1.5 Absolute Value

Informal Definition

Absolute value is the distance a number is from zero on the number line.

$$|14| = |-14| = 14$$

The absolute value of a number can never be negative!

Ex.1 Find the absolute value of the following:

$a)\ \left|-23\right|$

23

b) $\left|-4\pi\right|$

4π

Ex.2 Write without using absolute value signs:

a) $|7 - 11|$

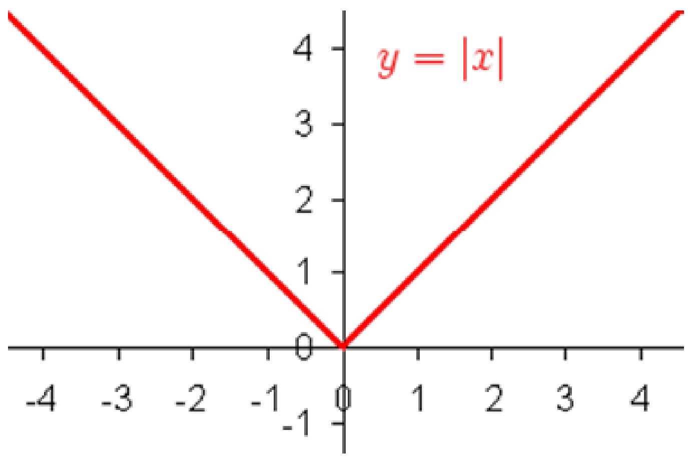
$11 - 7$

b) $|e - \pi|$

$\pi - e$

Formal Definition

$$|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$$

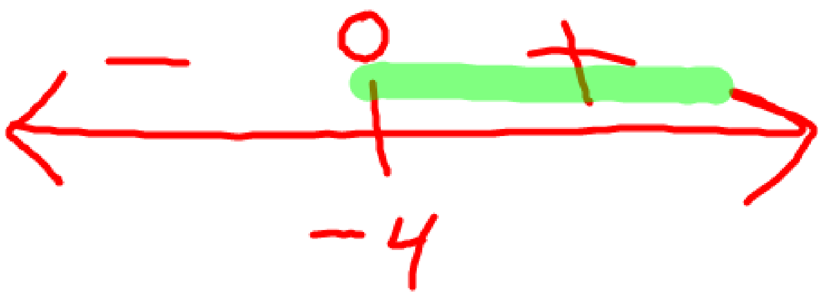


Removing Absolute Signs

Ex.3 Write an expression equivalent to the following without using absolute value signs.

a) $|x+4|$ $= \begin{cases} x+4 & \text{if } x \in [-4, \infty) \\ -(x+4) & \text{if } x \in (-\infty, -4) \end{cases}$

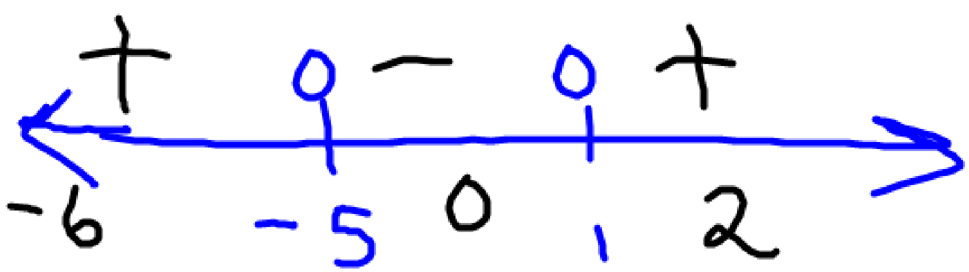
$x+4$



b) $\left|x^2+4x-5\right|$

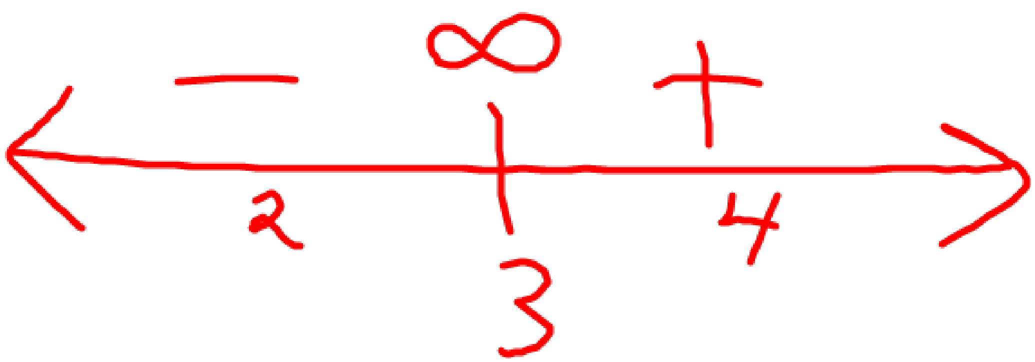
$$= \begin{cases} x^2+4x-5 & \text{if } x \in (-\infty,-5] \cup [1,\infty) \\ -(x^2+4x-5) & \text{if } x \in (-5,1) \end{cases}$$

$(x+5)(x-1)$



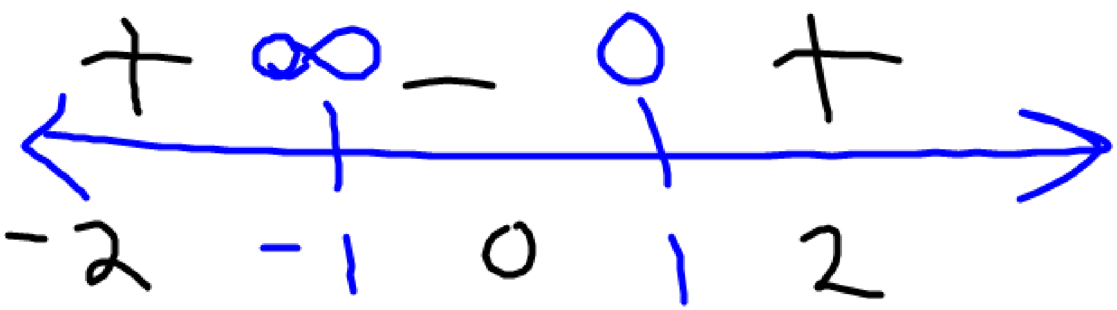
c) $\left|\frac{2}{x-3}\right| = \begin{cases} \frac{2}{x-3} & \text{if } x \in (3,\infty) \\ -\left(\frac{2}{x-3}\right) & \text{if } x \in (-\infty,3) \end{cases}$

$$\frac{2}{x-3}$$



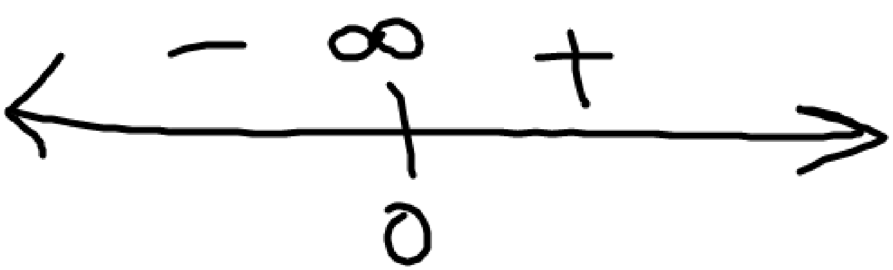
d) $$\left|\frac{x-1}{x+1}\right| = \begin{cases} \dfrac{x-1}{x+1} & \text{if } x \in (-\infty,-1) \cup [1,\infty) \\ -\left(\dfrac{x-1}{x+1}\right) & \text{if } x \in (-1,1) \end{cases}$$

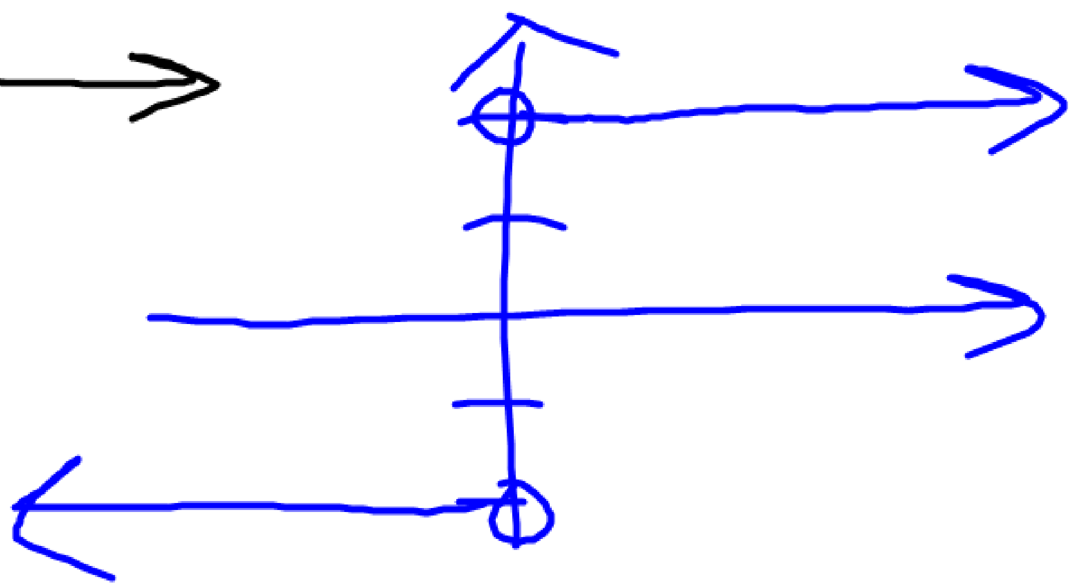
$$\frac{x-1}{x+1}$$



e) $\dfrac{2x}{|x|} = \begin{cases} \cancel{\dfrac{2x}{x}}\ 2 & \text{if } x \in (0,\infty) \\ \cancel{\dfrac{2x}{-x}}\ -2 & \text{if } x \in (-\infty,0) \end{cases}$

x





Before absolute value therapy, Jerome was feeling rather negative.

After absolute value therapy, Jerome was feeling much more positive.

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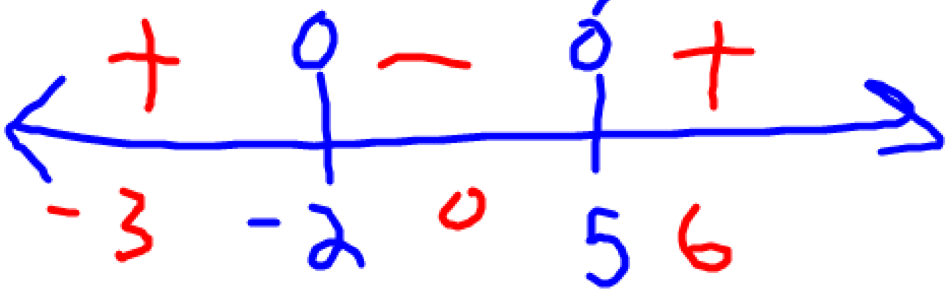
#'s 1-15 odd, 19, 21, 24, 29, 30, 31

24

$$|x^2-3x-10| = \begin{cases} x^2-3x-10 & \text{if } x \in (-\infty,-2] \cup [5,\infty) \\ -(x^2-3x-10) & \text{if } x \in (-2,5) \end{cases}$$

$$(x-5)(x+2)$$



Yesterday we worked on writing expressions by eliminating the absolute value signs.

Today we are going to learn how to solve absolute equations and absolute inequalities.

Ex.1 Solve the following equations and verify:

a) $|9\text{-}5x| = 6$

Two Cases

$9-5x=6$ OR $-(9-5x)=6$

$-5x=-3$

$x=3/5$

$-9+5x=6$

$5x=15$

$x=3$

Verify

$|9-5(\tfrac{3}{5})| \stackrel{?}{=} 6$

$|6|=6$ ✓

$|9-5(3)| \stackrel{?}{=} 6$

$|-6|=6$ ✓

b) $|15-2t| = t+6$

$15-2t = t+6$ OR $-(15-2t) = t+6$

$9 = 3t$

$3 = t$

$-15+2t = t+6$

$t = 21$

Verify

$|15-2(3)| = 3+6$

$|9| = 9$ ✓

Verify

$|15-2(21)| = 21+6$

✓ $|-27| = 27$

Ex.2 Solve the following inequalities. Write your solutions in interval notations.

a) $|-5x+4| \leq 11$

$\left[-\frac{7}{5}, 3\right]$

$-11 \leq -5x + 4 \leq 11$

$-15 \leq -5x \leq 7$

$3 \geq x \geq -\frac{7}{5}$

$-\frac{7}{5} \leq x \leq 3$

$$|6-5x| > 21$$

$6-5x > 21$ OR $6-5x < -21$

$-5x > 15$ $\quad$ $-5x < -27$

$x < -3$ $\quad$ $x > \frac{27}{5}$

$$(-\infty, -3) \cup (27/5, \infty)$$

b) $\left|\frac{5}{x-4}\right| > 1$

$x \neq 4$

$\hookrightarrow$ # line

$\cancel{(x-4)}\ \frac{5}{\cancel{x-4}} > 1(x-4)$ OR $-\left(\frac{5}{x-4}\right) > 1$

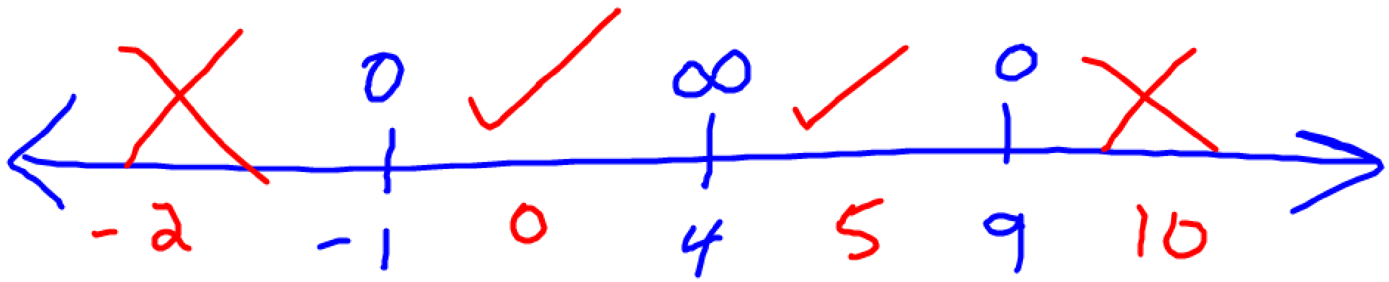
$5 > x-4$

$9 > x$

$\cancel{(x-4)}\ \frac{-5}{\cancel{x-4}} > 1(x-4)$

$-5 > x-4$

$-1 > x$

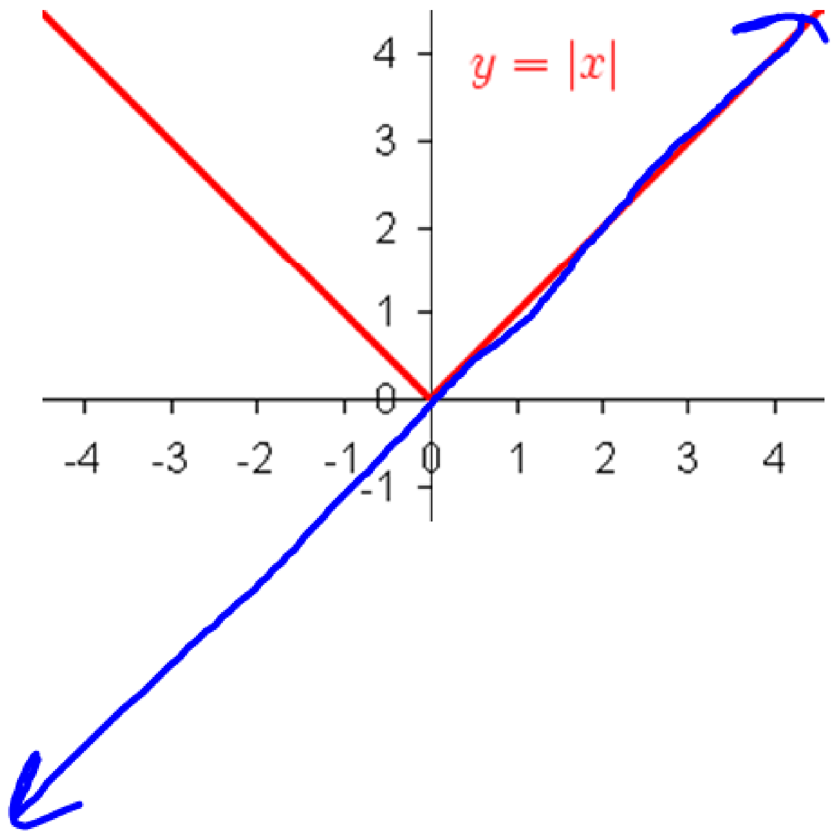


Test back original question

$$\left|\frac{5}{x-4}\right| > 1$$

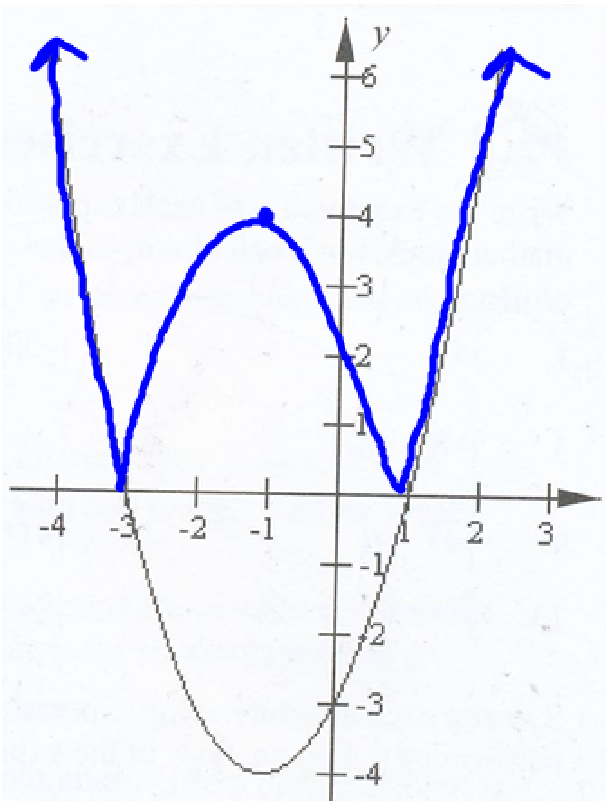
$$(-1,4)\cup(4,9)$$

Graphs of Absolute Value Functions



$y = x$

Say we have the graph of $y = x^2 + 2x - 3$.



How could we use this graph to graph the following?

$$y = \left| x^2 + 2x - 3 \right|$$

Assignment

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43, 45,49,51,53 ,56,57,59,

60, 66