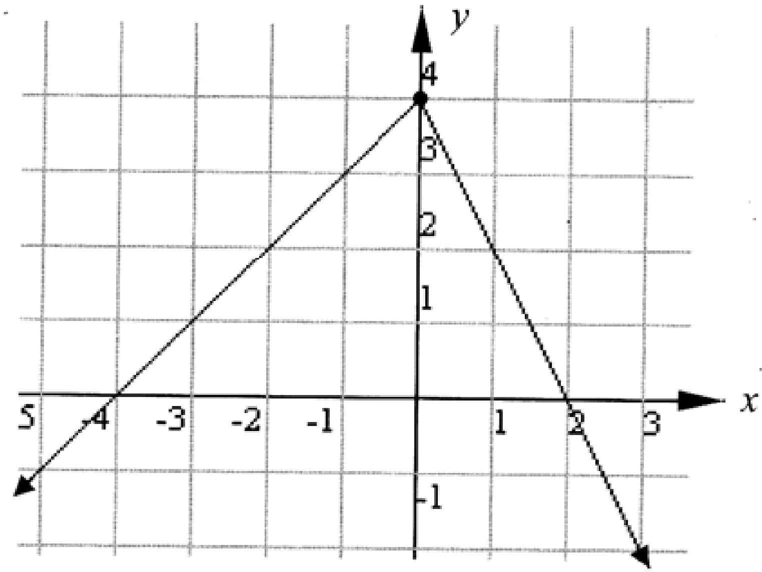

r
y
x
x² + y² = r²

$$y^2 = r^2 - x^2$$

$$y = \pm\sqrt{4\cancel{r}^2 - x^2}$$

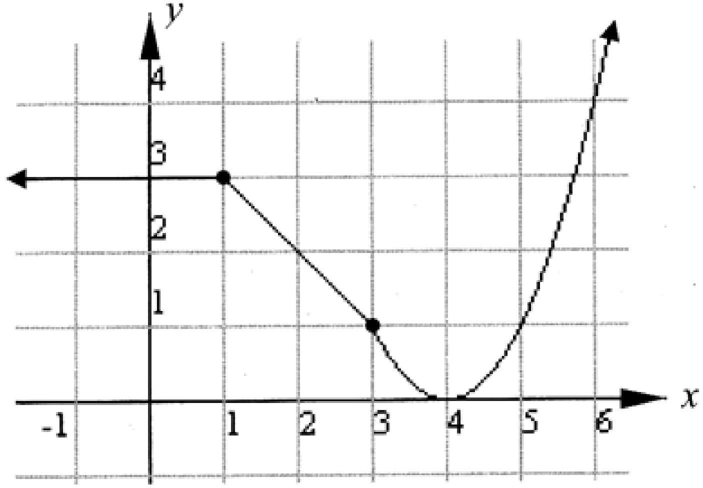
$$\pm\sqrt{16 - x^2}$$

(b)

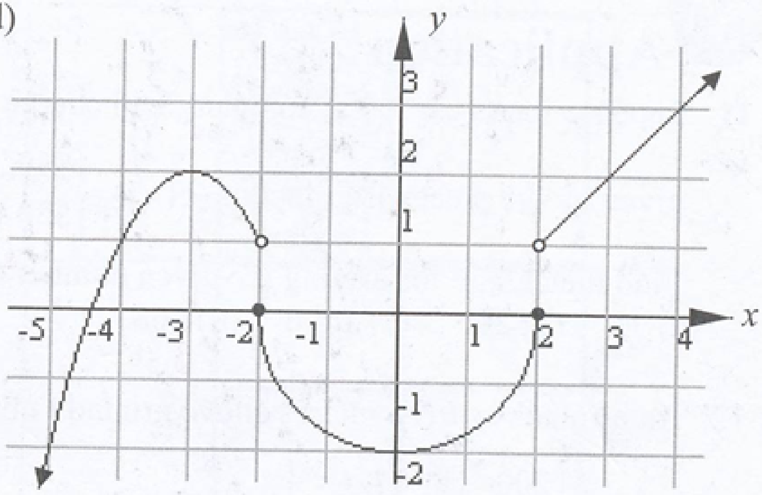


5. (Continued)

(c)



(d)



Combining Function Notation and Piecewise Functions

Let *f(x)* be defined as the following:

$$f(x)=\begin{cases} x-3, \text{ if } x\in(-\infty,2) \\ x^2+7, \text{ if } x\in[2,8) \\ \sqrt{x}, \text{ if } x\in[8,\infty) \end{cases}$$

Ex.5 Evaluate the following:

a) $f(6)$

$=(6)^2+7$

$=43$

b) $f(-4)$

$=-4-3=-7$

c) $f(11)$

$= \sqrt{11}$

d) $f(2)$

$= (2)^2 + 7$

$= 11$

Let *h(x)* be defined as the following:

$$h(x)=\begin{cases} x^2+1, \text{if } x\in(-\infty,0] \\ 2x-3, \text{if } x\in(0,8) \\ 3x+2, \text{if } x\in[8,\infty) \end{cases}$$

$\{-5, 8\}$

Ex.6 Find values of *x* such that:

a) $h(x)=26$

$x^2+1=26$

$x^2=25$

$x=\pm 5$

$x=-5$

$2x-3=26$

$2x=29$

~~$x=\frac{29}{2}$~~

$3x+2=26$

$3x=24$

$x=8$

b) $h(x) = 0$

$x^2 + 1 = 0$

~~$x^2 = -1$~~

$2x - 3 = 0$

$2x = 3$

$x = 3/2$

$3x + 2 = 0$

~~$x = -2/3$~~

Assignment

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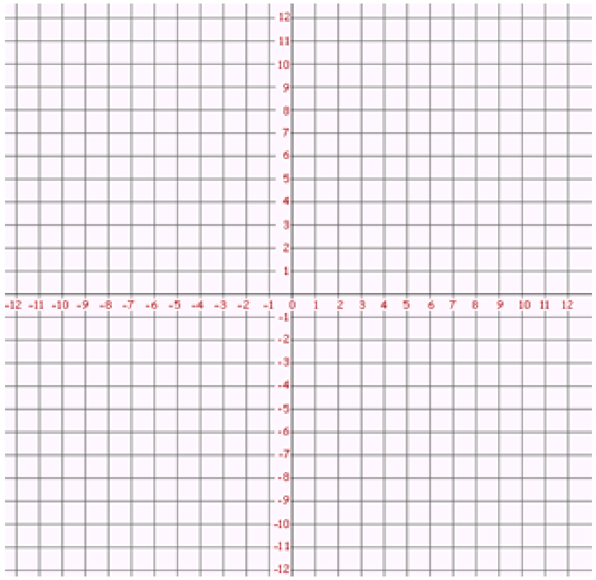
#'1,2,3,11,12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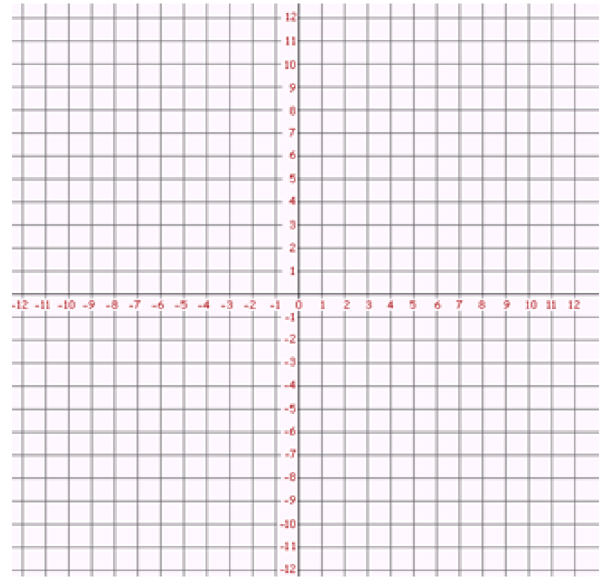
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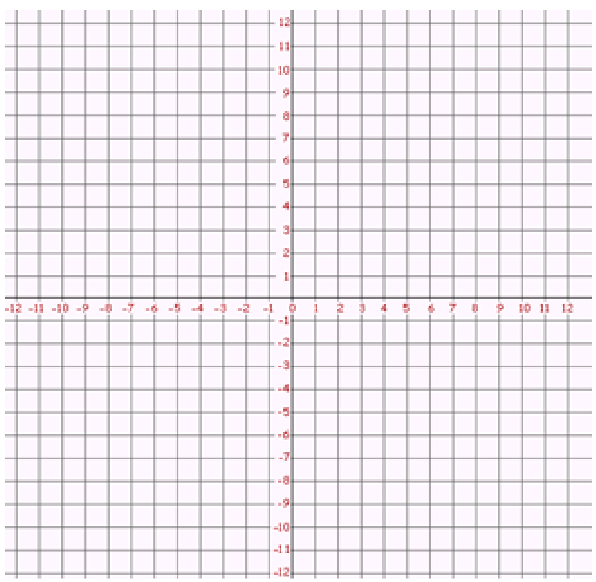
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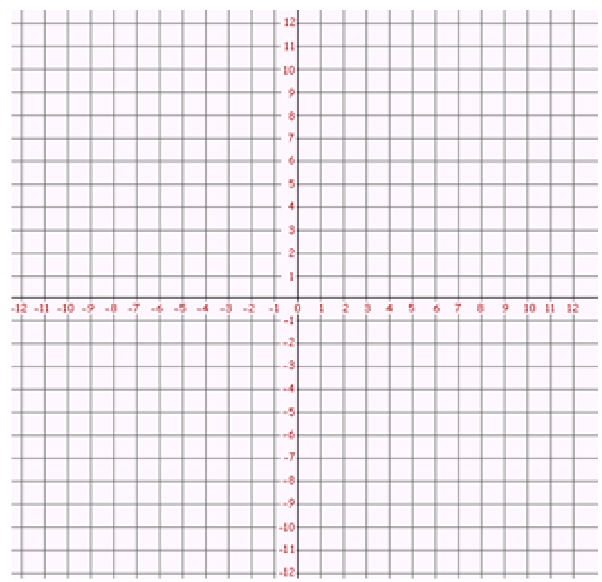
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